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Facets of tripartite entanglement

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Abstract. Tripartite entangled states of systems 1, 2 and 3 involving nonorthogonal states are used to reveal two hitherto unexplored quantum effects. The first shows that kinematic entanglement between the states of 1 and 2 can affect the result of dynamical interaction between 2 and 3, though 1 and 2 may be spatially separated so that they no longer interact. The second shows that if a residual interaction persists between 1 and 2 while 2 interacts with 3 to form an entangled state, the measurement of observables of 1 can be used to determine whether 2 has interacted with 3. This effect occurs even when the measurement on 1 is made long after the residual interaction between 1 and 2 has ceased to act. Such effects resulting from interplay between unitary dynamics and kinematic entanglement have interesting implications. In particular, we discuss the significance as regards what we call the dynamic version of Einstein locality.

Keywords. Tripartite entanglement; nonorthogonality; unitarity; residual interaction; Einstein locality.

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1. Introduction

Two or more quantum systems are said to be in an entangled or a nonfactorisable state if their joint wave function is *not* expressible as a product of individual wave functions but is instead a superposition of product states. The term entanglement was first introduced by Schroedinger [1] who described this as *the* characteristic trait of quantum mechanics - 'the one that enforces its entire departure from classical lines of thought.'

The demonstrations of quantum nonlocality require systems to be prepared in entangled states. Quantum mechanical correlations between results of measurments on entangled systems show testable violations of constraints derived from what is known as 'Einstein locality' or 'local realism'. Such violations have the counterintuitive implication that an individual outcome of a measurement on any one system cannot be specified independently of the parameters of measurements on its entangled partners, even if these spatially separated systems cease to interact once they are prepared in the entangled state [2].

Here we investigate the possibility of uncovering some new effects of quantum entanglement. The salient features are as follows:

(a) Discussions of quantum entanglement are usually based on *correlations* between measurements performed on the entangled systems. Thus it is only the *kinematic* component of the theory that is used in such discussions. In contrast, while the examples formulated in this work use tripartite entangled states of 1, 2 and 3 (referred to as *entangled entanglements* [3] since in such cases entanglement between any two systems is itself an entangled property), they also involve *dynamical* evolutions in a nontrivial way.

(b) *Global* aspects of entanglement in usual examples are manifested in terms of joint measurements on all the systems that are entangled. However, in the examples discussed here, the globality of entanglement is discernible by measurements confined to one sector *alone* (more specifically, *either* by measurements on system 1 or on systems 2 and 3).

(c) The use of entangled entanglements involving *nonorthogonal* states of the subsystems which make up the entangled states is a key ingredient in our examples. Here we may note that entangled nonorthogonal states have recently been used for showing violation of Bell's inequality [4]. Entanglements of nonorthogonal states are particularly relevant in the context of entangled coherent states [5].

(d) A key feature of our treatment is the consideration of *linearity* and *unitarity* requirements pertinent to the preparation of entangled entanglements of a tripartite system. This is viewed as a sequence of dynamical evolutions starting from entangled states of bipartite systems. The entangled entanglements thus prepared are used to exhibit their inherently *global* nature.

In essence, the effects pointed out are :

A. Joint properties of two systems, say, 2 and 3, once they have interacted, are shown to *depend* on *whether* the states of 2 are entangled with the states of a distant system, say, 1 with which 2 may have interacted earlier but with which 2 is presently *noninteracting.*

B. After 1 and 2 are prepared in an entangled state, let 2 interact with a third system 3. Then such an interaction can affect the expectation value of an observable of the system 1, even though system 1 gets widely separated from 2. This happens *provided a residual interaction* acts between the systems 1 and 2 *while* 2 interacts with 3; actual strength of this residual interaction is not relevant.

A particularly striking feature of the effect B is that it enables information about inner product between the states of 3 entangled with states of 2 and 1 to be transferred to an observer at a remote location who makes measurement on 1. This information transfer between 3 and 1 occurs although 3 has not interacted with 1 at any stage. This information is not sent via any direct or a classical channel and at the same time does not involve faster-than-light signalling because both residual interaction between 1 and 2 and the kinematic entanglement between 1, 2, and 3 are needed.

We begin by deriving some relevant theorems constraining the preparation of entangled entanglements involving mutually non-orthogonal states.

