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Supersymmetric unification at the millennium

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Abstract. We argue that the discovery of neutrino mass effects at super-Kamiokande implies a clear logical chain leading from the Standard Model, through the MSSM and the recently developed minimal left right supersymmetric models with a renormalizable see-saw mechanism for neutrino mass, to left right symmetric SUSY GUTS: in particular, SO(10) and $SU(2)_L \times SU(2)_R \times SU(4)_c$. The progress in constructing such GUTS explicitly is reviewed and their testability/falsifiability by lepton flavour violation and proton decay measurements emphasized. SUSY violations of the survival principle and the interplay between third generation Yukawa coupling unification and the structurally stable IR attractive features of the RG flow in SUSY GUTS are also discussed.

Keywords. Supersymmetry; R-parity; Pati–Salam; left–right.

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1. Introduction

This workshop is proceeding on the morning of the new millenium in the shadow of the towering cranes of a mammoth construction project. Thus it is entirely appropriate to review the progress in the ongoing 'petaproject' of the 'palazzo GUT' whose patroness (the santissima SUSY) has after 20 years of fervid courtship/worship still not lost her charisma for the high energy theory community. Although several 'sky-high' colonne of the SUSY-GUT palazzo are now firmly in place (force and third generation Yukawa unification, matter unification and the neutrino mass unification mass connection (nozze dei neutrini)) *il palazzo manca tetto e muri* (the palace still lacks a roof and walls). Thus it is still open to the gales of speculation habitual in its clime. On the other hand the discovery of neutrino mass effects at super-Kamiokande in the 100 milli eV range have made LR supersymmetric models and their GUT relatives hot favourites for a direct look through the windows of the palazzo. Finally, progress has been made in the last years of the millenium in identifying the characteristic doors through which we may enter the palazzo of SUSY unification: namely the generic signals provided by lepton flavour violation and proton decay. The questions of fermion masses of the first and second generations, fermion mixings etc are the walls and services of the palace whose emplacement is an open project for the new millenium.

In a recently published review talk I have covered the connections between the MSSM with R parity and LR supersymmetric unification in prolix detail [1]. On the other hand that review devoted little attention to the progress in understanding the gross features of the fermion mass spectrum and the generic lepton flavour violating signals for SUSY unification. Therefore the main points of the previous talk are here related telgraphically and the freed space used to discuss fresh topics within the space limitations imposed. My notations, abbreviations and conventions are those of that article [1] and will not be repeated here.

Let us begin with the implications of the super-Kamiokande discovery of beyond SM effects [2]. In the SM, neutrino mass can be introduced only via the non-renormalizable operator $(H^\dagger L)^2/M$. The presence of the scale M indicates new physics beyond the SM. Super-Kamiokande data interpreted as evidence of tau neutrino mass $\sim 10^{-1.5}$ eV implies the scale of the new physics leading to neutrino mass is $M \sim 10^{14\pm 0.5}$ GeV. Such a large scale can be plausibly separated from the electroweak scale by the introduction of supersymmetry which can prevent quadratic corrections ($\sim M^2$) to light boson masses. Thus one may argue that the heirarchy problem is no longer the hypothetical one posed by GUTs but is now posed by the SM and the super-Kamiokande data!

Since it offers an appealing and flexible escape route from the heirarchy problem, SUSY has enjoyed a 20 year old vogue that shows no sign of abating. There are good reasons for this fascination. As reviewed in detail in [1] it was predicted (in 1982!) [3,4] that only SUSY unification would be compatible with the data if it turned out that the top quark mass and Weinberg angle were as large as they were ultimately found to be (i.e $m_t \sim 200$ GeV and $\sin^2\theta_W \sim 0.23$). This has been strikingly vindicated by the precise EW data accumulated by LEP I and II. Furthermore the larger value of the unification scale in the SUSY case explains why baryon decay has not long been seen. Successful and unique gauge unification by minimal SUSY GUTs constitutes the first pillar of the 'Palazzo SUSY' and was the reason for the refocussing of interest on GUTs from the beginning of the nineties [5].

The unification of gauge couplings naturally begs for an explanation via spontaneous symmetry breaking of a larger GUT gauge symmetry in which the electroweak-QCD physics is embedded. The canonical and minimal possibilities for the GUT gauge symmetry are the Pati–Salam symmetry $G_{224} = G_{PS} =$ SU(2)$_L\times$ SU(2)$_R\times$ SU(4)$_c$ and SU(5) and also the rank 5 group Spin(10) (SO(10) with fermions) in which both can be embedded. Spin(10) is also minimal in that it is a simple gauge group which unifies the fermions of the SM, together with the right handed neutrino needed to explain neutrino mass effects convincingly, in a single irrep. It can accomodate several different breaking chains by use of appropriate Higgs multiplets and shall be the favoured model of this review.

