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Phenomenology of neutrino oscillations

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Abstract. The phenomenology of solar, atmospheric, supernova and laboratory neutrino oscillations is described. Analytical formulae for matter effects are reviewed. The results from oscillations are confronted with neutrinoless double beta decay.

Keywords. Neutrino oscillations; neutrino masses; matter effects; double beta decay.

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1. Introduction

In this talk, I shall try to give a bird's eye view of the current status of neutrino oscillations. Also I shall highlight local work wherever possible since phenomenological contributions have been made from the Institute of Mathematical Sciences on various aspects of neutrino oscillations.

Solar neutrino physics is 30-years old. Starting from the pioneering Cl experiments of Davis and collaborators [1], all the experiments have observed the depletion of the solar neutrinos as compared to the theoretically calculated flux from the standard solar model [2]. Atmospheric neutrino physics is much younger. The observation of the reduction in the ν_μ/ν_e ratio is only 10-years old. Nevertheless, atmospheric neutrino physics has won over solar neutrino physics, since the evidence for neutrino oscillation from the former is derived from ratios and is hence independent of the absolute flux. On the whole, very good evidence for oscillation exists from both fronts. There are still loose ends; for instance, the recoil energy spectrum of the solar neutrinos does not agree with the theoretically calculated one, even with oscillations.

Alternative ideas [3] must still be explored and ruled out, before neutrino oscillations are finally established as a part of physics. Nevertheless it is fair to state that oscillations are the natural explanation for the observed neutrino anomalies. For, oscillation is the most conservative explanation for the anomalies. It does not violate any known principle of physics and it does not invoke any exotic new physics. It only uses ordinary principles of quantum mechanics, especially the principle of superposition. In quantum mechanics, the neutrino states will mix and oscillate, if they have masses.

Hence, I think it is time to look at the next job. Attention must now shift from the oscillation itself, to determining the 6 fundamental parameters (2 mass differences, 3 mixing angles and one CP violating phase) describing the oscillation phenomena among the 3 flavours ν_e, ν_μ and ν_τ. Even after 30 years, none of these parameters is yet determined with any certainty. But, progress has recently been made in limiting their ranges or determining their orders of magnitude. One parameter, namely the mixing angle ϕ, has been bracketed rather closely.

It is important to stress the desirability of explaining neutrino anomalies and fitting the data using a three-neutrino framework. For, there exist three neutrinos. The high-quality data coming out from the great neutrino experiments of the present-day deserve to be treated by a realistic $3-\nu$ analysis rather than the $2-\nu$ toy model often used by the experimenters to give their results. This is the point of view with which we started our ν-phenomenology work at IMSc [4]. When we started, there were very few groups doing a $3-\nu$ analysis.

Having said the above, I have also to point out that Nature has helped the (lazy) $2-\nu$ people. First, it chose the mass hierarchy:

$$m_2^2 - m_1^2 << m_3^2 - m_2^2 \tag{1}$$

so that solar ν could be explained by $\delta m^2_{21} (\equiv m_2^2 - m_1^2)$ and atmospheric ν could be explained by $\delta m^2_{32} (\equiv m_3^2 - m_2^2)$. Next, the CHOOZ reactor experiment showed that one of the mixing angles (ϕ) is so small that the $3-\nu$ problem decouples into two effective $2-\nu$ problems.

Although neutrino oscillation is a direct consequence of quantum mechanics, it leads to a result of profound consequence for physics and astrophysics – the result that neutrinos have mass. That is the importance of the whole subject of neutrino oscillations. Neutrino mass is the only concrete evidence we have for physics beyond the Standard Model (SM) of high energy physics. Every other physics beyond the SM, that is being searched for, has remained speculative so far. Every other experimental signal for physics beyond the SM that appears now and then on the horizon has been disappearing in about 6 months to one year. Neutrino mass is the only evidence for physics beyond SM that has remained robust for the past 30 years.

2. Results

The neutrino flavour states $|\nu_\alpha\rangle (\alpha = e, \mu, \tau)$ are linear superpositions of the neutrino mass eigenstates $|\nu_i\rangle (i = 1, 2, 3)$ with masses m_i : $|\nu_\alpha\rangle = \Sigma_i U_{\alpha i} |\nu_i\rangle$ where U is the 3×3 unitary matrix :

$$U = \begin{pmatrix} c_\phi c_\omega & c_\phi s_\omega & s_\phi \\ -c_\psi s_\omega - s_\psi s_\phi c_\omega \mathrm{e}^{i\delta} & c_\psi c_\omega - s_\psi s_\phi s_\omega \mathrm{e}^{i\delta} & s_\psi c_\phi \mathrm{e}^{i\delta} \\ s_\psi s_\omega - c_\psi s_\phi c_\omega \mathrm{e}^{i\delta} & -s_\psi c_\omega - c_\psi s_\phi s_\omega \mathrm{e}^{i\delta} & c_\psi c_\phi \mathrm{e}^{i\delta} \end{pmatrix}, \tag{2}$$

where c and s stand for sine and cosine of the angle appearing as subscript.

From the oscillation phenomena, one has to determine the 6 parameters δm^2_{21}, $\delta m^2_{32}, \omega, \psi, \phi$ and δ. Most of the presently studied oscillation phenomena are insensitive to the CP-violation parameter δ. So let us ignore δ but we shall justify it at the end of the section.

