

$$L_i = (O_L^T)_{i\alpha} L_\alpha, \quad e_i^c = (O_R^T)_{i\alpha} e_\alpha^c, \quad (6)$$

where O_L is determined by requiring

$$O_L M_l O_R^T = \text{diagonal}. \quad (7)$$

The trilinear lepton number violating couplings get the following form in the new basis:

$$W = -\frac{\tan \theta_3}{\langle H_1 \rangle} [(O_L^T)_{3\alpha} L_\alpha] (m_\beta^l L_\beta e_\beta^c + m_i^D Q_i d_i^c). \quad (8)$$

The soft potential of these models also contains bilinear lepton number violating terms (B_i). But, these terms cannot be removed by the redefinition of the fields due to the inequality of the leptonic and the Higgs mass terms as well as the corresponding B parameters at the weak scale. In general, the B parameters can be typically of $O(M_{\text{SUSY}})$ leading to unacceptably large neutrino mass. A natural way of suppressing this large mass is to consider universal soft masses and B terms at the high scale. Exploiting the universality, the scalar potential in the basis where the bilinear \mathcal{L} terms are rotated away from the superpotential can be written as

$$\begin{aligned} V = & (m_{H_1}^2 + \mu^2)|H_1^0|^2 + (m_{H_2}^2 + \mu^2)|H_2^0|^2 + m_{\tilde{\nu}_3}^2 |\tilde{\nu}_3|^2 + m_{\tilde{\nu}'_2}^2 |\tilde{\nu}'_2|^2 + m_{\tilde{\nu}'_1}^2 |\tilde{\nu}'_1|^2 \\ & - [\mu H_2^0 (B H_1^0 + c_3 s_3 \tilde{\nu}_3 (\Delta B_H - s_2^2 \Delta B_L) - c_2 s_2 s_3 \tilde{\nu}_2 \Delta B_L) + \text{H.c.}] \\ & + [-c_2 s_2 \Delta m_L (s_3 H_1^0 + c_3 \tilde{\nu}_3) \tilde{\nu}_2^* + s_3 c_3 \tilde{\nu}_3 H_1^{0*} (\Delta m_H - s_2^2 \Delta m_L) + \text{H.c.}] \\ & + \frac{1}{8} (g^2 + g'^2) (|H_1^0|^2 + |\tilde{\nu}_1|^2 + |\tilde{\nu}_2|^2 + |\tilde{\nu}_3|^2 - |H_2^0|^2)^2, \end{aligned} \quad (9)$$

where

$$\begin{aligned} m_{H_1}^2 &= m_{H_1}^2 + s_3^2 \Delta m_H - s_2^2 s_3^2 \Delta m_L, \\ m_{\tilde{\nu}_3}^2 &= m_{\tilde{\nu}_3}^2 - s_3^2 \Delta m_H - s_2^2 c_3^2 \Delta m_L, \\ m_{\tilde{\nu}'_2}^2 &= m_{\tilde{\nu}'_2}^2 + s_2^2 \Delta m_L, \\ B &= B_\mu + s_3^2 \Delta B_H - s_2^2 s_3^2 \Delta B_L, \end{aligned} \quad (10)$$

and

$$\Delta m_{L,H} \equiv (m_{\tilde{\nu}'_3}^2 - m_{\tilde{\nu}'_2, H_1}^2), \quad \Delta B_{L,H} \equiv (B_3 - B_{2,\mu}). \quad (11)$$

In writing the above, we have neglected the contributions coming from the first two generation Yukawa couplings. The parameters $\Delta m_{L,H}$ and $\Delta B_{L,H}$ are equal to zero at the high scale, due to the universality condition, but are nonzero at the weak scale due to the differences in RG scaling of the soft terms of the Higgs and the leptons and within the leptonic generations. The relevant RG equations for these parameters given in [8,9,11] show that these parameters are proportional to the down quark and the tau lepton Yukawa couplings.