The singular properties of the neutrino (it is the only neutral fermion of the SM, it is bereft of a chiral partner, and its minimal mass operator – the $d = 5$ operator above – violates the anomaly free global $B - L$ symmetry of the SM), the indications of its mass found by super-Kamiokande as well as the explanation of the solar neutrino deficit via neutrino oscillations all obtain a natural and elegant explanation in terms of a 'see-saw' between the neutrino and its righthanded partner [6]. The extra field ν_R is naturally and necessarily present in LR symmetric GUTs where the maximal parity violation of the SM is viewed as arising from spontaneous breaking of a parallel SU(2)$_R$ symmetry present at high energies. The 'fatso' scale of the seesaw is moreover precisely the scale M_{B-L} at which the *gauged* $B - L$ symmetry of LR models breaks to the weakly violated global symmetry of the SM. Explicitly, the presence of $\nu_l^c \sim \nu_R^*$ allows one to write the mass term and Yukawa interaction (absent in the SM) as

$$(M_{\nu^c})_{ij}\nu^c_{Li}\nu^c_{Lj} + h_{ij}H^\dagger L_i\nu^c_{Lj}. \tag{1}$$

Integrating out the heavy neutrino gives the neutrino mass operator

$$-h_{ij}(M_{\nu^c}^{-1})_{jk}h_{kl}H^\dagger L_i H^\dagger L_l. \tag{2}$$

This is the type I see-saw [6]. In case the left neutrinos acquire (small!) Majorana masses M_{ν_L} from some other source then they will add to the above contribution giving the so called type II see saw mechanism [7]. The mass M_{ν^c} may itself be taken to arise via spontaneous symmetry breaking of the $B-L$ symmetry via vevs of SU(2)$_R\times$ U(1)$_{B-L}$ multiplets: which must be triplets if the interaction is to be renormalizable, but may be taken to be doublets if one permits higher dimensional operators to do the job.

R-parity and LR symmetry

The presence of scalar partners of the fermions of the SM in the MSSM allows renormalizable interactions which violate the crucial $B-L$ symmetry which protects the current universe from evaporation into radiation. These violations are thus strongly constrained by our persistence. The simplest way to remove them is by noting that in their absence the MSSM enjoys the discrete mutiplicative symmetry $R_p=(-)^{3(B-L)+2S}$ [8]. Imposing this symmetry thus furnishes a natural if, *ad hoc*, justification for the $B-L$ symmetry of the SM. The form of the symmetry now leads to an unexpected brownie point for LR theories: R_p is effectively a part of their gauge symmetry. Moreover if one implements the see-saw using a renormalizable term to generate the ν^c mass then the required Higgs fields Δ^c are SU(2)$_R$ triplets with $B-L=-2$ and *their vevs cannot violate R parity*. Explicitly, the couplings are

$$h_{ij}^a L_i\phi_a L_j^c + f_{ij}L_i\Delta L_j + f_{ij}^c L_i^c\Delta^c L_j^c + \mathrm{h.c.} \tag{3}$$

These give a large Majorana mass to the ν_L^c fields when the Δ^c field develops a large vev, while the vev of the $\phi(2,2,0,1)$ field which is dominantly responsible for EW SSB gives rise to a Dirac mass term between ν_L and ν_L^c. Thus a seesaw mechanism occurs very naturally as a consequence of the hierarchy between SU(2)$_L\times$ U(1)$_Y$ and SU(2)$_R\times$ U(1)$_{B-L}$ breaking scales. Since the scalar potential in general allows couplings of the form

$$V=M^2\Delta^2+\Delta\phi^2\Delta^c+\cdots. \tag{4}$$

It follows that once ϕ,Δ^c acquire vevs $\sim M_W, M$ respectively Δ acquires a vev due to the linear term generated:

$$<\Delta>\sim M_W^2/M \tag{5}$$

so that the seesaw is in general of type II.

2. Minimal SUSY LR models

A significant advance [9] has been the realization that when – as is generically the case and as is now experimentally indicated – the scale of $B-L$ violation is $M_{B-L}>>M_W$, phenomenological constraints and the structure of the SUSY vacuum ensure that R-parity is preserved. Viable minimal LR supersymmetric models (MSLRM) have been constructed in detail [9–11] and embedded in GUTS while retaining these appealing properties.