Under the hierarchy assumption of eq. (1), one can show that the solar neutrino problem depends only on $\delta m^2_{21}, \omega$ and ϕ while the atmospheric neutrino problem depends only on $\delta m^2_{32}, \psi$ and ϕ. It is this simplification that allows us to analyse the two problems within

a $3-\nu$ framework under reasonable control and one gets a fairly broad and stable set of allowed regions in the 5-parameter space [4,5]. If we temporarily put ϕ as zero, then the two problems decouple and the results are the following:
There are three solutions of the solar neutrino problem:

1. MSW-small angle : $\delta m^2_{21} \approx 10^{-5}\mathrm{eV}^2, \sin^2 2\omega \approx 10^{-3}$
2. MSW-large angle : $\delta m^2_{21} \approx 10^{-5}\mathrm{eV}^2, \sin^2 2\omega \approx 1$
3. Vacuum oscillations : $\delta m^2_{21} \approx 10^{-10}\mathrm{eV}^2, \sin^2 2\omega \approx 1$

Atmospheric neutrinos problem has the solution : $\delta m^2_{32} \approx 10^{-3}\mathrm{eV}^2,\ \sin^2 2\psi \approx 1.$

The non-observation of any depletion of $\bar{\nu}_e$ in the reactor experiment at CHOOZ [6] turns out to be a crucial result. Interpreted in the $3-\nu$ framework [7], with the hierarchy assumption of eq. (1) it implies $\phi < 9^\circ$, which is a very powerful constraint and this result justifies the neglect of ϕ in the analysis of solar and atmospheric ν. This constraint is independent of CP violation.

One can show that CP violation always occurs in the combination $\sin\phi \mathrm{e}^{\pm i\delta}$. So, in view of the CHOOZ result, CP violation is suppressed in all neutrino oscillation phenomena.

In the next few sections, we shall briefly describe how these results were obtained.

3. Oscillations in vacuum and matter

The probability of a neutrino of flavour α to be observed with flavour β after a distance of travel L in vacuum is given by

$$P(\nu_\alpha \to \nu_\beta) \;=\; \delta_{\alpha\beta} - 4\sum_i \sum_{j>i} U_{\alpha i}U_{\beta i}U_{\alpha j}U_{\beta j}\sin^2\left[\frac{1.27\delta m^2_{ij}L}{E}\right], \qquad (3)$$

where U is defined in eq. (2), δm^2_{ij} is in eV^2, L is in metres, neutrino energy E is in MeV and CP violation is ignored.

In matter (especially of varying density), the above formulae are drastically changed because of the famous Mikheyev–Smirnov–Wolfenstein (MSW) effect. We consider the propagation of the neutrinos through solar matter. Let a neutrino of flavour α be produced at time $t = t_0$ in the solar core. Its state vector is $|\Psi_\alpha(t_0)\rangle = |\nu_\alpha\rangle = \sum_i U^c_{\alpha i}|\nu^c_i\rangle$ where $|\nu^c_i\rangle$ are the mass eigenstates with mass eigenvalues m^c_i and mixing matrix elements $U^c_{\alpha i}$ in the core of the sun. The neutrino propagates in the sun adiabatically up to t_R (the resonance point), makes nonadiabatic Landau–Zener transition $i \to j$ at t_R with probability amplitude M^{LZ}_{ji}, propagates adiabatically up to t_1 (the edge of the sun) and propagates as a free particle up to t_2 when it reaches the earth.

The state vector at t_2 is

$$|\Psi_\alpha(t_2)\rangle \;=\; \sum_{i,j} |\nu_j\rangle M^{LZ}_{ji}U^c_{\alpha i}\exp\left(-i\int_{t_R}^{t_2}\epsilon_j(t)\mathrm{d}t - i\int_{t_0}^{t_R}\epsilon_i(t)\mathrm{d}t\right), \qquad (4)$$

where $\epsilon_i(t)\left\{= E + m^2_i(t)/2E\right\}$ are the matter-dependent energy eigenvalues in the sun up to t_1 and vacuum eigenvalues for $t_1 < t < t_2$. The probability for detecting a neutrino of flavour β on the earth is

$$|\langle \nu_\beta | \Psi_\alpha(t_2)\rangle|^2 = \sum_{iji'j'} U^*_{\beta j} U_{\beta j'} M^{LZ}_{ji} M^{LZ^*}_{j'i'} U^c_{\alpha i} U^{c^*}_{\alpha i'}$$
$$\exp\left\{ -i \int_{t_R}^{t_2} (\epsilon_j - \epsilon_{j'}) \mathrm{d}t - i \int_{t_0}^{t_R} (\epsilon_i - \epsilon_{i'}) \mathrm{d}t \right\}. \qquad (5)$$

Next comes the crucial step [8,9] of averaging over t_0 and t_2 and the assumption that the oscillations are rapid enough so that the averaged exponential in this equation can be replaced by $\delta_{ii'}\delta_{jj'}$. Calling this averaged probability as $P^D_{\alpha\beta}$ (the probability for a ν_α produced in the sun to be detected as a ν_β on the earth at daytime), we get

$$P^D_{\alpha\beta} = \sum_{ij} |U_{\beta j}|^2 |M^{LZ}_{ji}|^2 |U^c_{\alpha i}|^2. \qquad (6)$$

4. Solar, atmospheric and reactor neutrinos

Extensive literature exists on the solar [4,10] and atmospheric neutrinos [5,11]. So, we shall be brief.