The terms linear in the sneutrino fields in the soft potential would now generate vacuum expectation values for the sneutrinos. Since, these VEVs are found to be much smaller than the Higgs vacuum expectation values, we can determine them analytically as

$$\begin{aligned}
 \langle \tilde{\nu}_2 \rangle &\approx \frac{c_2 s_2 s_3}{(m_{\tilde{\nu}_2}^2 + D)} (v_1 \Delta m_L - \mu v_2 \Delta B_L), \\
 \langle \tilde{\nu}_3 \rangle &\approx \frac{c_3 s_3}{(m_{\tilde{\nu}_3}^2 + D)} (v_1 (-\Delta m_H + s_2^2 \Delta m_L) - \mu v_2 (-\Delta B_H + s_2^2 \Delta B_L)).
 \end{aligned} \tag{12}$$

As we have neglected the first two generation Yukawa couplings, only two of the sneutrinos attain $v_{\tilde{\nu}s}$. Due to these $v_{\tilde{\nu}s}$, neutrinos now mix with the neutralinos giving rise to a tree level mass to one of the neutrinos. In the limit where the sneutrino $v_{\tilde{\nu}s}$ are much smaller than the Higgs $v_{\tilde{\nu}s}$, one can use the seesaw approximation [7] to determine the tree level neutrino mass matrix. In the charged lepton mass basis this matrix is given as

$$M_0 = m_0 O_L \begin{pmatrix} 0 & 0 & 0 \\ 0 & s_\phi^2 & s_\phi c_\phi \\ 0 & s_\phi c_\phi & c_\phi^2 \end{pmatrix} O_L^T, \tag{13}$$

where O_L has been defined above and

$$\tan \phi = \frac{\langle \tilde{\nu}_2 \rangle}{\langle \tilde{\nu}_3 \rangle} \tag{14}$$

and

$$m_0 = \frac{\mu (c g^2 + g'^2) (\langle \tilde{\nu}_2 \rangle^2 + \langle \tilde{\nu}_3 \rangle^2)}{2(-c\mu M + 2M_W^2 c_\beta s_\beta (c + \tan^2 \theta_W))}. \tag{15}$$

As noted earlier, the tree level mass matrix eq. (13) gives mass to only one of the neutrinos. The other two masses are generated at the one loop level through the loops involving lepton number violating Yukawa couplings of eq. (8). These masses take the following form in the charged lepton mass basis where we have retained only the dominant contribution [9,16]:

$$(M_{1b})_{\alpha\beta} = m_{1b} (O_L)_{\alpha 3} (O_L)_{\beta 3}, \tag{16}$$

$$M_{1\tau} = m_{1\tau} \begin{pmatrix} O_{L_{13}}^2 & O_{L_{13}} O_{L_{23}} & 0 \\ O_{L_{13}} O_{L_{23}} & O_{L_{23}}^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{17}$$

where

$$m_{1b,1\tau} = N_c \frac{m_{\tilde{b},\tau}^3}{16\pi^2 v_1^2} \tan \theta_3^2 \sin \phi_{b,\tau} \cos \phi_{b,\tau} \ln \left(\frac{M_{b_2,\tau_2}^2}{M_{b_1,\tau_1}^2} \right), \tag{18}$$

and $N_c = 3, 1$ for the \tilde{b} and $\tilde{\tau}$. We see that the 1-loop contribution also depends on the down quark and the tau lepton Yukawa couplings. Thus, in the limit where the down quark and the tau lepton Yukawa couplings tend to zero, there will be no neutrino masses in the model. The total mass matrix including the 1-loop corrections is given by

$$\mathcal{M}_\nu = M_0 + M_{1b} + M_{1\tau}. \quad (19)$$

The above mass matrix is written in the mass basis of the charged leptons. It has the following simple form in the basis:

$$O_L^T \mathcal{M}_\nu O_L \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_0 s_\phi^2 + m_{1\tau} N_2^{-4} c_2^2 s_2^2 c_3^2 & m_0 s_\phi c_\phi + m_{1\tau} N_2^{-4} c_2 s_2^3 c_3^3 \\ 0 & m_0 s_\phi c_\phi + m_{1\tau} N_2^{-4} c_2 s_2^3 c_3^3 & m_0 c_\phi^2 + m_{1b} + m_{1\tau} N_2^{-4} s_2^4 c_3^4 \end{pmatrix}. \quad (20)$$

