PRAMANA
— journal of
physics

Vol. 58, No. 2
February 2002
pp. 285–292

Quantum spin-glass transition in the two-dimensional electron gas

SUBIR SACHDEV
Department of Physics, Yale University, P.O. Box 208120, New Haven, CT 06520-8120, USA

Abstract. We discuss the possibility of spin-glass order in the vicinity of the unexpected metallic state of the two-dimensional electron gas in zero applied magnetic field. An average ferromagnetic moment may also be present, and the spin-glass order then resides in the plane orthogonal to the ferromagnetic moment. We argue that a quantum transition involving the destruction of the spin-glass order in an applied in-plane magnetic field offers a natural explanation of some features of recent magnetoconductance measurements. We present a quantum field theory for such a transition and compute its mean field properties.

Keywords. Spin glasses; quantum phase transition; ferromagnetism; electron gas.

PACS Nos 73.40.-c; 71.30.+h; 75.10.Jm; 75.10.Nr

The pioneering experiments of Kravchenko, Pudalov, Sarachik, and others [1] have demonstrated that the low density electron gas in two dimensions (2DEG), in the absence of an applied magnetic field, exhibits some fascinating strong correlation physics. The initial indications of this new regime came from an unexpected decrease in the resistivity as the temperature was lowered on the electrons in a silicon inversion layer. Experimentally it is of course impossible to definitively rule out that there will be an eventual upturn of this resistivity at some exponentially small temperature, and that the true ground state is always an insulator. Nevertheless, the single-electron-transistor measurements of Ilani *et al* [2] show that there is a remarkable qualitative change in the fluctuations of the local compressibility at the same densities, and there must be a corresponding transition in the quantum ground state. A description of this correlation-induced transformation remains an important theoretical challenge.

Useful guidance is obtained from numerical studies of the disorder-free 2DEG at the same densities. These indicate that the system is not too far from a Wigner crystallization transition, and that non-trivial spin correlations are present [3]. A study of an effective spin Hamiltonian [4,5] for the Wigner crystal ground state shows instabilities to ferromagnetism [6] and to a spin singlet ground state which is an attractive candidate for fractionalization.

Turning to the disordered 2DEG, we can reasonably expect that there will be regions of the sample where the system is locally ferromagnetic [7]. However, in other regions the ring exchange terms may prefer a spin singlet state, and these should couple the different ferromagnetic regions so that they align in different directions. On these grounds, we propose here that the ground state has spin-glass order in the vicinity of the transition in the conductivity, possibly with a concomitant average ferromagnetic moment; if the ferromagnetic moment is non-zero, the spin-glass order resides in a plane (in spin space) orthogonal to the direction of the moment. We expect that the spin-glass character is stronger on the metallic side, while the ferromagnetic character of the ground state becomes stronger as the density of electrons is reduced, and this is partially responsible for the decrease in the conductivity. Other arguments for the proximity of the spin-glass order, which were based directly on the theory of disordered metal, were presented in [8].

Our proposal is also motivated by the recent experimental studies of the magnetization of 2DEG [9–11]. In particular, the measurements by Vitkalov *et al* [9] in an in-plane ('parallel') magnetic field, H, indicate that there is a well-defined critical field H_σ, dependent on the density of 2DEG, at which the magnetoconductance displays a kink-like feature, and which they identify with a quantum phase transition. For a clean 2DEG it is natural to interpret H_σ as a saturation field, above which all electrons have their spin polarized in the direction of the applied field. In a disordered 2DEG there is no reason for all regions of the sample to respond in the same manner, and there will always be local configurations in which the electrons prefer to form low spin clusters at these relatively weak fields. Naively, one would then conclude that the magnetization of a disordered 2DEG should evolve smoothly as a function of the applied field, without non-analytic structure at any critical field H_σ. A simple way out of this dilemma is to search for some other order parameters associated with the preserved symmetry of spin rotations about the direction of the applied field. Spin-glass order in the plane perpendicular to the magnetic field is our proposed candidate: this order is present for $H < H_\sigma$, but vanishes for $H > H_\sigma$ when the polarization of many spins along the direction of the applied field reduces their 'canting' enough to disrupt temporal memory of their orientation in the orthogonal plane.