2. Two theorems

Let the systems 1 and 2 be initially prepared in an entangled state

$$\Psi_0(1,2) = a\psi_+(1)\phi_-(2) + b\psi_-(1)\phi_+(2), \tag{1}$$

where, in general, neither $\langle\psi_+(1) \mid \psi_-(1)\rangle = \gamma_1$ nor $\langle\phi_+(2) \mid \phi_-(2)\rangle = \gamma_2$ need to vanish.

System 2 is then spatially separated from 1 and made to interact locally with system 3. The initial state of the combined system of 1, 2 and 3 is given by

$$\Psi_i(1,2,3) = \Psi_0(1,2)\chi_0(3), \tag{2}$$

where $\chi_0(3)$ is the initial state of system 3. Starting from $\Psi_i(1,2,3)$ and allowing the system to evolve, let an entangled entanglement be prepared of the form

$$\Psi_f(1,2,3) = a\psi_+(1)\phi_-(2)\chi_+(3) + b\psi_-(1)\phi_+(2)\chi_-(3), \tag{3}$$

where $\langle\chi_+(3) \mid \chi_-(3)\rangle = \gamma_3$ also need not vanish.

Let the *total* evolution from $\Psi_i(1,2,3)$ to $\Psi_f(1,2,3)$ be represented by a unitary linear operator U so that

$$U\psi_+(1)\phi_-(2)\chi_0(3) = \psi_+(1)\phi_-(2)\chi_+(3) \tag{4}$$

and

$$U\psi_-(1)\phi_+(2)\chi_0(3) = \psi_-(1)\phi_+(2)\chi_-(3), \tag{5}$$

whence due to the *linearity* of U

$$U\Psi_i(1,2,3) = \Psi_f(1,2,3),$$

where Ψ_i and Ψ_f are given by eqs (2) and (3) respectively.

From eqs (4) and (5) by taking the inner products of both sides and requiring the unitarity of U we get

$$\gamma_1\overline{\gamma_2} = \gamma_1\overline{\gamma_2}\gamma_3. \tag{6}$$

Now an important point is that even if the global unitarity condition as given by eq. (6) is satisfied, the final state *cannot* be prepared for the particular case $\gamma_1 = 0$, $\gamma_2, \gamma_3 \neq 0$ if 1 and 2 have *ceased* to interact *before* 2 interacts with 3. This prohibition arises because the total unitary evolution after the interaction between 1 and 2 ceases to act has the factorisable decomposition $U = U(1) \otimes U(2,3)$ where $U(1)$ acts in the Hilbert space $H(1)$ while $U(2,3)$ acts in the tensor product Hilbert space $H(2) \otimes H(3)$. This implies that $U(2,3)$ is required to be unitary *in addition* to the unitarity of total U, which in turn leads to the above mentioned constraint. To see this let $U(2,3)$ be such that

$$U(2,3)\phi_-(2)\chi_0(3) = \phi_-(2)\chi_+(3) \tag{7}$$

and

$$U(2,3)\phi_+(2)\chi_0(3) = \phi_+(2)\chi_-(3) \tag{8}$$

which lead to

$$U(2,3)(\phi_-(2) + \phi_+(2))\chi_0(3) = \phi_-(2)\chi_+(3) + \phi_+(2)\chi_-(3) \tag{9}$$

by the linearity of $U(2,3)$. Now, using the unitarity of $U(2,3)$ we get from (7) and (8) that

$$\overline{\gamma_2} = \overline{\gamma_2}\gamma_3 \tag{10}$$

which cannot be satisfied for *both* $\gamma_2, \gamma_3 \neq 0$. This is true irrespective of whether γ_1 vanishes or not.

Nevertheless, the preparation of the state (3) with $\gamma_1 = 0$, $\gamma_2, \gamma_3 \neq 0$ becomes allowed *if* a *residual interaction* acts between 1 and 2 *while* 2 interacts with 3. This residual interaction could either be the 'tail' of the interaction generating the entangled state $\Psi_0(1,2)$ or some other interaction such as a long-range Coulomb interaction. As long as this residual interaction between 1 and 2 does not vanish during the interaction between 2 and 3, the unitarity of $U(2,3)$ is *not* required as an additional constraint. It is *only* the global unitarity condition (6) which needs to be satisfied.