From the above, we see that one of the neutrinos remains massless in the limit of vanishing first two generation Yukawa couplings. The eigenvalues of the lower 2×2 block of eq. (20) give the two neutrino masses in this approximation. One of the eigenvalues is essentially the tree level contribution whereas the other is essentially the loop induced contribution. In general, the tree level contribution dominates over the loop contribution giving rise to the hierarchical pattern in the neutrino masses. Addition of the first two generation Yukawa couplings would give rise to a much smaller mass compared to these two masses [8,9,11].

The total mixing is determined by the product of the matrices,

$$U = O_L O_\nu^T, \quad (21)$$

where O_ν is the matrix diagonalizing the RHS of eq. (20). O_L is basically dependent on the ratios of the R -parity violating parameters whereas, O_ν depends both on R -parity violating parameters and the soft supersymmetry breaking parameters. The contribution to the mixing from O_ν is small unless the loop level mass is comparable to the tree level mass. Thus, if all the R -parity violating parameters are of similar magnitude, large mixing is natural in these models. From the above we see that, the neutrino mass spectrum depends crucially on the soft sector of the theory and hence on the way supersymmetry is broken. We consider here two popular models of supersymmetry breaking namely, minimal supergravity and minimal messenger model (MMM) of gauge mediated supersymmetry breaking and study the neutrino mass spectrum in these scenarios.

2.2 Constrained MSSM

As mentioned earlier, universality at the high scale for the soft parameters leads naturally to small neutrino masses at the weak scale. Constrained MSSM i.e, minimal supergravity with universal boundary conditions at M_{GUT} provides an appropriate framework from this point of view. The total parameters of the above model in the CMSSM framework are the standard CMSSM parameters m , M_2 , $\tan \beta$, A and the $\text{sign}(\mu)$ along with the R -parity violating parameters ϵ_i or the three angles s_1, s_2, s_3 .

It is found that the loop mass $m_{1\tau}$ contributes negligibly to the neutrino mass spectrum, throughout the CMSSM parameter space [8,9]. In the limit $m_{1\tau} \ll m_{1b}, m_0$ approximate expressions for the neutrino masses can be given as [9]

$$\begin{aligned} m_{\nu_3} &\sim m_0 + m_{1b}, \\ m_{\nu_2} &\sim \frac{m_0 m_{1b} s_\phi^2}{m_0 + m_{1b}}, \end{aligned} \quad (22)$$

where the approximation $s_\phi^2 \ll 1$ is used in obtaining the second line of the above. One of the neutrinos remains massless in this approximation. The main features of the neutrino mass spectrum in this framework can be summarized as follows :

- For most of the CMSSM parameter space, the tree level contribution dominates over the loop contribution giving rise to the hierarchical pattern in the mass spectrum [8,9].
- But, there also exist regions in the CMSSM parameter space, where the two contributions to the sneutrino $vevs$, (eq. (12)) cancel each other for one particular sign of μ . In these regions, the loop contribution, m_{1b} can become comparable to the tree level mass, m_0 and cancellations among m_0 and m_{1b} can take place. Here, the two neutrinos form a pseudo-Dirac pair with a common mass $m_0 s_\phi$ relevant for solutions of the neutrino anomalies [9].
- The mixing matrix is given by the product $O_L O_\nu^T$. Even in the case where O_ν allows only small mixing, large mixing can be generated from O_L , as it depends only on the ratios of the R -parity violating parameters [9].
- Even though the neutrino masses are suppressed in these scenarios, they are typically of the $O(\text{MeV})$. Thus, the R -parity violating parameters have to be chosen to be much suppressed compared to the typical order of the supersymmetric breaking scale if neutrino masses are to be of the right order to solve the neutrino anomalies [8,9].
- Numerical results from Hempfling [8] show that solutions for solar (either with MSW conversion or vacuum oscillations) and the atmospheric neutrino anomalies can be accommodated naturally within these models.