Since the argument for R-parity exactness is so simple and general we present it first in isolation before going on to the details of the MSLRMs. Given $M_{B-L} >> M_W \sim M_S$ (the scale of SUSY breaking) it immediately follows that the scalar partners of the ν_L^c fields also have positive mass squares $\sim M_{B-L}^2$ and hence are protected from getting any vevs: modulo effects suppressed by these large masses. Thus when the ν_L^c superfield is integrated out the effective theory is the MSSM with R-parity and with $B-L$ violated only by (the SUSY version of) the highly suppressed $d=5$ neutrino mass operator. As a result if, for any reason, the scalar $\tilde{\nu}_L$ were to obtain a vev [12] the low energy theory would contain an almost massless scalar (i.e a 'doublet' (pseudo) Majoron) in its spectrum. However such a pseudo-Majoron is conclusively ruled out by LEP. These arguments based on decoupling are very robust, see [1,9–11] for details. To sum up, quite generally:

The low energy effective theory of MSLRMs with a renormalizable is the MSSM with exact R parity so the lightest supersymmetric particle (LSP) is stable .

The detailed structure of MSLRMs has been established in the series of papers cited above for the generic case (now favored strongly by experiment) when the scale of right handed ($B-L$ breaking) physics is high. We conclude that the fields of generic MSLRMs (in addition to the supersymmetrized anomaly free set of fields of the LR model $(Q, Q^c, L, L^c, \phi, \Delta, \bar{\Delta}, \Delta^c\bar{\Delta}^c)$) are one of the following:

(a) Introduce a pair $\Omega(3,1,0,1) \oplus \Omega^c(1,3,0,1)$ of SU(2)$_{L/R}$ triplet fields. Then one can achieve SSB of the LR symmetry via the Ωs and separately the SSB of the $B-L$ symmetry, at an independent scale M_{B-L} by the $\Delta^c, \bar{\Delta}^c$ fields.

(b) Stay with the minimal set of fields, but (reasoning that small non-renormalizable corrections must be counted when the leading effects are degenerate) include the next order $d=4$ operators allowed by gauge invariance in the superpotential. These operators are of course suppressed by some large scale M and may be thought to arise either from Planck scale physics $(M \sim M_{\rm Planck})$ or when one integrates out heavy fields in some GUT in which the LR model is embedded $(M \sim M_X)$. The principal effect of allowing such terms is that the charge breaking flat direction is lifted and one obtains a phenomenologically viable low energy effective theory (with characteristic additional fields at the scale M_{B-L}^2/M_R: see below).

(c) Finally one may introduce a parity odd singlet in either of cases (a), (b) which we shall for convenience refer to as cases (a′) and (b′). This case is not academic or non-minimal since such parity odd singlets (POS) arise very naturally when one embeds these models in SO(10).

The symmetry breaking in these models has been analysed in considerable detail using the help of the theorem [13] which labels the vacuum manifold of SUSY theories by the chiral gauge invariants left unfixed by the vanishing of the F terms on the vacuum manifold. One then finds that the above parity preserving scenario is convincingly realized.

Survival principle violations

The detailed analysis of symmetry breaking also allows one to calculate the mass spectrum of the theory [14,1]. Besides, the usual particles of the SM and their superpartners at M_S

one finds that certain superfields associated with SU(2)$_R\times$ U(1)$_{B-L}$ breaking remain relatively light and for favourable values of the parameters may even be detectable at current or planned accelerators. In cases (a) and (a') one finds that a complete supermultiplet with the quantum numbers of $\Omega(3,1,0,1)$ has a mass M_{B-L}^2/M_R. If $M_{B-L} << M_R$ then these particles could be detectable. However given the expectation of $M_{B-L} > 10^{14}$ GeV from neutrino mass this does not appear to be a likely possibility. In cases (b) and (b') (which one may consider as the truly minimal alternative) one finds instead that the entire slew of fields $\Delta, \bar{\Delta}, \delta^c_{--}, \bar{\delta}^c_{++}, \delta^c_0 + \bar{\delta}^c_0, H'_u, H'_d$ have masses $\sim M_R^2/M$. If, for instance, $M \sim 10^{19}$ GeV and $M_R \sim 10^{11}$ GeV then these particles could conceivably be detectable specially because they include exotic particles with charge 2 which are coupled to the usual light fermions of the model. In the above H'_u, H'_d are a pair Higgs doublets left over after the fine-tuning to keep one pair of doublets light out of the four (i.e two bidoublets) introduced to allow sufficient freedom in the tree level Yukawa couplings.

The *reason* for the lightness (noticed from the beginning [17]) of the exotic super multiplets has been under appreciated in the past. It is due to a generic feature of SUSY lagrangians where gauge invariance and the constraint of renormalizability of the superpotential often leads to an absence of cubic couplings for some fields.