The experimental results on the total solar neutrino flux from the three types of detectors (Cl, Gl and H_2O) are the following [12]: $R_{\mathrm{Cl}} = 0.33 \pm 0.028$; $R_{\mathrm{Ga}} = 0.56 \pm 0.05$; $R_{\mathrm{SK}} = 0.475 \pm 0.015$ where we have given the ratios of the experimental rates to the theoretical rates without neutrino oscillations calculated in the standard solar model (SSM). One can see that the statistical uncertainty is the least for the super-Kamioka (SK) water Cerenkov detector, which is thus presaging the era of precision neutrino physics. Since the three types of detectors are sensitive to different regions of the solar neutrino spectrum, the above three numbers already contain some spectral information. Ascribing the above observed depletion factors to oscillation and using the formulae in eqs (3) or (6), the best fits for the neutrino parameters can be obtained, with the results already discussed in §2. However, the recoil electron energy spectrum as measured by SK does not fit [13] with any oscillation scenerio, at the higher energy end where the observed number is too large (but with large errors).

Since the neutral current (NC) weak interaction is flavour-blind, the flux of solar ν detected through NC node would be the total $(\nu_e + \nu_\mu + \nu_\tau)$ flux and hence is independent of oscillation and would be a test of the standard solar model. Also, the ratio CC/NC, would be a test of neutrino oscillation independent of the uncertainties of the solar models. Therefore great expectations have been raised by the SNO detector [14] which will soon give the first results. Another exciting avenue will open when Borexino [15] starts its operation, since it is the first detector zeroing in on the monochromatic ν line from the Be^7 decay in the sun.

Cosmic rays impingent on the atmosphere produce hadrons (especially pions) that decay, resulting in a wide spectrum of $\nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e$ ranging in energy up to about 100 GeV, with the calculated ratio $R_{\mathrm{cal}} = (N_{\nu_\mu} + N_{\bar{\nu}_\mu})/(N_{\nu_e} + N_{\bar{\nu}_e})$ approximately equal to 2, whereas the experimentally measured ratio is nearer to unity. More precisely, r defined as $R_{\mathrm{obs}}/R_{\mathrm{cal}}$ is about 0.6. This is the atmospheric neutrino problem whose solution is the oscillation of $\nu_\mu(\bar{\nu}_\mu)$ into $\nu_\tau(\bar{\nu}_\tau)$.

Abundant data from super-Kamioka detector [16] is now available on the zenith angle dependence as well as on the energy-distribution of the ν_μ and ν_e events. All these are consistent with ν_μ oscillating into ν_τ over the distance scale of about 10,000 km; so it is for the upward-going ν_μ travelling through the earth that the effect is most dominant. The effect of earth matter in the atmospheric neutrino problem is not significant [5] at the present level of accuracy and hence one can use eq. (3). The resulting neutrino parameters were given in §2.

Although the analysis is performed in terms of ratios such as ν_μ/ν_e or up/down and hence is relatively insensitive to the rather large uncertainties that exist in the primary cosmic ray flux and spectrum, further improvement in our knowledge of the latter will be essential for the complete understanding of all atmospheric neutrino data.

Also, it is worth pointing out that, if the above explanation of atmospheric neutrino anomaly is correct, then the upward-going atmospheric neutrino beam has the approximate composition of $\nu_e : \nu_\mu : \nu_\tau = 1 : 1 : 1$. The direct detection of this ν_τ through production of τ will be a crucial test.

Nuclear fission reactors are powerful sources of $\bar{\nu}_e$. The detector was placed about 1 km away from the CHOOZ power reactor [6] and the observed $\bar{\nu}_e$ flux was compared with the calculated flux : $\phi_{\rm obs}/\phi_{\rm cal} = 0.98 \pm 0.04 \pm 0.04$. If $\delta m^2_{32} >> \delta m^2_{21}$ and $\delta m^2_{21} \lesssim 10^{-5}$ eV^2, the $3-\nu$ formula of eq. (3) reduces [7] to the $2-\nu$ formula

$$P(\bar{\nu}_e \to \bar{\nu}_e) \; = \; 1 - \sin^2 2\phi \sin^2 \frac{1.27\delta m^2_{32} L}{E}. \tag{7}$$

Comparison with the CHOOZ result yields $\phi < 12^\circ$ for $\delta m^2_{32} \gtrsim 10^{-3}$ eV^2. CHOOZ have now improved their limit to $\phi < 9^\circ$.

For the first time, a negative result on neutrino oscillations from laboratory experiment has given a constraint of significance in the context of solar and atmospheric neutrinos. That is the importance of the CHOOZ experiment. Independent confirmation of this result has come from the Palo Verde experiment, although with less statistics. Hopefully the dependence on the calcualted flux $\phi_{\rm cal}$ will be removed in the future by placing another detector near the reactor.

5. Neutrinos through the earth and the moon

Neutrino oscillation is a complex phenomenon depending on many unknown parameters (six parameters for three flavours ν_e, ν_μ and ν_τ) and a considerable amount of experimental work and ingenuity will be required before the neutrino problem is solved.

Whenever physicists are confronted with a beam of unknown properties, they pass it through different amounts of matter. Nature has fortunately provided us with such opportunities (see figure 1): (a) Neutrinos produced in the solar core pass through solar matter; (b) solar neutrinos detected at night pass through earth; (c) solar neutrinos detected during a solar eclipse pass through the moon; (d) solar neutrinos detected at the far side of earth during a solar eclipse pass through the moon and earth; (e) upward going atmospheric neutrinos pass through the earth. To these we may add two more experiments of the future: (f) Long-base-line experiments of accelerator and reactor produced neutrinos and (g) detection of geophysical neutrinos [17].