2.3 MMM or minimal messenger model

The minimal messenger model of gauge mediated supersymmetry breaking [17] is characterized by the vanishing of B and A parameters at the high scale $X \sim \text{few hundred TeV}$. The mass terms for the leptons and one of the Higgs (H_1) are equal at the high scale, whereas the B_i terms are assumed to vanish, thus giving rise to universality in the soft terms relevant for the neutrino mass spectrum. The framework is more constrained than the CMSSM, as the neutrino mass spectrum is now essentially controlled by only four parameters namely Λ and the three R -parity violating couplings, ϵ_i .

The parameter $s_\phi^2 \ll 1$ over most of the parameter space. The approximate eigenvalues for the neutrinos are given as [11]:

$$\begin{aligned} m_{\nu_3} &\sim m_0 + m_{1b}, \\ \frac{m_{\nu_2}}{m_{\nu_3}} &\sim c_2^2 s_2^2 \frac{m_{1\tau}}{m_0 + m_{1b}}. \end{aligned} \quad (23)$$

The mixing is essentially dictated by O_L as the mixing from O_ν is very small. Thus, we see that the features of the neutrino mass spectrum like hierarchical structure and large mixing continue to stay in this framework also. But, due to the very constrained nature of this framework, it has the following consequences when one tries to find solutions for the neutrino anomalies [11]:

- It has been found that solutions for the both the solar (either with MSW conversion or vacuum oscillations) and atmospheric neutrino problems can be accommodated in this model separately.
- Unlike in the case of CMSSM, there exists no regions in the MMM parameter space where the loop mass becomes comparable to the tree level mass. This gives rise to a strong hierarchy within the mass eigenvalues. Consequently, there is no parameter space where simultaneous solutions for solar (with MSW conversion) and atmospheric neutrino problems can be realized.
- Though the large hierarchy between the mass eigenvalues suits more for simultaneous solutions of solar (vacuum oscillations) and atmospheric neutrino problems, two large mixings are difficult to realize within the allowed parameter space.

It has been pointed out in [11] that a non-minimal version of this model, which allows for a negative μ parameter can reduce the large hierarchy within the mass eigenvalues and in fact one can have regions in the parameter space where simultaneous solutions for the atmospheric and the solar (with MSW conversion) neutrino problems can be realized. These versions of the models are intricately related to models which try to solve the μ problem in gauge mediated supersymmetry breaking scenarios [11].

3. Trilinear R_p violation

Let us now study the structure of neutrino masses in the presence of trilinear lepton number violating couplings in the superpotential. Here, we consider only λ' couplings to be present in the superpotential. We will comment on the presence of λ couplings later. Thus, the R -parity violating part of the superpotential is given as

$$W_{R_p} = \lambda'_{ijk} L_i Q_j d_k^c. \quad (24)$$

The λ' couplings are 27 in number and a priori it appears that there is a lot of arbitrariness in the model. It was pointed out by Drees *et al* [18] that this is not the case and a meaningful study of the neutrino masses in these models can be made if one assumes all the λ' to be of the same order. Though this model has much larger number of parameters, the neutrino mass spectrum has several similar features to the bilinear R -parity violating models.

3.1 The framework

The trilinear R -parity violating couplings also give rise to neutrino masses both at the tree level as well as at the loop level similar to that of bilinear R -parity violating models. This is because even in the presence of only trilinear lepton number violating couplings in the superpotential, bilinear soft lepton number violating terms are generated in the soft potential due to RG scaling at the weak scale [12,19]. Thus, at the weak scale, these couplings are present in the scalar potential given as

$$V = m_{\tilde{\nu}_i}^2 |\tilde{\nu}_i|^2 + m_{H_1}^2 |H_1^0|^2 + m_{H_2}^2 |H_2^0|^2 + [m_{\nu_i H_1}^2 \tilde{\nu}_i^* H_1^0 - \mu B_\mu H_1^0 H_2^0 - B_i \tilde{\nu}_i H_2^0 + \text{H.c}] + \frac{1}{8}(g_1^2 + g_2^2)(|H_1^0|^2 - |H_2^0|^2)^2 + \dots \quad (25)$$