(The discussion of this present paper will restrict attention to the case where $H_\sigma > 0$. Experimentally, it is known that H_σ decreases as the density of electrons decreases, and a possible reason for this is the increase in the spontaneous ferromagnetic moment which acts as an effective magnetic field. It is possible that there is a density at which H_σ reaches zero: this corresponds to a transition at $H = 0$, driven by tuning the density, between a ferromagnetic state with spin-glass order in the transverse plane and an ordinary ferromagnet. The theory for this density-tuned transition will be similar to the field-tuned transition described below, but will not be presented here: it follows by combining the results below with those of [12–14].)

The presence of a critical point at $H = H_\sigma$ also leads to a natural explanation of another puzzling feature of the data. For the density at which H_σ was small, it was found that the characteristic H-width of the magnetoconductance feature scaled roughly as the absolute temperature, $\sim k_{\rm B}T$. (We have absorbed a factor of $g\mu_{\rm B}$ in the definition of H, where $\mu_{\rm B}$ is the Bohr magneton and g is the g-factor of electrons in the silicon conduction band.) A trivial system whose thermodynamics depends only on H/T are the isolated free spins. However, given the large exchange interaction energies between the electrons, it appears extremely unlikely that there is a sufficient density of isolated free moments to lead to a significant change in the magnetoconductance. Indeed there is a 'catch-22' here: if the moments are really isolated enough to behave as free spins, their coupling to the itinerant electrons is weak and they have a negligible effect on the conductance. The presence of a spin-glass quantum critical point at $H = H_\sigma$ offers an alternative route to obtaining a characteristic field of order T. Indeed it can be argued on rather general grounds that if the quantum critical point obeys the hyperscaling property, and if H couples to a conserved total spin, then the characteristic field scale will be of order $k_{\rm B}T$ [15].

It is now appropriate to mention some very interesting recent observations of glassy behavior, persisting in the metallic phase, by Bogdanovich and Popović [16], which appear to be consistent with our proposal. These authors made the connection to glassy behavior in models of charge transport of spinless electrons [17], while here we will focus only on the spin degrees of freedom.

We will now propose a quantum field theory for the transition at $H = H_\sigma$. We will follow a general approach to quantum spin-glasses developed in [12] and reviewed in chapter 16 of [18]. In the presence of an applied magnetic field, there is only a $U(1)$ symmetry of spin rotations about the direction of the applied field, and we are interested in singularity in the dependence of the conserved 'charge' of this symmetry (the magnetization) on the applied field. A general theory for such transitions was presented in [19] and in ch. 11 of [18]. Here we will extend this theory to random systems with spin-glass order by combining it with the methods of [12].

Let us assume that the field H is applied in the z direction, and let $S_\alpha(r,\tau)$ with $\alpha = x,y$ be the component of the electron spin density in the orthogonal x,y plane at the spatial point r and at imaginary time τ. We use the standard replica method to treat the quenched disorder: consequently, we introduce replica indices $a = 1,\ldots,n$, and the spin density $S_{\alpha a}(r,\tau)$. The quantum theory for spin-glass order is expressed in terms of the order parameter functional $Q^{ab}_{\alpha\beta}(r,\tau_1,\tau_2)$ which is

$$Q^{ab}_{\alpha\beta}(r,\tau_1,\tau_2) \sim S_{\alpha a}(r,\tau_1) S_{\beta b}(r,\tau_2). \tag{1}$$

By applying the methods developed in [12,18] to spin-glass order in the x,y plane in the presence of an applied magnetic field we obtain the following low order terms in the effective action for $Q^{ab}_{\alpha\beta}(r,\tau_1,\tau_2)$:

$$\begin{aligned}
\mathscr{S}_{\rm sg} = \int \mathrm{d}^d r \Bigg\{ & \frac{1}{\kappa}\int \mathrm{d}\tau \sum_a \left[\frac{1}{2}\left(-i\frac{\partial}{\partial\tau_1} + i\frac{\partial}{\partial\tau_2}\right)\varepsilon_{\alpha\beta} \right. \\
& \left. + \left(H - H^0_\sigma\right)\delta_{\alpha\beta} \right] Q^{aa}_{\alpha\beta}(x,\tau_1,\tau_2)\Bigg|_{\tau_1=\tau_2=\tau} \\
& + \frac{1}{2}\int \mathrm{d}\tau_1 \mathrm{d}\tau_2 \sum_{ab} \left[\nabla Q^{ab}_{\alpha\beta}(r,\tau_1,\tau_2)\right]^2 \\
& - \frac{\kappa}{3}\int \mathrm{d}\tau_1 \mathrm{d}\tau_2 \mathrm{d}\tau_3 \sum_{abc} Q^{ab}_{\alpha\beta}(r,\tau_1,\tau_2) Q^{bc}_{\beta\rho}(r,\tau_2,\tau_3) Q^{ca}_{\rho\alpha}(r,\tau_3,\tau_1) \\
& + \frac{1}{2}\int \mathrm{d}\tau \sum_a \left[u\, Q^{aa}_{\alpha\beta}(r,\tau,\tau) Q^{aa}_{\alpha\beta}(r,\tau,\tau) + v\, Q^{aa}_{\alpha\alpha}(r,\tau,\tau) Q^{aa}_{\beta\beta}(r,\tau,\tau)\right] \Bigg\},
\end{aligned} \tag{2}$$

where $d = 2$ is the spatial dimensionality, summation of the spin indices α,β,ρ over x,y is implied, $\varepsilon_{\alpha\beta}$ is the antisymmetric tensor, H^0_σ is the mean-field value of the critical field (which will be renormalized by fluctuations), κ,u,v are the coupling constants, and we have omitted an additional quadratic term which has no influence on the mean-field theory, but does play an important role in the violation of hyperscaling in the perturbative fluctuations [12]. In a metal with an appreciable contribution of low energy spin excitations associated with the particle-hole continuum, there will also be an additional dissipative term in the action, as discussed in [20]: in the present situation with strong local correlations we believe this is unlikely to be the case, and so have omitted such a term above.

An important property of (2) is that the field H couples to a conserved $U(1)$ charge: consequently changes in H can be absorbed by a time-dependent gauge transformation which is equivalent to a transformation into a 'rotating reference frame' [15,18]. More specifically, if we generalize the action to a time-dependent field $H(\tau)$, then it is invariant under the infinitesimal transformation

$$Q^{aa}_{\alpha\beta}(r,\tau_1,\tau_2) \to Q^{aa}_{\alpha\beta}(r,\tau_1,\tau_2) - \varepsilon_{\alpha\gamma}\phi(\tau_1)Q_{\gamma\beta}(r,\tau_1,\tau_2) - \varepsilon_{\beta\gamma}\phi(\tau_2)Q_{\alpha\gamma}(r,\tau_1,\tau_2)$$
$$H(\tau) \to H(\tau) - i\partial_\tau\phi(\tau), \tag{3}$$

where $\phi(\tau)$ is infinitesimal. If the critical point at $H = H_\sigma$ satisfies strong hyperscaling properties, then (3) implies that the free energy density $\mathscr{F}_{\rm sg}$ obeys

$$\mathscr{F}_{\rm sg} = \mathscr{F}_0 + T^{\mu}\Phi\left(\frac{H-H_\sigma}{T^{\varphi}}\right), \tag{4}$$

where the exponents $\mu = 1 + d/z$, $\varphi = 1$, z is the dynamic critical exponent, and Φ is a scaling function. Taking the H derivative of (4) we obtain the magnetization

$$M = M_0 - T^{\mu-\varphi}\Phi'\left(\frac{H-H_\sigma}{T^{\varphi}}\right), \tag{5}$$

where M_0 is the background ferromagnetic magnetization present for $H \gg H_\sigma$. The onset of spin-glass order causes a decrease in this magnetization as H is lowered. A related scaling form should also hold for the magnetoconductance.

We illustrate the above behavior of the magnetization by a simple mean field analysis of $\mathscr{S}_{\rm sg}$ in (2). Unfortunately these mean field results do not satisfy hyperscaling properties appropriate to any value of d, and the reasons for this are similar to those discussed at length in [12] for other quantum spin-glasses. The actual situation for realistic spin-glasses remains an open problem, as an analysis of fluctuations about the mean field solutions leads to a runaway flow to strong coupling. If our proposal is indeed the correct explanation for the experimental data [9], then the hyperscaling prediction $\varphi = 1$ must be valid for $d = 2$.