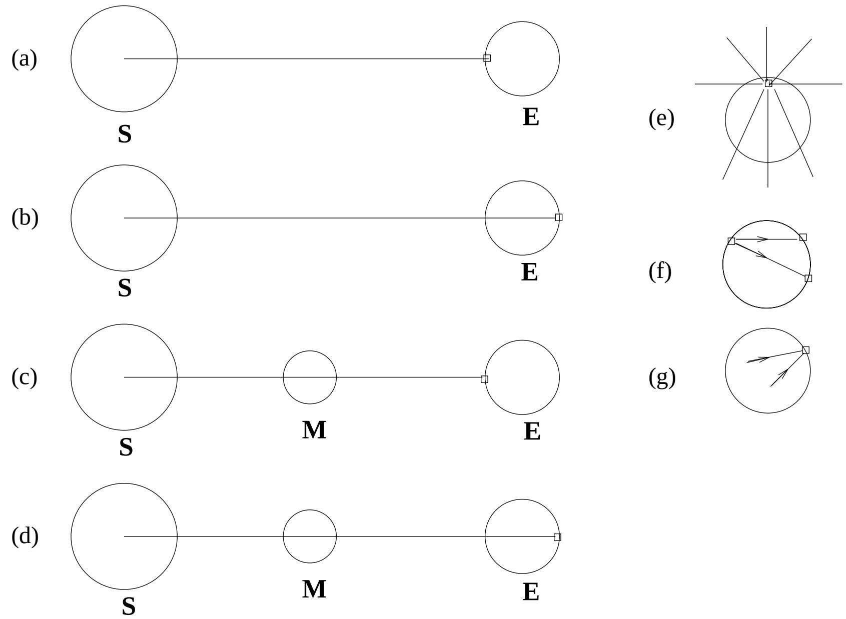

Figure 1. Neutrinos passing through various pieces of matter.

It is possible to treat these effects analytically. The analytical formula for (a) was already given in eq. (6). We shall now derive the formulae for (b) the night effect [18], (c) the eclipse effect [21] and (d) the double eclipse effect [21].

5.1 *The night effect*

Starting with $|\Psi_\alpha(t_2)\rangle$ on the surface of the earth given by eq. (4), we multiply the right-hand side by $\sum_k |\nu_k^E\rangle\langle\nu_k^E|(=1)$ where $|\nu_k^E\rangle(k=1,2,3)$ is the complete set of matter dependent mass eigenstates just inside the earth. If the neutrino propagates adiabatically up to t_3 on the other side of the earth (we shall soon correct for nonadiabatic jumps during the propagation), the state vector at t_3 is

$$|\Psi_\alpha(t_3)\rangle = \sum_{i,j,k} |\nu_k^E\rangle\langle\nu_k^E|\nu_j\rangle M_{ji}^{LZ} U_{\alpha i}^c \exp\left\{-i\int_{t_2}^{t_3}\epsilon_k \mathrm{d}t - i\int_{t_R}^{t_2}\epsilon_j \mathrm{d}t - i\int_{t_0}^{t_R}\epsilon_i \mathrm{d}t\right\}. \tag{8}$$

This expression automatically contains $\langle\nu_k^E|\nu_j\rangle$ which is the probability amplitude for nonadiabatic transition $j \to k$ at the vacuum-earth boundary and we shall call it M_{kj}^E:

$$M_{kj}^E = \langle\nu_k^E|\nu_j\rangle = \sum_\sigma \langle\nu_k^E|\nu_\sigma\rangle\langle\nu_\sigma|\nu_j\rangle = \sum_\sigma U_{\sigma k}^E U_{\sigma j}^*, \tag{9}$$

where ν_k^E and U^E are mass eigenstates and mixing matrix just inside the earth.

Averaging the probability $|\langle\nu_\beta|\Psi_\alpha(t_3)\rangle|^2$ over t_R results in the desired incoherent mixture of mass eigenstates of neutrinos reaching the surface of the earth. Calling this average probability as $P_{\alpha\beta}^N$ (the probability for ν_α produced in the sun to be detected as ν_β in the earth at night), we can write the result as

$$P_{\alpha\beta}^N = \sum_j P_{\alpha j}^S P_{j\beta}^E, \tag{10}$$

where $P_{\alpha j}^S$ is the probability of ν_α produced in the sun being detected as ν_j (mass eigenstate) as it enters the earth and $P_{j\beta}^E$ is the probability of ν_j entering the earth to be detected as ν_β after it propagates through the earth. These are given by

$$P_{\alpha j}^S = \sum_i |M_{ji}^{LZ}|^2 |U_{\alpha i}^c|^2, \tag{11}$$

$$P_{j\beta}^E = \sum_{k,k'} U_{\beta k}^{E^*} U_{\beta k'}^E M_{kj}^E M_{k'j}^E \exp\left(-i\int_{t_2}^{t_3} (\epsilon_k - \epsilon_{k'})\mathrm{d}t\right). \tag{12}$$

It is important to note that the factorization of probabilities in eq. (10) (which has been derived here as a consequence of the averaging over t_R), is valid only for mass eigenstates in the intermediate state. An equivalent statement of this result is that the density matrix is diagonal only in the mass-eigenstate representation and *not* in the flavour representation.