This important feature of symmetry breaking and mass spectra in SUSY theories has recently been emphasized in [15]. Normally the masses of the non-goldstone parts of a Higgs multiplet get masses of the same magnitude as the symmetry breaking vev. Thus it is considered reasonable when undertaking RG analyses to assume that the mass spectrum obeys this 'survival principle' and compute the RG flow even without calculating the mass spectrum explicitly. However it was noticed long ago [17] that there tend to be large supermultiplets which remain light after high scale SUSY breaking involving vevs for their GUT partners. The reason for this can be clarified by a simple example. Consider a Wess–Zumino model with a single chiral superfield in which, for some reason, the superpotential contains a quadratic and higher than cubic terms (suppressed by powers of some large mass $M >> m$) but no cubic term

$$W = m\Phi^2 + \sum_{n>0} \frac{\phi^{n+3}}{M^n}. \tag{6}$$

When Φ gets a vev ($\sim \sqrt{mM}$) the effective cubic coupling is thus $\sim (\frac{\langle\Phi\rangle}{M})^{n_{\min}} << 1$ so that the mass of the residual non-goldstone super multiplet is $<< \langle\Phi\rangle$.

For a gauge example consider a U(1) model with two fields ϕ, $\bar{\phi}$ with opposite charges. Then the superpotential takes the form

$$W = m\phi\bar{\phi} - \frac{(\phi\bar{\phi})^2}{2M}. \tag{7}$$

This then implies that the vevs of both fields are $\sqrt{mM}$ while the effective cubic coupling $\sim \sqrt{m/M}$ is very small if the scale $M >> m$ as is natural to suppose for non-renormalizable couplings. Indeed one finds that while one multiplet ($\Phi - \bar{\Phi}$) is the super Higgs and has a mass $g < \phi > \sim g\sqrt{mM}$ the other $\Phi + \bar{\Phi}$ has a much smaller mass $\sim m$. These considerations apply directly in SUSY LR models since the Δ multiplets (in SO(10) the 126's) have no cubic couplings. This leads to light doubly charged exotics and makes it abundantly clear that RG analyses based on a blind application of the survival principle can be erroneous. These observations can obviously have critical implications for low energy SUSY phenomenology.

3. LR SUSY guts

As we have seen, LR SUSY models are natural candidates for SUSY unification which accommodates neutrino mass. Thus it is natural to consider further unification in which the various factors of the LR symmetry group are unified with each other. The two most appealing possibilities are unification within the Pati–Salam group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)_C$ and SO(10). The multiplets **45, 210** of SO(10) contain parity odd singlets [16] and the Pati–Salam gauge group is a subgroup of SO(10). Thus the study of SO(10) unification teaches one much about the Pati–Salam case as well. Therefore we [14,1] have re-examined SO(10) SUSY unification keeping in view the progress in understanding of LR SUSY models detailed above and developed a minimal SO(10) theory of R-parity and neutrino mass with the appealing features of automatic R-parity conservation. A detailed and explicit study of the SSB at the GUT scale and various possible intermediate scales was performed. The mass spectra in various cases could be explicitly computed. In particular the light supermultiplets with possibly low intermediate scale masses ($\sim M_R^2/M_{PS}, M_{PS}^2/M_X$ etc.) that often arise in SUSY GUTS (see [17] for an early example involving SO(10)) were determined. With these computed (rather than assumed spectra) a preliminary one-loop RG survey of coupling constant unification in such models was carried out. We find that an appealing and viable model may be constructed using the Higgs multiplets $\mathbf{45}(\boldsymbol{A}), \mathbf{54}(\boldsymbol{S}), \mathbf{126}(\boldsymbol{\Sigma}), \overline{\mathbf{126}}(\bar{\boldsymbol{\Sigma}}), \mathbf{10}(\boldsymbol{H})$. The reader is referred to [1,14] for details.

4. Fermion masses, Yukawa unification and IR attractors

The striking and consistent unification of gauge couplings in the MSSM naturally raises the question of the behaviour of the other parameters of the MSSM at high energies. Even in the SM there are already ~ 15 other parameters. Allowing for neutrino masses and mixings and for SUSY breaking parameters raises this number to $\sim 10^2$. Thus a separation of parameters into different classes corresponding to the gross and fine structure of the model is necessary to make progress. After gauge couplings and vector boson masses the most important parameters are the fermion masses. These are conveniently divided into three subproblems:

(a) Third generation masses/Yukawa couplings: Since the third generation masses, in particular m_{top} are so much larger than the others it is plausible that they are connected with the gross structure of the fermion mass matrix and of symmetry breaking.

(b) The second and first generation masses and the mixing matrix in the quark sector obey: $m_{u,d,c,s.e.\mu} << m_{t,b,\tau}, V_{\mathrm{CKM}} << 1$ and may plausibly be considered to arise as a next order effect due to radiative or other suppressed corrections. Recently some progress has been made along these lines in models closely related to the ones favoured here [18].