The above considerations are now summarised in the form of following theorems:

Theorem 1. Consider that the preparation of the state $\Psi_f(1,2,3)$ given by eq. (3) proceeds from $\Psi_i(1,2,3)$ given by (2) in which 1 is entangled with 2 but 3 does not interact with either 1 or 2. Subsequently, 2 is spatially separated from 1 and made to interact with 3 *after* 1 and 2 are ensured to be mutually *noninteracting*. Then the preparation of $\Psi_f(1,2,3)$ is *not* allowed by the unitarity requirements if *both* γ_2, $\gamma_3 \neq 0$. If *either* γ_1 or γ_2 is nonzero, the preparation of $\Psi_f(1,2,3)$ is allowed by the unitarity considerations *provided* that the other one vanishes. This is independent of whether γ_3 vanishes.

Theorem 2. Next consider the situation where starting from $\Psi_i(1,2,3)$ given by (2) system 2 interacts with 3 *while* there is still a *residual interaction* persisting between systems 1 and 2. Under this condition, the preparation of $\Psi_f(1,2,3)$ given by (3) where both γ_2 and γ_3 are nonzero, is allowed in conformity with the global unitarity requirement *if* γ_1 vanishes.

The above theorems will now be used to demonstrate the following curious features of quantum entanglement.

3. The new effects

A. The entangled pure state is composed of pairs of basis states for the subsystems with $\gamma(1) \neq 0$, $\gamma(2) = 0$, and $\gamma(3)$ can be zero or not. Here we show that kinematic entanglement between the states of spatially separated and mutually non-interacting systems 1 and 2 affects the outcome of local interaction between 2 and 3, even though the same unitary operator $U'(2,3)$ governs the interaction between 2 and 3 whether systems 1 and 2 are in an entangled state or not. The effect is manifested through any physical quantities $\alpha(2)$ and $\beta(3)$ that are not diagonal in the prepared bases for 2 and 3. The expectation value of $\alpha(2) \otimes \beta(3)$ depends on $\gamma(1)$ when 1 and 2 had previously been in an entangled state. Our argument runs as follows:

If the initial state of system 3 is χ_0 (taken to be normalised), then let the interaction between 2 and 3 be represented by $U'(2,3)$ which is chosen such that

$$U'(2,3)(\phi_-(2)\chi_0(3)) = \phi'_-(2)\chi'(3), \tag{11}$$

$$U^{'}(2,3)(\phi_{+}(2)\chi_{0}(3)) = \phi^{'}_{+}(2)\chi^{''}(3), \tag{12}$$

where

$$\langle\phi_{-}(2) \mid \phi_{+}(2)\rangle = \langle\phi^{'}_{-}(2) \mid \phi^{'}_{+}(2)\rangle\langle\chi^{'}(3) \mid \chi^{''}(3)\rangle. \tag{13}$$

Now, suppose that system 2 is initially prepared in the pure state ϕ_0 given by

$$\phi_0(2) = c\phi_{-}(2) + \mathrm{d}\phi_{+}(2), \tag{14}$$

where $\langle\phi_{-}(2)|\phi_{+}(2)\rangle$=0. Then starting from the initial state $\phi_0(2)\chi_0(3)$, using $U^{'}(2,3)$ and satisfying the unitary requirement (13), the entangled state

$$\Psi_f(2,3) = U^{'}(2,3)\phi_0(2)\chi_0(3) = c\phi^{'}_{-}(2)\chi^{'}(3) + \mathrm{d}\phi^{'}_{+}(2)\chi^{''}(3), \tag{15}$$

where either $\langle\phi^{'}_{-}(2) \mid \phi^{'}_{+}(2)\rangle$ or $\langle\chi^{'}(3) \mid \chi^{''}(3)\rangle$ or both of them vanish.

Subsequently, calculating the expectation value of a product of two dynamical variables, say, $\alpha(2)$ and $\beta(3)$ pertaining to the systems 2 and 3 respectively, one obtains with respect to the state $\Psi_f(2,3)$given by (15)

$$\begin{aligned}\langle\alpha(2)\beta(3)\rangle = {} & \mid c \mid^2 \langle\phi^{'}_{-}(2) \mid \alpha(2) \mid \phi^{'}_{-}(2)\rangle\langle\chi^{'}(3) \mid \beta(3) \mid \chi^{'}(3)\rangle \\ & + \mid d \mid^2 \langle\phi^{'}_{+}(2) \mid \alpha(2) \mid \phi^{'}_{+}(2)\rangle\langle\chi^{''}(3) \mid \beta(3) \mid \chi^{''}(3)\rangle \\ & + 2\mathrm{Re}\, \overline{c}d\langle\phi^{'}_{-}(2) \mid \alpha(2) \mid \phi^{'}_{+}(2)\rangle\langle\chi^{'}(3) \mid \beta(3) \mid \chi^{''}(3)\rangle. \end{aligned} \tag{16}$$