During the day, put $t_3 = t_2$ so that $P_{j\beta}^E$ becomes $|U_{\beta j}|^2$ and so eq. (10) reduces to eq. (6). One can justify [8] the averaging over t_0 and t_2 by the facts that the neutrinos are produced over an extended region in the solar core and they are detected over an extended region or time since the detector is moving with the earth. While averaging over t_0 and t_2 is equivalent to averaging over t_R as far as $P_{\alpha\beta}^D$ is concerned, it is not so for $P_{\alpha\beta}^N$, but we have adopted the latter method for $P_{\alpha\beta}^N$ because of its simplicity in giving us the factored probability expression in eq. (10).

However, two points have to be made: (i) For $P_{\alpha\beta}^N$, it is not justified to average over t_2 or t_3 since we would like to detect the neutrinos during a narrow time-bin in the night. (ii) Averaging over t_R (as we have done) may be partially justified since the result may be effectively the same for energy-integrated rates. However, this argument does not apply for Borexino [15], where the monochromatic Be^7 neutrino line spectrum will be detected.

Next we show how to take into account nonadiabatic jumps during the propagation inside the earth. Consider ν propagation through a series of slabs of matter, density varying inside each slab smoothly but changing abruptly at the junction between adjacent slabs. The state vector of the neutrino at the end of the nth slab $|n\rangle$ is related to that at the end of $(n-1)$th slab $|n-1\rangle$ by $|n\rangle = F^{(n)} M^{(n)} |n-1\rangle$ where $M^{(n)}$ describes the nonadiabatic jump occuring at the junction between $(n-1)$th and nth slabs while $F^{(n)}$ describes the adiabatic propagation in the nth slab. They are given by

$$M_{ij}^{(n)} = \langle\nu_i^{(n)}|\nu_j^{(n-1)}\rangle = (U^{(n)^\dagger} U^{(n-1)})_{ij}^*, \tag{13}$$

$$F_{ij}^{(n)} = \delta_{ij} \exp\left(-i\int_{t_{n-1}}^{t_n} \epsilon_i(t)\mathrm{d}t\right), \tag{14}$$

where the indices (n) and $(n-1)$ occurring on ν and U refer respectively to the nth and $(n-1)$th slabs at the junction between these slabs. Also note that $M^{(1)}$ is the same as M^E defined in eq. (9). Defining the density matrix at the end of the nth slab as $\rho^{(n)} = |n\rangle\langle n|$, we have the recursion formula

$$\rho^{(n)} = F^{(n)} M^{(n)} \rho^{(n-1)} M^{(n)\dagger} F^{(n)\dagger}. \tag{15}$$

Starting with $\rho^{(0)} = |\nu_j\rangle\langle\nu_j|$ (i.e. ν_j entering the earth), we can calculate $\rho^{(N)}$ at the end of the Nth slab using eq. (15). The probability of observing ν_β at the end of the Nth slab is

$$P^E_{j\beta} = \langle\nu_\beta|\rho^{(N)}|\nu_\beta\rangle = (U^{(N)} \rho^{(N)^*} U^{(N)^\dagger})_{\beta\beta}. \tag{16}$$

This formula (which reduces to eq. (12) for $N = 1$) can be used for the earth modeled as consisting of $(N+1)/2$ concentric shells, with the density varying gradually within each shell.

We have already referred to the detection of solar ν through the neutral current mode for bypassing the uncertainties of the solar models. Yet another way would be the detection of the night effect. An asymmetry between the night and day rates would be an unambiguous signal for neutrino oscillations independent of the details of the solar models. The recent results from SK [22,23] for this asymmetry is at the level of 0.06 ± 0.03 and is hence consistent with zero (at 2σ). Even the absence of the effect contains important information since it helps to rule out certain regions of neutrino parameter space in an unambiguous manner.

The night effect is bound to exist at some level and the accummulated data will soon reveal its magnitude. Further, since the neutrino samples different amount of matter in the earth during a single night and also during the period of a year, the data accumulated in various bins at different times of the night contain an enormous amount of information on neutrino parameters. We have stressed the importance of analyzing this time-of-night variation [18] and recent results from SK [22,23] do suggest such a variation.

Many interesting physical effects are contained in the analytical formulae already presented. As an example, we shall mention what we may call 'vacuum oscillations in matter'. For $\phi \approx 0$, we get [18] the following simple formula relating the survival probability in the night and day:

$$P^N_{ee} = P^D_{ee} + \left(1 - 2P^D_{ee}\right)\left(P^E_{2e} - \sin^2\omega\right)\frac{1}{\cos 2\omega}, \tag{17}$$

where

$$\begin{aligned} P^E_{2e} &= \sin^2\omega_E + \sin 2\omega_E \sin 2(\omega_E - \omega)\sin^2 \frac{1}{4E}\int_{t_1}^{t_2}\left[m_2^2(t) - m_1^2(t)\right]\mathrm{d}t \\ &\approx \sin^2\omega + 2(\omega_E - \omega)\sin 2\omega \sin^2\frac{\delta m^2_{21} L}{4E}. \end{aligned} \tag{18}$$

Here ω_E is the mixing angle just below the surface of the earth and L is the distance the neutrino travels inside the earth. In arriving at the approximate expression for P^E_{2e} given in eq. (18), we have assumed that $\delta m^2_{21} >> A(\equiv 2\sqrt{2}G_F NE)$, N being the electron number density inside the earth. Under this approximation of small matter effect, P^E_{2e}

and hence P_{ee}^N will exhibit vacuum type oscillations as a function of the distance travelled within earth, but their amplitude will be controlled by matter density (since $(\omega_E - \omega)$ is of order $A/\delta m_{21}^2$). Such regular oscillations were indicated in the earlier numerical calculations [24] for appropriate choice of parameters and their interpretation is clear from our analytical formulae. This effect can perhaps be detected at the Borexino (however, see the remark made above concerning the average over t_R).