(c) As discussed above neutrino masses and mixings are an independent issue due to the neutrino's peculiar properties. Moreover they inevitably imply physics beyond the SM and hence fall in an independent category. The question of how the large neutrino mixing angles apparently favoured by data could be compatible with GUTs and the small mixings in the quark sector has attracted much model building attention recently. However these discussions lie out of the minimalistic gaze of this review.

In the MSSM there is an important new parameter freedom necessarily present: the ratio of the vevs of the two Higgs doublets $\tan\beta = v_u/v_d$. This freedom breaks the strict correlation between the Yukawa couplings $h_{t,b,\tau}$ and the corresponding masses $m_{t,b,\tau}$. The minimal versions of the basic types of GUT models imply strict relations between Yukawa couplings at the unification scale and the consistency of these relations with the low energy data is then an important constraint on these models.

In the case of the Pati–Salam model with fermion masses $(2,2,4)_F(2,2,\bar{4})_F\ \langle(2,2,1)\rangle$ arising from bidoublet vevs and also in minimal SO(10) models with fermion masses $16_F 16_F\langle 10_H\rangle$ the relations $h_t = h_b = h_\tau$ follow naturally. SU(5) invariant mass terms $10_t 10_{t^c}\langle\bar{5}_{\bar{H}}\rangle + 10_{b,\tau}5_{\tau^c,b^c}\langle 5_H\rangle$ however imply only $h_b = h_\tau$.

The implications of the so called Pendleton–Ross [19] fixed point, the Hill [20] effective fixed point, and their manifold generalizations, for our understanding of the labyrinthine question of fermion masses are so profound, and shed so much light on these vexed questions that I will review them in some detail. My treatment is largely based on the excellent review of Schrempp and Wimmer [21].

Consider first the evolution of the couplings h_t, g_3, ignoring other couplings. It is convenient to form the ratio $\rho_t = h_t^2/g_3^2$ and to trade the evolution parameter μ for the asymptotically free coupling g_3 with which it is monotonically correlated. The evolution equation at one loop is

$$\frac{\mathrm{d}\rho_t}{\mathrm{d}\ln g_3^2} = -2\rho_t\left(\rho_t - \frac{7}{18}\right). \tag{8}$$

Clearly there is a fixed point at $\rho_{tf} = 7/18$ and is easily seen to correspond to a top quark mass of $126\sin\beta$ GeV.

Furthermore, it can be shown [20,21] that for sufficiently large initial value ρ_0 that the low energy value of ρ obeys the Hill bound

$$\rho < \frac{7}{18}\left(1 - \left(\frac{\alpha_3^0}{\alpha_3}\right)^{7/9}\right)^{-1}. \tag{9}$$

This bound functions as an effective attractor for the IR flow which slows down markedly in its vicinity.

Next one can introduce the coupling h_b and write the corresponding RG equations

$$\begin{aligned}\frac{\mathrm{d}\rho_t}{\mathrm{d}\ln g_3^2} &= -2\rho_t\left(\rho_t + \frac{\rho_b}{6} - \frac{7}{18}\right)\\ \frac{\mathrm{d}\rho_b}{\mathrm{d}\ln g_3^2} &= -2\rho_b\left(\rho_b + \frac{\rho_t}{6} - \frac{7}{18}\right).\end{aligned} \tag{10}$$

Thus one now has the following structural features:

- infrared attractive fixed point (**0**): $\rho_t = \rho_b = 1/3$.
- strongly attractive infrared attractive fixed line (**1**): see figure 1.

- less attractive IR fixed line (**2**): $\rho_t = \rho_b$ (intersects **1** in **0**).
- Hill effective fixed line (**3**): outer arc in figure 1.

Note that except *on* the fixed line **2** the flow is first attracted by the fixed line **1** and then along it to the fixed point.

When one switches on the other gauge couplings g_1, g_2 the fixed point **0** generalizes to $\rho_1 = \rho_2 = \rho_\tau = 0, \rho_t = \rho_b = 1/3$ while the structures **0, 1, 2, 3** become embedded in an IR attractive surface onto which the flow proceeds before going onto **1** and then along it into **0** unless initially (and then always) on **2**. The role of the attractive fixed structures in inducing insensitivity to the initial values of the couplings at high energies is dramatically illustrated by considering a range of $\rho_{t0}, \rho_{b0} \in [0, 25]$ i.e initial values at $\mu_0 = \mathrm{e}^{t_0} \sim 10^{16}$ GeV. This entire range of intial values flows into $\rho_t, \rho_b \in [0, 1]$ at $\mu \sim 10^2$ GeV. Indeed most of the parameter range (say $[0.5, 25]$) flows into a narrow band between the fixed line **1** and the Hill quasi-attractor **3**.