By comparison, consider that *before* interacting with 3, system 2 gets entangled with 1 resulting in their combined state being given by

$$\psi^{'}_{i}(1,2) = c\psi_{+}(1)\phi_{-}(2) + \mathrm{d}\psi_{-}(1)\phi_{+}(2), \tag{17}$$

where $\langle\psi_{+}(1) \mid \psi_{-}(1)\rangle \neq 0$ but $\langle\phi_{-}(2) \mid \phi_{+}(2)\rangle = 0$. Note that (17) can be prepared from $\phi_0(2)$ given by (14) and $\psi_0(1)$ by a linear unitary operator $U_0(1,2)$ such that

$$U_0(1,2)\phi_{-}(2)\psi_0(1) = \psi_{+}(1)\phi_{-}(2), \tag{18}$$

$$U_0(1,2)\phi_{+}(2)\psi_0(1) = \psi_{-}(1)\phi_{+}(2). \tag{19}$$

Subsequent to the preparation of the state (17), assume that 1 and 2 have ceased to interact, then let 2 interact locally with 3 which is initially in the state $\chi_0(3)$. Using $U'(2,3)$ given by (11) and (12), the unitary considerations still permit the preparation of an entanglement between the states of 2 and 3 similar to that given by (15) but with the crucial difference that now the entangled state of 2 and 3 is itself entangled with the states of 1, the composite state being given by

$$\Psi^{'}_{f}(1,2,3) = U^{'}(2,3)\psi^{'}_{i}(1,2)\chi_0(3), \tag{20}$$

whence using (17), (11), (12) we get

$$\Psi^{'}_{f}(1,2,3) = c\psi_{+}(1)\phi^{'}_{-}(2)\chi^{'}(3) + \mathrm{d}\psi_{-}(1)\phi^{'}_{+}(2)\chi^{''}(3). \tag{21}$$

Note that *Proposition* 1 allows preparation of the state Ψ'_f given by (21) for $\langle\psi_+(1) \mid \psi_-(1)\rangle \neq 0$ if either $\langle\phi'_-(2) \mid \phi'_+(2)\rangle$ or $\langle\chi'(3) \mid \chi''(3)\rangle$ or both of them vanish.

Now, if the expectation value of $\alpha(2) \otimes \beta(3)$ is measured in the state given by (21), one obtains

$$\begin{aligned}\langle\alpha(2)\beta(3)\rangle = {} & \mid c \mid^2 \langle\phi'_-(2) \mid \alpha(2) \mid \phi'_-(2)\rangle\langle\chi'(3) \mid \beta(3) \mid \chi'(3)\rangle \\ & + \mid d \mid^2 \langle\phi'_+(2) \mid \alpha(2) \mid \phi'_+(2)\rangle\langle\chi''(3) \mid \beta(3) \mid \chi''(3)\rangle + 2\mathrm{Re}\, \bar{c}d \\ & + \langle\phi'_-(2) \mid \alpha(2) \mid \phi'_+(2)\rangle\langle\chi'(3) \mid \beta(3) \mid \chi''(3)\rangle \\ & \quad \langle\psi_+(1) \mid \psi_-(1)\rangle. \end{aligned} \tag{22}$$

That the outcome of a local interaction between 2 and 3 has been affected is clear from the difference between the expectation values (16) and (22), even though the *same* $U'(2,3)$ operates on 2 and 3. Note that in both these cases system 2 is prepared in the *same* initial state $\phi_0(2)$ given by (14) and finally 2 interacts with 3. However, in one of these cases, in-between these two events there is an interaction between 1 and 2 which produces an entangled state of 2 and 1. Though this *interaction* ceases *before* 2 interacts with 3, the difference between (16) and (22) shows up in the interference term which in (22) is multiplied by an additional factor given by inner product between the states of 1.

Provided that 1, 2, and 3 are in a tripartite entangled state, it is thus possible to obtain information about the states of 1 through local measurements on 2 and 3, no matter how far they are from 1. The result of interaction between 2 and 3 retains a signature of the kinematical entanglement of 1, 2, and 3 even when 1 is no longer interacting with either 2 or 3. A straightforward calculation shows that the same signature is present in the reduced density matrix for 2 and 3.