It is particularly important to see the effect of the core of the earth [19]. A detector situated near the equator, such as one in South India [20] can do this.

5.2 *The eclipse effect*

The above calculation can be extended to include the effect of the moon [21]. We can consider both the case of the single eclipse when the neutrino passes through the moon only and the case of the 'double eclipse' when it passes through the moon and the earth and gets detected on the night-side. We present the results only. We get [21], for the single eclipse,

$$P_{\alpha\beta}^M = \sum_j P_{\alpha j}^S P_{j\beta}^M, \tag{19}$$

where

$$P_{j\beta}^M = \sum_{\ell k \ell' k'} U_{\beta\ell}^* U_{\beta\ell'} M_{kj}^M M_{k'j}^{M^*} M_{k\ell}^{M^*} M_{k'\ell'}^M$$
$$\exp\left\{-i\left(\epsilon_k^M - \epsilon_{k'}^M\right) d_M - i\left(\epsilon_\ell - \epsilon_{\ell'}\right) r\right\} \tag{20}$$

and for the double eclipse

$$P_{\alpha\beta}^{ME} = \sum_j P_{\alpha j}^S P_{j\beta}^{ME}. \tag{21}$$

where

$$P_{j\beta}^{ME} = \sum_{\substack{\ell k p \\ \ell' k' p'}} U_{\beta p}^{E^*} U_{\beta p'}^E M_{p\ell}^E M_{p'\ell'}^{E^*} M_{kj}^M M_{k'j}^{M^*} M_{k\ell}^{M^*} M_{k'\ell'}^M$$
$$\exp\left\{-i(\epsilon_k^M - \epsilon_{k'}^M) d_M - i(\epsilon_\ell - \epsilon_{\ell'}) r - i(\epsilon_p^E - \epsilon_{p'}^E) d_E\right\}. \tag{22}$$

Here, the superscripts M and E refer to the moon and earth respectively and we have assumed constant densities for simplicity (but the expressions can be easily generalized to include variable densities and discrete jumps in densities); d_M, d_E and r denote the diameter of the moon, diameter of the earth and the earth-moon distance respectively; M^M and M^{M^*} are the non-adiabatic jump probability amplitudes at the vacuum–moon interface and the moon–vacuum interface respectively defined analogously to eq. (9). We again note the convenient factorization in the results of eqs (19) and (21).

Our calculations [21] show considerable enhancements in the neutrino counting rate during the eclipse – even as high as 100%. However, since the counting rates are currently no more than about one per hour, the enhancement during the hour or two of the duration of the eclipse is hard to see at the present detectors. Perhaps we have to wait for the next generation of detectors.

6. Neutrinos from supernovae

The observation of neutrinos from the supernova SN 1987A was an exciting event and it spurred much activity in this field. Since we now have some idea of the mass-differences and the mixing angles from the study of solar, atmospheric and reactor neutrinos, we may ask what effect do the oscillations have on the neutrinos from supernovae and whether such effects can be observed in the neutrino detectors during a supernova event in the future. Here we shall restrict ourselves to focussing attention on one such important signal discussed recently [25].

Consider the thermal or cooling phase of the supernova when all the three flavours of neutrinos and antineutrinos are emitted. We denote the flux of ν_α and $\bar{\nu}_\alpha$ produced in the core by F^0_α and $F^0_{\bar{\alpha}}$ respectively. For all practical purposes one can put [26]

$$F^0_\mu = F^0_{\bar{\mu}} = F^0_\tau = F^0_{\bar{\tau}} \equiv F^0_x. \tag{23}$$

If $P_{\alpha\beta}$ is the probability for α changing to flavour β during propagation through the supernova, then, the fluxes of ν_e and $\nu_\mu + \nu_\tau$ coming out of the supernova are given by

$$\begin{aligned} F_e &= F^0_e P_{ee} + F^0_\mu P_{\mu e} + F^0_\tau P_{\tau e} \\ &= F^0_e - (1 - P_{ee})(F^0_e - F^0_x) \\ 2F_x &= F_\mu + F_\tau = 2F^0_x + (1 - P_{ee})(F^0_e - F^0_x), \end{aligned} \tag{24}$$

where we have used eq. (23) and the constraint $\sum_\alpha P_{\alpha\beta} = 1$. A similar analysis for the antineutrinos gives

$$\begin{aligned} F_{\bar{e}} &= F^0_{\bar{e}} - (1 - P_{\bar{e}\bar{e}})(F^0_{\bar{e}} - F^0_x) \\ 2F_{\bar{x}} &= 2F^0_x + (1 - P_{\bar{e}\bar{e}})(F^0_{\bar{e}} - F^0_x). \end{aligned} \tag{25}$$

Let us now calculate P_{ee} and $P_{\bar{e}\bar{e}}$. The variation of the three matter-dependent mass eigenvalues for the neutrinos and antineutrinos as functions of matter-density ρ are schematically depicted in figure 2. The ν_e has the decomposition in matter:

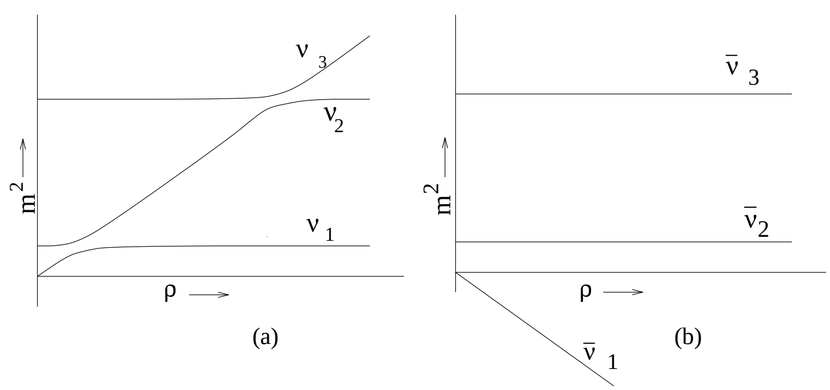

Figure 2. (**a**) Mass squares of the three neutrinos as functions of matter density ρ, with two MSW resonances, (**b**) same for antineutrinos where there are no resonances.

$$|\nu_e\rangle = \cos\phi_m \cos\omega_m |\nu_1^m\rangle + \cos\phi_m \sin\omega_m |\nu_2^m\rangle + \sin\phi_m |\nu_3^m\rangle, \tag{26}$$

where 'm' denotes matter. In the dense core of the supernova, $\phi_m \to \pi/2$ and so the ν_e is emitted as ν_3^m in the fireball: $|\nu_e\rangle = |\nu_3^m\rangle$.

For the parameters relevant for supernovae, one can show [25] that the Landau–Zener nonadiabatic jump probabilities at the two MSW resonances depicted in figure 2 are vanishingly small as long as $\sin\phi \geq 10^{-2}$. Hence, the neutrino state vector $|\nu_3^m\rangle$ evolves adiabatically and ends up as $|\nu_3\rangle$ as it emerges out of the supernova. Since $\langle\nu_e|\nu_3\rangle = \sin\phi$, we have $P_{ee} = \sin^2\phi \approx 0$ where we have used the result $\phi < 9°$ from the CHOOZ reactor experiment.

For the antineutrinos, we start with

$$|\bar{\nu}_e\rangle = \cos\bar{\phi}_m \cos\bar{\omega}_m |\bar{\nu}_1^m\rangle + \cos\bar{\phi}_m \sin\bar{\omega}_m |\bar{\nu}_2^m\rangle + \sin\bar{\phi}_m |\bar{\nu}_3^m\rangle \tag{27}$$

and since $\bar{\phi}_m \to 0, \bar{\omega}_m \to 0$, at high densities, we see that, when produced, $|\bar{\nu}_e\rangle = |\bar{\nu}_1^m\rangle$ and $\bar{\nu}_1^m$ emerges from the supernova as $\bar{\nu}_1$. Using $\langle\bar{\nu}_e|\bar{\nu}_1\rangle = \cos\phi\cos\omega$, we therefore get $P_{\bar{e}\bar{e}} = \cos^2\phi\cos^2\omega \approx 1$ or $1/2$ for $\phi < 9°$ and the small or large ω solar solution respectively.

Substituting these results into eqs (24) and (25), we get the changed neutrino fluxes due to oscillations:

$$F_e \approx F_x^0; \quad 2F_x \approx F_e^0 + F_x^0, \tag{28}$$

$$F_{\bar{e}} \approx F_{\bar{e}}^0; \quad F_{\bar{x}} \approx F_{\bar{x}}^0, \qquad \text{(for small } \omega) \tag{29}$$

$$F_{\bar{e}} \approx \frac{1}{2}(F_{\bar{e}}^0 + F_{\bar{x}}^0); \quad 2F_{\bar{x}} \approx \frac{1}{2}(F_{\bar{e}}^0 + 3F_{\bar{x}}^0). \qquad \text{(for large } \omega) \tag{30}$$

The changes in the neutrino detection rates arising from these have been calculated [25], but the important signal for oscillation is contained in eq. (28) which states that ν_x (i.e ν_μ or ν_τ) are converted into ν_e. Since the original average energies of ν_e and ν_x in the supernova are 12 MeV and 24 MeV, respectively, the average energy of ν_e is shifted upwards by the oscillation. This can be detected by the charged current mode of ^{16}O in the H_2O detector which has a threshold of 15.4 MeV. The rate for this mode can be enhanced by as much as two orders of magnitude as a consequence of oscillation [25]. If one can construct a detector with ^{16}O or ^{12}C without protons, it will be ideal since otherwise the $\bar{\nu}_e p$ absorption reaction is dominant.

7. Majorana neutrinos and global analysis

If neutrinos are Majorana fermions, then neutrinoless double beta decay is allowed. However the latter has not been seen yet and the experimental limits on it are getting stronger. The strongest upper limit so far comes from the germanium experiment and it is [27]

$$\left|\sum_j m_j U_{ej}^2 \eta_j\right| < 0.2\,\text{eV} \quad (\text{at } 90\%\text{CL}), \tag{31}$$

where $\eta_j (= \pm 1)$ is the CP parity (apart from a factor i) of the Majorana neutrino ν_j. The mixing matrix for Majorana neutrinos is

$$U = \begin{pmatrix} c_\omega c_\phi & s_\omega c_\phi e^{-i\delta_1} & s_\phi e^{-i\delta_2} \\ -s_\omega c_\psi e^{i\delta_1} - c_\omega s_\psi s_\phi e^{i(\delta_2+\delta_3)} & c_\omega c_\psi - s_\omega s_\psi s_\phi e^{i(\delta_3+\delta_2-\delta_1)} & s_\psi c_\phi e^{i\delta_3} \\ s_\omega s_\psi e^{i(\delta_1-\delta_3)} - c_\omega c_\psi s_\phi e^{i\delta_2} & -c_\omega s_\psi e^{-i\delta_3} - s_\omega c_\psi s_\phi e^{i(\delta_2-\delta_1)} & c_\psi c_\phi \end{pmatrix}. \tag{32}$$