One can use the trivial relation $\tan\beta = \frac{m_t}{m_b}\sqrt{\frac{\rho_b}{\rho_t}}$ to trade the fixed lines **1, 3** for lines in the $\tan\beta - m_t$ plane to obtain figure 2. We know that almost any initial values will flow into the narrow band between the lines **1, 3**. Thus this figure offers us a structurally stable insight into the structure of the parameter space of the MSSM. It is worth mentioning that the effect of including the electroweak gauge couplings is to narrow this band and make the Hill line **3** more attractive [21]. From figure 2 it is clear that there are two narrow ranges of $\tan\beta$ ($\tan\beta \in [1, 4]$ or $\tan\beta \in$ [42–66]) compatible with experimental value of $m_t \sim 175$ GeV. These ranges have a strong dependence on m_b^{expt} but only a weak dependence on m_s, α_s. Figure 3 illustrates various possibilities.

The role of the fixed lines **1, 3** in permitting the viability of GUT initial conditions like $h_\tau^0 = h_b^0$ can be clarified by considering the RG equation for the ratio $R = h_\tau / h_b$,

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{R}{16\pi^2}\left(h_t^2 - \frac{16}{3}g_3^2 + 3(h_b^2 - h_\tau^2) + \frac{4}{3}g_1^2\right). \tag{11}$$

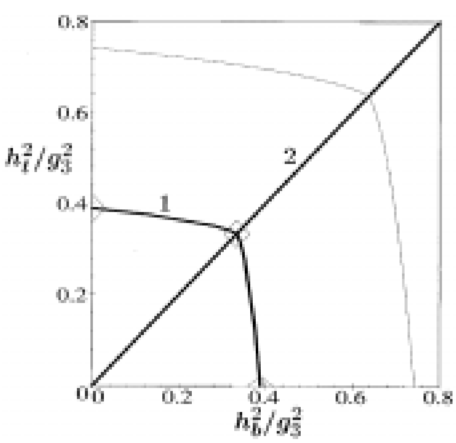

Figure 1. Fixed lines in the $\rho_t - \rho_b$ plane. The outer arc is the Hill effective fixed line (**3**). After ref. [21].

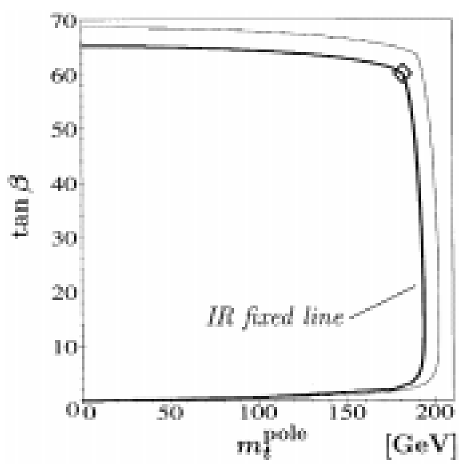

Figure 2. The lines **1** (heavy line) **3** (outer line) translated to the $\tan\beta - m_t$ plane. After ref. [21].

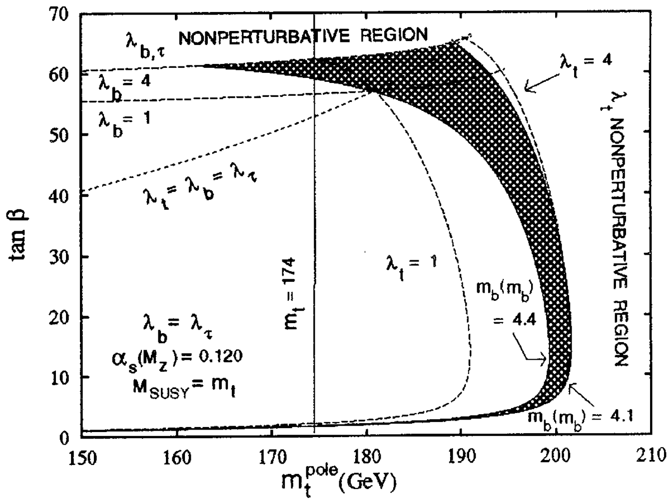

Figure 3. Contours of constant m_b in the $\tan\beta - m_t$ plane subject to tau-bottom unfication. Read $h_{b,\tau,t}$ for $\lambda_{b,\tau,t}$ (after Barger, Berger and Ohmann see ref. [21]).