Such a dependence of the local dynamical behaviour of 2 and 3 on their global entanglement with the states of a far away system has *no* analogue in the case of classical correlated systems. Thus this reveals a *nonclassical* aspect of quantum entanglement involving kinematic as well as dynamic interdependence of systems.

B. The two systems 1 and 2 have been initially prepared in an entangled state $\Psi_0(1,2)$ given by

$$\Psi_0(1,2) = a\psi_+(1)\phi_-(2) + b\psi_-(1)\phi_+(2), \tag{23}$$

where $\langle\psi_+(1) \mid \psi_-(1)\rangle = \gamma_1 = 0$, $\langle\phi_+(2) \mid \phi_-(2)\rangle = \gamma_2 \neq 0$. We analyse what happens if the system 2 is now allowed to interact with 3, where the inner product $\gamma(3) \neq 0$.

The key difference between cases A and B is that in case A systems 1 and 2 are no longer interacting when the interaction between 2 and 3 commences, thus, the dynamical evolution of 2 and 3 is entirely independent of system 1. However, in case B we assume that a *residual interaction* persists between 1 and 2 while 2 interacts with 3. Then it is *only* the global unitarity of total U that is relevant. Thus Proposition 2 can be used, viz. that an entangled entanglement of the form (3) can be prepared with $\gamma_1 = 0$, $\gamma_2, \gamma_3 \neq 0$. Of course, once this form of entangled entanglement is prepared, systems 1 and 2 may separate so that they become non-interacting.

Now, consider the measurement of an observable quantity $Q(1)$ associated with system 1 which is represented by a Hermitian operator acting in the Hilbert space $H(1)$. The expectation value of $Q(1)$ calculated for the state $\Psi_f(1,2,3)$ given by (3) is as follows:

$$\begin{aligned}\langle Q(1)\rangle = &\mid a\mid^2 \langle\psi_+(1)\mid Q(1)\mid\psi_+(1)\rangle \\ &+\mid b\mid^2 \langle\psi_-(1)\mid Q(1)\mid\psi_-(1)\rangle \\ &+ 2\mathrm{Re}\,\overline{a}b\gamma_2\overline{\gamma_3}\langle\psi_+(1)\mid Q(1)\mid\psi_-(1)\rangle,\end{aligned} \tag{24}$$

where $\gamma_2 = \langle\phi_+(2)\mid\phi_-(2)\rangle$, $\gamma_3 = \langle\chi_+(3)\mid\chi_-(3)\rangle$.

On the other hand, if 2 does *not* interact with 3, the expectation value of $Q(1)$ is calculated in the two-system entangled state $\Psi_0(1,2)$ given by (23). Then one gets

$$\begin{aligned}\langle Q(1)\rangle = &\mid a\mid^2 \langle\psi_+(1)\mid Q(1)\mid\psi_+(1)\rangle \\ &+\mid b\mid^2 \langle\psi_-(1)\mid Q(1)\mid\psi_-(1)\rangle \\ &+ 2\mathrm{Re}\,\overline{a}b\gamma_2\langle\psi_+(1)\mid Q(1)\mid\psi_-(1)\rangle.\end{aligned} \tag{25}$$

Comparing (24) with (25) it is seen that *whether* 2 has interacted with 3 can be found out by measurement of $\langle Q(1)\rangle$ on system 1, provided $\langle\psi_+(1)\mid Q(1)\mid\psi_-(1)\rangle \neq 0$. The difference between (25) and (24) is proportional to $\mathrm{Re}\,\overline{a}b\gamma_2(1-\overline{\gamma_3})$. This means that *if* 2 interacts with 3, the information concerning γ_3, the inner product between the states of 3, can be transferred to a distant observer performing a measurement of $\langle Q(1)\rangle$ on system 1. Such information is discernible by any observer measuring $\langle Q(1)\rangle$ who already knows about the preparation of the initial state $\Psi_0(1,2)$.

While this form of 'communication' does *not* require any external channel and occurs essentially via the three-system entangled state, the *necessity* of a residual interaction acting between the systems 1 and 2 while 2 evolves by interacting with 3 is rather crucial. An important feature is that once the required entangled entanglement has been prepared, $Q(1)$ may be measured on 1 *well after* 1 has been spatially separated from 2. Thus this effect is discernible by measurements on 1 *even when* 1 is no longer interacting with 2. In other words, information about inner product between the states of 3 may be transferred to a distant observer making measurements on 1 even if 1 is dynamically independent of both 2, 3 and although at *no* earlier stage 1 had interacted with 3.