The relevant RG equations which determine the bilinear soft lepton number violating terms at the weak scale are [12,19,20]:

$$\begin{aligned} \frac{dB_i}{dt} &= B_{\epsilon_i} \left(-\frac{1}{2}Y^\tau - \frac{3}{2}Y^t + \frac{3}{2}\tilde{\alpha}_2 + \frac{3}{10}\tilde{\alpha}_1 \right) \\ &\quad - \frac{3}{16\pi^2} \mu h_k^D \lambda'_{ikk} \left(\frac{1}{2}B_\mu + A_{ikk}^{\lambda'} \right), \\ \frac{dm_{\nu_i H_1}^2}{dt} &= m_{\nu_i H_1}^2 \left(-2Y^\tau - \frac{3}{2}Y^b \right) - \frac{3}{32\pi^2} h^D \\ &\quad \lambda'_{ikk} \left(m_{H_1}^2 + m_{L_i}^2 + 2m_{kk}^{2Q} + 2A_{ikk}^{\lambda'} A_{kk}^D + 2m_{kk}^{2D^c} \right). \end{aligned} \quad (26)$$

Since these terms are absent at the M_{GUT} scale and only generated at the weak scale, the solutions of these equations can be represented as

$$\begin{aligned} B_i &= \lambda'_{ipp} h_{ip}^d \kappa_{ip}, \\ m_{\nu_i H_1}^2 &= \lambda'_{ipp} h_{ip}^d \kappa'_{ip}, \end{aligned} \quad (27)$$

where p is summed over the generations and the parameters κ, κ' represent the running of the standard parameters [5] present in the RGE's from the high scale to the weak scale. The above soft potential eq. (25) would now give rise to sneutrino *vevs* given as

$$\langle \tilde{\nu}_i \rangle = \frac{B_i v_2 - m_{\nu_i H_1}^2 v_1}{m_{L_i}^2 + \frac{1}{2}m_Z^2 \cos 2\beta}. \quad (28)$$

As noted earlier, the sneutrino *vevs* so generated will now mix the neutrinos with the neutralinos thus giving rise to a tree level neutrino mass matrix given as

$$\mathcal{M}_{ij}^0 = \frac{\mu(cg^2 + g'^2) \langle \tilde{\nu}_i \rangle \langle \tilde{\nu}_j \rangle}{2(-c\mu M_2 + 2M_w^2 c_\beta s_\beta (c + \tan \theta_w^2))}. \quad (29)$$

We can factorize the λ' couplings in the above using eq. (27) and rewrite it as

$$\mathcal{M}_{ij}^0 = \lambda'_{ikk} h_{kk}^d \lambda'_{jpp} h_{pp}^d m_0. \quad (30)$$

In the above, m_0 contains all the rest of the parameters which are dependent solely on the soft SUSY breaking parameters [21]. The loop induced mass due to the presence of trilinear couplings in the superpotential [16] is given as

$$\begin{aligned} \mathcal{M}_{ij}^{\text{loop}} &= \frac{3}{16\pi^2} \lambda'_{ilk} \lambda'_{jkl} h_l^d \sin \phi_k \cos \phi_k \ln \frac{M_{2k}^2}{M_{1k}^2}, \\ &= \lambda'_{ilk} \lambda'_{jkl} h_{ll}^d h_{kk}^d m_l, \end{aligned} \quad (31)$$

where m_l now parameterizes all the quantities which are dependent on the soft SUSY breaking parameters. We see from the eqs (30), (31) that both the tree level and the loop level neutrino masses are proportional to the down quark Yukawa couplings. Moreover, they almost have a similar structure. The only difference being that the tree level mass

matrix depends only on diagonal (λ'_{ikk}) couplings, where as loop mass matrix contains contributions from both diagonal and non-diagonal ($\lambda'_{ijk}, j \neq k$) couplings. Exploiting this fact, the total mass matrix can be written as

$$\mathcal{M}^\nu = (m_0 + m_l) \lambda'_{ikk} h_{kk}^d \lambda'_{jpp} h_{pp}^d + m_l h_{22}^d h_{33}^d A_{ij}, \quad (32)$$

where

$$A_{ij} = \lambda'_{i23} \lambda'_{j32} + \lambda'_{i32} \lambda'_{j23} - \lambda'_{i22} \lambda'_{j33} - \lambda'_{i33} \lambda'_{j22}. \quad (33)$$