This first order DE has *two* conditions on it set by the GUT condition $R^0 = 1$ and the experimental constraint $R(m_t) = m_b/m_\tau$. To satisfy them the value of h_t^0 (the free parameter in (11)) must be fine tuned. On the other hand h_t has its own RGE and final condition to satisfy. The role of the IR attractive structures is precisely to make the flow insensitive to the value of h_t^0 provided it is greater than some minimum value. If one further imposes the equality of $h_b^0 = h_\tau^0 = h_t^0$ as required by minimal SO(10) one finds that one flows into the close vicinity of the IR fixed point **0** and $\tan\beta \sim 50$. *So that SUSY* SO(10) *models prefer the high* $\tan\beta$ *solution*.

The state of the art in the study of the RG structure of SUSY GUT models can be glimpsed from the results of a recent paper [22] which also analyses the compatibility of the above discussion with radiative electroweak symmetry breaking (EWRSB). Recall that if the top Yukawa coupling h_t is large enough then $m_{H_u}^2$ flows to smaller values and can turn negative leading to EWRSB [23]. The breaking is constrained to obey $m_{H_d}^2 - m_{H_u}^2 > M_Z^2$. However when one attempts to combine this scenario, considered by many to be one of the most attractive bonuses of the MSSM, with the picture of the structurally stable attractor governing the third generation fermion masses one runs into an obstacle. It has been shown [24] that minimal SUSY SO(10) with universal soft SUSY breaking terms suffers from problems in obtaining simultaneous consistency with experimental bounds the branching ratio for $b \to s\gamma$ and the cosmological bound $\Omega_{\rm LSP}h^2 < 10^{-1}$ since the LSP is essentially the Bino. Also $m_{H_d}^2 < m_{H_u}^2$ so that there is no EWRSB. However it was soon shown [25] that non-universality in the soft breaking terms introduced naturally via the small D term vev $< D_X >\sim M_S$ when SO(10) breaks to the MSSM (here $X = -2Y + 5(B-L)$ is the diagonal generator broken while reducing the rank of the gauge group from 5 to 4) was sufficient to split the initial values of the soft masses for $H_u, H_d, \tilde{Q}, \tilde{U}, \tilde{D}$ etc so that the constraints of the model with universal soft terms were evaded and EWRSB was achievable.

The recent work of [22] has shown that this program can indeed be implemented successfully over a considerable region of the soft SUSY parameter space. The authors of [22] carried out a scan of the soft SUSY parameter space in the minimal SO(10) model i.e they generated random initial values for $m_{1/2}, M_{16}, m_{10}, m_D, A_0$ (the soft trilinear coupling) and integrated the two loop RG equations for 26 couplings and masses. For an appreciable range of parameters they found that (i) the electroweak gauge couplings unify exactly and the strong coupling is within 10%, (ii) $h_{t,b,\tau}$ unify within 5% yielding a value of $\tan\beta = 48 \pm 4$, (iii) the LSP is mostly a Higgsino so that the constraints from the $b \to s\gamma$ and $\Omega_{\rm LSP}$ are obeyed, (iv) EWRSB is achieved. A representative plot of one of their successful models is shown in figure 4. The successful gauge and Yukawa unification is obvious as is the non-universality in the soft masses at the high scale and the EWRSB at low energies. The outlook for these models is thus very favourable since they combine so many desirable features and yet run the gauntlet of experimental and cosmological constraints successfully.

Further recent work on generation of realistic fermion masses and mixings for the first and second generations via radiative corrections is very promising and appealing [18]. However a detailed scan of the parameter space at the level of the above analysis of [22] remains to be done.

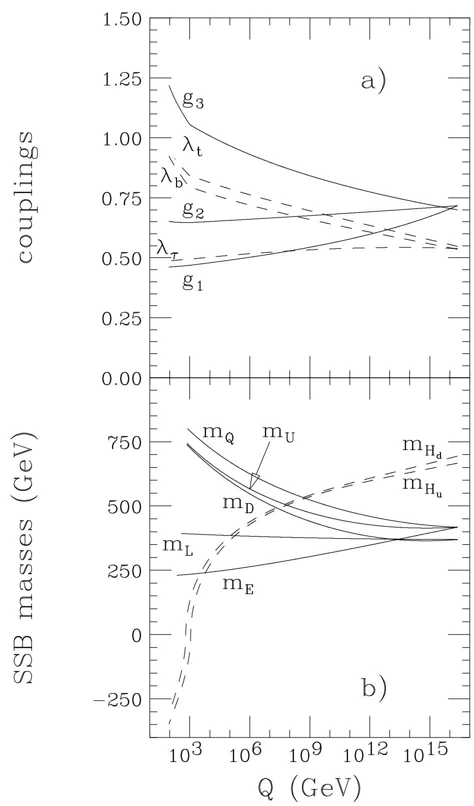

Figure 4. RG flow in a minimal SO(10) model. After ref. [22].