We shall now argue that the form of information transfer entailed by the effect B signifies violation of what may be called a dynamical version of Einstein locality.

4. Quantum violation of dynamic Einstein locality

A succinct statement of Einstein locality, in the words of Einstein, is: '.......... the real factual situation of the system S_2 is independent of what is done with the system S_1 which is spatially separated from the former' [6]. While explaining the root of his faith in such a condition, Einstein had remarked: 'If this axiom were to be completely abolished, the idea of the existence of (quasi) enclosed systems, and thereby the postulation of laws which can be checked empirically in the accepted sense would become impossible' [7].

It is interesting that motivation underpinning Einstein locality is *not* the relativistic requirement of no faster-than-light signalling but rather a consideration related to a methodological principle. Einstein's statements concerning the locality condition do not invoke the notion of spacelike separation. Instead the notion of 'isolation' or 'independence' of a system constitutes the central theme. Now note that *implicit* in Einstein locality is the following assumption:

Any effect on a system induced by an action-at-a-distance that is mediated by an interaction with any other system falls off with decreasing strength of the interaction so that if spatial separation between two systems is ensured to be large compared to the range of interaction between them, any one can be considered (in some limiting way) to be 'isolated' or 'independent' of the other.

It is in the above sense that the phrases '(quasi) enclosed system' and 'spatial separation' occurring in Einstein's relevant statements need to be interpreted. This is brought out explicitly by what we call *dynamic Einstein locality*.

If an action on a system is mediated via an interaction, magnitude of the induced effect can be made *arbitrarily small* by reducing the relevant interaction's strength.

Any 'local' model (with or without realist import) is usually taken to satisfy the above feature. On the other hand, there is a more common form of the locality condition that may be called *kinematic version of Einstein locality*:

No action-at-a-distance can occur for *non-interacting* systems although they may be correlated or their states may be entangled.

It is the above version that Bell alludes to while deriving his famous inequality: 'It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a *distant* system with which it has interacted in the *past*, that creates the essential difficulty' [8]. Note that this is distinct from the condition Bell later called 'local causality' which pertains essentially to spacelike separated measurements [9].

The formalism of quantum mechanics satisfies kinematic Einstein locality for the observable statistical results *(signal locality)* but is *incompatible* with the kinematic version at *individual level* if the result of an individual measurement is assumed to be specified, either deterministically or stochastically ('realist' condition). This latter feature is known as the quantum violation of locality in conjuction with the notion of 'realism'.

What the effect B discussed in §2 shows is the quantum violation of dynamic Einstein locality *(without* any 'realist' condition) at the level of statistical results. This is because for the entanglement mediated action-at-a-distance entailed by the effect B, it suffices that residual interaction between 1 and 2 be finite and nonnegligible compared to the interaction between 2 and 3. Actual strength of the residual interaction has no bearing on the effect, although in the absence of any residual interaction the preparation of tripartite pure state entanglement required for this effect is ruled out by the local unitarity requirement.

The effect B is thus essentially *nonclassical* and *nonlocal* that is mediated by quantum entanglement and that requires the presence of an interaction but does *not* depend on its strength. This may therefore be viewed as quantum nonlocality exhibited by a tripartite entanglement involving nonorthogonal pairs of states prepared consistent with the relevant unitarity conditions.

The fact that the effect B is *independent* of the strength of residual interaction, however, raises the vexed question as to 'how' within the theory one can unambiguously specify the *cut-off* separation between 1 and 2 beyond which they can be regarded dynamically independent. In other words, it needs to be made precise at which stage the preparation of tripartite entanglement giving rise to the effect B becomes forbidden by the local unitarity requirement.

5. Conclusion

The curious features of tripartite pure state entanglement revealed in this work suggest that studies on entanglement can be enriched by including the possibility of entangled nonorthogonal states, together with the consideration of dynamical evolutions that are subjected to unitarity requirements which constraint the preparation of entangled states. In particular, ramifications of the effect B call for more probing in the context of a specific dynamical model of interaction giving rise to the entanglement between orthogonal and nonorthogonal pairs of states required for this effect. Investigation to this end is in progress [10].

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