In writing the above, we have neglected contributions coming from $O(h_1^d, h_2^d)$ terms. The first matrix in eq. (32) has only one eigenvalue which is basically equivalent to the tree level contribution. The total mass matrix has the following eigenvalues:

$$\begin{aligned} m_{\nu_1} &\approx m_l h_{22}^d h_{33}^d \delta_1, \\ m_{\nu_2} &\approx m_l h_{22}^d h_{33}^d \delta_2, \\ m_{\nu_3} &\approx (m_0 + m_l)(a_1^2 + a_2^2 + a_3^2), \end{aligned} \quad (34)$$

where

$$\begin{aligned} a_i &= \lambda'_{ikk} h_{kk}^d, \\ \delta_1 &= (c_1^2 A'_{11} - 2c_1 s_1 A'_{12} + s_1^2 A'_{22}), \\ \delta_2 &= (c_1^2 A'_{11} - 2c_1 s_1 A'_{12} + s_1^2 A'_{22}). \end{aligned} \quad (35)$$

The A'_{ij} are functions of λ' given in [19]. From eq. (34) we see that one of the mass eigenvalues is basically the tree level mass and the other two are governed by the loop level contributions. The ratio of the two eigenvalues is given by

$$\begin{aligned} \frac{m_{\nu_2}}{m_{\nu_3}} &\sim \frac{m_l h_{22}^d}{m_0 h_{33}^d} \left(\frac{\delta_2}{\sum_i \lambda'^2_{i33}} \right) \\ &\approx \frac{m_l}{m_0} \frac{m_s}{m_b}. \end{aligned} \quad (36)$$

The second line in the above is under the assumption that all the λ' 's are of the same order. In this limit, we see that the ratio of the mass eigenvalues is completely independent of the R -parity violating parameters and only depends on the soft SUSY breaking parameters. The total mixing is governed by

$$K = U U' = \begin{pmatrix} c_1 c_2 - s_1 s_2 c_3 & s_1 c_2 + c_1 s_2 c_3 & s_2 s_3 \\ -s_2 c_1 - s_1 c_2 c_3 & -s_1 s_2 + c_1 c_2 c_3 & c_2 s_3 \\ s_1 s_3 & -s_3 c_1 & c_3 \end{pmatrix}, \quad (37)$$

where

$$\begin{aligned} s_2 &= \frac{a_1}{\sqrt{a_1^2 + a_2^2}}, \\ s_3 &= \frac{(a_1^2 + a_2^2)^{\frac{1}{2}}}{\sqrt{a_1^2 + a_2^2 + a_3^2}}, \end{aligned} \quad (38)$$

and $\tan 2\theta_1 = (2A'_{12}/A'_{11} - A'_{22})$.

From the above, we see that the mixing is determined by the ratios of the λ' couplings. In the limit, where all the λ' couplings are of the same order large mixing is natural in these models.

3.2 Neutrino anomalies and constrained MSSM

Within the CMSSM framework, the tree level contribution dominates over the loop level contribution for large regions of the parameter space leading to hierarchical pattern for the neutrino mass spectrum. However, similar to the case of bilinear R -parity violating scenarios, for one particular sign of μ , there exists regions in the CMSSM parameter space where the two contributions to the tree level mass cancel each other. In these regions, the loop mass becomes comparable to the tree level mass reducing the hierarchy of the mass spectrum. Thus, large hierarchy and large mixing continue to be the main features of these models just as in the case of models with bilinear lepton number violation. The main implications for solutions of the solar and atmospheric neutrino anomalies are summarized as below:

- The ratio m_l/m_0 which is typically of $O(10^{-3})$ [19] together with m_s/m_b produces the right hierarchy in mass eigenvalues (eq. (36)) for simultaneous solutions for atmospheric and solar neutrino problems (vacuum oscillations). Though, the mixing between all the states is naturally large constraints from the CHOOZ experiment can be accommodated without fine tuning of the parameters [19]. The λ' is typically required to be of the order 10^{-4} .
- Solutions for the solar neutrino problem with MSW conversion (large mixing angle) and atmospheric neutrino problem can be simultaneously accommodated in this model for regions of the CMSSM parameter space where, $m_l \approx m_0$. As noted above, this is possible in the parameter space where the two contributions to the sneutrino vev cancel each other [19].