5. Lepton flavour violating signals for SUSY guts

A notable development in the basic picture of SUSY GUTS in the nineties has been the realization [26] that in contrast to non-SUSY theories SUSY GUTS inevitably imply very exotic lepton flavour violating signals suppressed only by inverse powers of the low scale M_S and by CKM mixing angles.

The basic argument is very simple. Firstly in the SM the $B, L_{e,\mu,\tau}$ symmetries are 'accidental' consequences of the gauge symmetries. When the SM is grand unified the fact that GUT gauge symmetries rotate quarks into leptons imples that baryon number and the three lepton numbers are violated. However since they are symmetries of the SM the effects of the violation are represented by effective operators suppressed by inverse powers $M_{\rm GUT}$.

On the other hand, in the MSSM these quantum numbers are no longer automatic and it is necessary to introduce R parity ($R_p = (-)^{3(B-L)+2S}$!) to forbid the violation of B and $L_{\rm total}$ by $d = 4$ operators. *However, since $L = L_e + L_\mu + L_\tau$, this symmetry does not ensure that the individual lepton flavours are conserved*! Therefore, in SUSY GUTs the presence of lepton flavour violation in the soft SUSY breaking sector will inevitably leak into the fermion interaction. Now since this effect must vanish as $M_S \to \infty$ it follows that it is suppressed only by inverse powers of M_S! Even if the soft SUSY breaking terms are flavour blind at M_G (to obey FCNC constraints) the effects of the large top Yukawa communicate the lepton flavour violation to the low energy fermion couplings [26]. Thus, for example, in minimal SUSY SU(5) one finds that the branching ratios for the exotic processes $\mu \to e\gamma, \tau \to \mu\gamma$ are within an order of magnitude of the current experimental limits due to mixing between the different right handed sleptons. As one considers more general non-renormalizable superpotentials with couplings between GUT Higgs and three fermion representations suppressed by the Planck mass (in order to fit the light quark masses [27]) one finds that the left handed sleptons also mix and lead to further exotic processes such as $\mu \to e\gamma$ and electric dipole moments for the neutron. For large values of $\tan\beta$ the rates are further enhanced. Furthemore, when one considers models with a ν^c – as one must to make sense of neutrino masses – left slepton mixing contributions again arise and raise the value of exotic branching ratios such as $\mu \to e\gamma, \tau \to \mu\gamma$.

The importance of these *generic* signals of SUSY unification can hardly be overemphasized. Indeed the thrust of this talk has precisely been that the 'zeroth order' version of lepton flavor violation, namely the effects of neutrino mass, calls naturally for SUSY. It is then gratifying to note that conversely SUSY unification can signal its presence clearly through generic lepton flavour violating effects whose decoupling scale is M_S itself and that these signals are already on the threshold of verifiability/falsifiability with current dectectors. The reader is referred to [1] for the situation regarding proton decay, regarding which we have nothing fresh to relate.

6. Conclusions

To summarize: besides the well established and appreciated unification of representations and gauge couplings achieved by SUSY GUTs, one also now sees that:

- There is a clear logical chain leading from the SM with neutrino mass to the minimal supersymmetric LR models with renormalizable seesaw mechanisms developed in detail recently.

- These MSLRMs have the MSSM with R-parity and seesaw neutrino masses and have quasi exact B–L as their effective low energy theory. They can also have light Higgs triplet supermultiplets in their low energy spectra, due to violations of the survival principle in SUSY theories, leading to very distinctive experimental signatures.
- They can be embedded in GUTS based on the PS group or SO(10). The former case may be more suitable in stringy scenarios which so far disfavor light (on stringy scales) SO(10) GUT Higgs of dimension > 54.
- The SSB in the SO(10) SUSY GUT has been worked out explicitly and the mass spectra calculated. This allowed us to perform a RG analysis based on calculated spectra leading to the conclusion that $M_X \geq 10^{15.5}$ GeV while $M_R, M_{PS} \geq 10^{13}$ GeV. With M_X at its lower limit the $d = 6$ operators for nucleon decay can become competitive with the $d = 5$ operators raising the possibility that observation of $p \rightarrow \pi^0 e^+$ need not rule out SUSY GUTS after all.
- Fermion mass spectra can be compatible with charged fermion mass data and neutrino mass values suggested by neutrino oscillation data from super-Kamiokande and solar neutrino oscillation experiments.
- Dimension five operators in theories with seesaw lead to a remarkable connection between neutrino masses and nucleon decay which constrains these models fairly tightly and makes them testable by upcoming nucleon stability measurements.
- Further work on the doublet triplet splitting problem, the question of fermion mass spectra, two loop RG analysis etc. is required.
- *Thus LR SUSY seesaw models and their GUT generalizations look good. The show has just begun but it aint over till the fat neutrino sings*!

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