There are three CP-violating phases for Majorana neutrinos, in contrast to the case of Dirac neutrinos where there is only one phase (see eq. (2)). However, for oscillation phenomena, only one combination of the three phases occurs and so oscillations cannot distinguish between Majorana and Dirac neutrinos.

Is it possible to combine the very important constraint on the neutrino masses and mixings provided by eq. (31) with the information already derived from the solar, atmospheric and reactor neutrinos? The answer is yes, provided we make some assumption about the neutrino mass scale. Until two years ago, cosmologists had claimed that their analysis of the data on the anisotropies of the cosmic microwave background radiation and the large scale structure of the universe require the presence of some hot component (presumably massive neutrinos) in the dark matter and their best fit was [28]

$$\sum_{j=1}^{3} m_j \approx \text{ a few eV}. \tag{33}$$

Many years ago [29], I had formulated the law that allowed only a one-way traffic between high energy physics and cosmology: high energy physics $\rightarrow$ cosmology. We violated this law when we [30] used the cosmological result of eq. (33) in neutrino physics and punishment came in the form of the observation [31] of high red shift supernovae and their interpretation in terms of a nonvanishing cosmological constant. The hot dark-matter component is no longer favoured by cosmologists. So, we now have to regard eq. (33) merely as a cosmological assumption about the neutrino mass scale.

Since the oscillations of the solar and atmospheric neutrinos imply mass differences which are much smaller than the cosmological scale of eq. (33), we can take all the three neutrinos as almost degenerate in mass: $m_i \approx m_\nu \approx 1$ eV (for $i = 1, 2, 3$) and so eq. (31) becomes

$$|(\eta_1 \cos^2 \omega + \eta_2 \sin^2 \omega e^{-2i\delta_1}) \cos^2 \phi + \eta_3 \sin^2 \phi e^{-2i\delta_2}| < 0.2. \tag{34}$$

This constraint can be analysed for all possible choices of δ_1, δ_2 and η_i and the allowed regions for the mixing angles ω and ϕ can be mapped out [30]. One fact can be immediately noted. Since $\phi < 9°$ according to the reactor experiment, small values of ω cannot be consistent with eq. (34). So we have the important conclusion: If the cosmological assumption of m_ν in the eV scale is correct and if the small-ω solution turns out to be the correct solution of the solar ν problem, then neutrinos cannot be Majorana fermions.

8. LSND and the fourth neutrino

Since all the results of the solar, atmospheric and reactor neutrino experiments could be consistently explained within the framework of three neutrinos, it seemed that all that was required was the resolution of the three-fold ambiguity of the solar neutrino solutions and more precision neutrino experiments to pin down the fundamental neutrino parameters.

Table 1. Long base-line neutrino experiments.

Expt.	Baseline	L (km)	$\langle E_\nu \rangle$ GeV	Δm^2 (eV^2) probed	Status
K2K	KEK $\rightarrow$ Kamioka	250	1.4	10^{-3}	Taking data
	FNAL $\rightarrow$ Soudan	730	10	10^{-2}	To start in 2002
	CERN $\rightarrow$ Gransasso	732	20	10^{-2}	To start in 2005
Kamland	Reactors $\rightarrow$ Kamioka	160	3×10^{-3}	10^{-5}	To start in 2001

But a spanner was thrown into the works by the LSND experiments [32] reporting positive results on $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and $\nu_\mu \rightarrow \nu_e$. Since the base-line length of these experiments is as short as 29 m, the implied δm^2 is in the rangle 1–10 eV^2. It is difficult to incorporate this result within the three-neutrino framework and so most theorists have decided to ignore the LSND result, citing the fact that it has not yet been confirmed by an independent experiment. The independent experiment KARMEN has not confirmed the LSND result, but KARMEN [33] has not ruled out the full parameter space allowed by LSND either. A real confirmation or ruling out has to await the Mini BOONE experiment at FNAL, a long agonizing 4 years away.

If the LSND result is correct, we need a 4th neutrino, but since the known invisible width of Z is completely exhausted by ν_e, ν_μ and ν_τ, the new neutrino has to be a singlet under SU(2) and be sterile under known interactions. The natural mass hierarchy would be to place ν_4 a few eV above the known three neutrinos, but this is contradicted [34] by a combination of known experimental data, unless ν_4 decays [35].

9. Future

Is it possible to confirm or refute the results on neutrino oscillations claimed by the solar and atmospheric neutrino observations using laboratory experiments? That would be one of the chief goals of the long base-line neutrino experiments that are being planned (see table 1). One can see that sensitivities up to the level of Δm^2 needed for solar and atmospheric neutrinos will be reached in these experiments. Even larger baselines can be contemplated.

Finally, there are prospects of constructing muon storage rings that will function as neutrino factories and these promise to take neutrino physics to a new era. Hopefully these as well as the long baseline experiments will lead to a determination of the neutrino parameters. Of course, entirely new phenomena could also be discovered.

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