At the mean-field saddle-point, we may make the following ansatz for $Q^{ab}_{\alpha\beta}(r,\tau_1,\tau_2)$ based on the requirements of translational invariance in space and time (we henceforth set $k_{\rm B} = 1$):

$$Q^{ab}_{\alpha\beta}(r,\tau_1,\tau_2) = T\sum_{\omega_n}\widetilde{D}^{ab}_{\alpha\beta}(\omega_n){\rm e}^{-i\omega_n(\tau_1-\tau_2)}, \tag{6}$$

where ω_n is a Matsubara frequency, and

$$\widetilde{D}^{ab}_{\alpha\beta}(\omega_n) = \delta_{ab}D_{\alpha\beta}(\omega_n) + \delta_{\alpha\beta}\frac{\delta_{\omega_n,0}}{T}q_{\rm EA}. \tag{7}$$

The first, replica-diagonal term in (7) is the local dynamic spin susceptiblity in the x,y plane, while $q_{\rm EA}$ is the Edwards–Anderson spin-glass order parameter. For simplicity we have assumed a replica-symmetric form for the spin-glass order – replica symmetry is not broken for the terms included in (2), but is broken when higher order terms are included: this is as discussed in [12]. Note also that the second term in (7) also includes a contribution of $q_{\rm EA}$ along the replica diagonal – this is unlike the usual procedure in the theory of classical spin-glasses; the diagonal contribution here accounts for the long-time limit of the local spin correlation function. Further analysis is simplified by rewriting the $D_{\alpha\beta}$ in 'circularly-polarized' components:

$$D_{xx} = D_{yy} = \frac{1}{2}\left(D_{+-} + D_{-+}\right),$$
$$D_{xy} = -D_{yx} = \frac{i}{2}\left(D_{+-} - D_{-+}\right). \tag{8}$$

Inserting (6)–(8) into (2) we obtain the following expression for the free energy density after taking the limit $n \to 0$:

$$\begin{aligned}\mathscr{F}_{\rm sg} = \mathscr{F}_0 &+ \frac{T}{\kappa}\sum_{\omega_n}\Big\{(-i\omega_n + H - H_\sigma)D_{+-}(\omega_n) \\ &\qquad + (i\omega_n + H - H_\sigma)D_{-+}(\omega_n) + 2q_{\rm EA}(H - H_\sigma)\Big\} \\ &- \frac{\kappa T}{3}\sum_{\omega_n}\Big\{D_{+-}^3(\omega_n) + D_{-+}^3(\omega_n)\Big\} \\ &- \kappa q_{\rm EA}\Big\{D_{+-}^2(0) + D_{-+}^2(0)\Big\} \\ &+ \frac{v}{2}\left[2q_{\rm EA} + T\sum_{\omega_n}\Big\{D_{+-}(\omega_n) + D_{-+}(\omega_n)\Big\}\right]^2 \\ &+ u\left[q_{\rm EA} + T\sum_{\omega_n}D_{+-}(\omega_n)\right]\left[q_{\rm EA} + T\sum_{\omega_n}D_{-+}(\omega_n)\right];\end{aligned} \tag{9}$$

we have replaced H_σ^0 by H_σ anticipating the mean-field position of the critical point. It is now straightforward to determine the saddle-point of $\mathscr{F}_{\rm sg}$ with respect to variations in $D_{+-}(\omega_n)$, $D_{-+}(\omega_n)$, and $q_{\rm EA}$. We find two classes of solutions describing the paramagnetic and spin-glass phases respectively, and we will discuss their properties in turn.