It should be noted that attempts have been made to understand the neutrino anomalies in these models without including the tree level contribution [18]. In such a case, the hierarchy of the mass eigenvalues (eq. (36)) is dependent only on m_s/m_b , which is in the right range for simultaneous solutions of the solar ν problem (MSW conversion) and the atmospheric neutrino problem. Addition of the tree level contribution increases this hierarchy making it more natural to find regions in parameter space where simultaneous solutions for the solar neutrino (with vacuum oscillations) and atmospheric ν problems can be found. Moreover, since large mixing is natural in these models, additional assumptions on the structure of λ' couplings viz., discrete symmetries [18] or hierarchical [22] patterns are required if one looks for solutions for solar neutrino problem with small angle MSW conversion. An analysis of the appropriate values for the products of the trilinear couplings required to explain the existing neutrino anomalies and a comparison with the existing bounds from other experiments has been reported in [23].

We have so far concentrated on λ' couplings alone. The same discussion will go through if one considers λ couplings in the superpotential. The tree level mass would continue to dominate over the loop mass, but the structure of mixing matrix would be different from here due to the antisymmetry of the λ couplings.

4. Discussion

We have considered here supersymmetric theories with both bilinear and trilinear lepton number violation separately and studied the structure of neutrino masses and mixing in

these models. Natural large mixing and hierarchical masses are the main features of the neutrino mass spectrum in these models. The main results are:

- The bilinear soft \mathcal{L} terms play an important role in determining the neutrino mass spectrum in both the bilinear and trilinear lepton number violation scenarios. They give rise to sneutrino $vevs$ which then lead to the generation of tree level mass to the neutrino. Since this mass dominates over the loop mass for most of the parameter space, these terms play a key role in determining the hierarchy of the mass spectrum.
- The mixing is essentially controlled by ratios of the R -parity violating parameters. Large mixing can be typically achieved in both trilinear and the bilinear R -parity violating scenarios, by choosing the R -parity violating parameters to be of the same order in magnitude.
- Assuming universal boundary conditions at the SUSY breaking scale gives rise to small neutrino masses. In these cases, the neutrino masses are proportional to bottom and the tau Yukawa couplings. In the limit where these Yukawa couplings tend to zero, neutrino masses vanish in these models. Additional suppression is required through R -parity violating parameters, if the masses are required to be in the right range to explain the neutrino anomalies.
- In the CMSSM framework, there exists regions in the parameter space where the two contributions to the sneutrino vev cancel each other. In these regions, the loop level contribution becomes comparable to the tree level contribution. In the bilinear R_p violating scenario, this leads to the formation of a pseudo-Dirac pair, whereas in the trilinear R_p violating scenario, this reduces the hierarchy between the masses, making it possible to incorporate simultaneous solutions for MSW and atmospheric ν problems.
- The CMSSM framework with bilinear R -parity violation provides a natural model where solutions for solar and atmospheric neutrino problems can be realized. Simultaneous solutions for solar and atmospheric neutrino problems are difficult to realize in much more constrained framework of MMM of gauge mediated models with bilinear R -parity violation, though one can solve them separately.
- In the CMSSM framework with trilinear lepton number violation, it is more natural to find parameter space where simultaneous solutions for the solar (with vacuum oscillations) and atmospheric neutrino problems can be realized. Simultaneous solutions for solar (MSW, large angle) and atmospheric ν problems can be found in regions where the tree level mass is comparable to the loop level mass. To incorporate MSW conversion (small angle) one has to resort to additional conditions on the λ' couplings such as discrete symmetries or hierarchical patterns.

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