In the paramagnetic phase we have $q_{\rm EA} = 0$, and

$$D_{+-}(\omega_n) = -\frac{1}{\kappa}\sqrt{-i\omega_n + \Delta},$$
$$D_{-+}(\omega_n) = -\frac{1}{\kappa}\sqrt{i\omega_n + \Delta} \tag{10}$$

where the energy Δ is determined by the solution of

$$\Delta = H - H_\sigma - (u + 2v)T\sum_{\omega_n}\sqrt{-i\omega_n + \Delta}. \tag{11}$$

The condition $\Delta \geq 0$ delineates the boundary of the paramagnetic phase. At $T = 0$, the Matsubara summation in (11) becomes a frequency integral which evaluates to zero (the expressions in (10), (11) only aim to capture the singular low frequency behavior, and it is assumed that the high frequency form is such that contours of frequency integration can be freely closed at complex infinity). So $\Delta = H - H_\sigma$ at $T = 0$, which demonstrates that the paramagnetic phase is stable only for $H > H_\sigma$. For $T > 0$, (11) can be analyzed using methods that have been discussed in some detail in ch. 15 of [18]; for $H - H_\sigma$ small, its solution can be written as

$$\Delta + (u+2v)T\sqrt{\Delta} = H - H_\sigma + (u+2v)T^{3/2}\Xi\left(\frac{H-H_\sigma}{T}\right), \tag{12}$$

with the function Ξ given by

$$\Xi(y) = \frac{1}{\pi}\int_0^\infty \sqrt{\Omega}\,\mathrm{d}\Omega\,\mathscr{P}\left(\frac{1}{\mathrm{e}^{\Omega+y}-1}\right) + \theta(y)\sqrt{y}, \tag{13}$$

where $\mathscr{P}$ denotes a principle part and $\theta(y)$ is the unit step function. The expression (13) was derived for $y > 0$ ($H > H_\sigma$) but has been written in a manner which defines it for real y. Despite appearances, the function $\Xi(y)$ is actually analytic at $y = 0$, and indeed it is analytic at all real values of y; for small y, $\Xi(y) = \zeta(3/2)/(2\sqrt{\pi}) + 0.411958y + \cdots$. For $y < 0$, the results (12), (13) apply in the paramagnetic portion of the phase diagram present at $T > 0$, $H < H_\sigma$ (see figure 1). At the critical field, $H = H_\sigma$, (12) predicts that $\Delta = (u+2v)\Xi(0)T^{3/2}$; application of hyperscaling to this quantum critical region would have implied $\Delta \sim T \times$ a function of $(H - H_\sigma)/T$, and so it is evident that hyperscaling is not obeyed by the mean-field theory. Imposing the condition $\Delta = 0$ in (12) determines the mean-field boundary of the spin-glass phase as $H = H_\sigma - (u+2v)\Xi(0)T^{3/2}$ at small T: this leads to the phase diagram in figure 1 – the structure of the crossovers is very similar to those discussed earlier for other spin-glasses, and the reader is referred to ch. 15 of [18] for a review. Note that the paramagnetic phase extends to $H < H_\sigma$ for $T > 0$, and that the expression (12) remains valid in this regime where $y < 0$. The free energy in the paramagnetic phase is also easily obtained from (9)–(11), and we obtain

$$\mathscr{F}_{\mathrm{sg}} = \mathscr{F}_0 - \frac{(\Delta - H + H_\sigma)^2}{\kappa^2(u+2v)} - \frac{4T^{5/2}}{3\pi\kappa^2}\int_0^\infty \frac{\Omega^{3/2}\,\mathrm{d}\Omega}{\mathrm{e}^{(\Omega+\Delta)/T}-1}. \tag{14}$$

The T and H dependence of the magnetization is determined by taking a H derivative of (14): we find

$$M = M_0 \quad \text{as } T \to 0 \text{ for } H > H_\sigma, \tag{15}$$

up to exponentially small terms, indicating that the magnetization is effectively saturated in the paramagnetic phase in this mean-field theory, as indicated in figure 2. Fluctuations will induce a more appreciable variation in the magnetization even at $T = 0$, as strict saturation is not possible in a disordered system.

Finally, we describe the saddle point of (9) in the spin-glass phase. Here we find the simple solution

$$\begin{aligned}
D_{+-}(\omega_n) &= -\frac{1}{\kappa}\sqrt{-i\omega_n}, \\
D_{-+}(\omega_n) &= -\frac{1}{\kappa}\sqrt{i\omega_n}, \\
q_{\mathrm{EA}} &= \frac{H_\sigma - H}{\kappa(u+2v)} + \frac{T}{\kappa}\sum_{\omega_n}\sqrt{-i\omega_n} \\
&= \frac{H_\sigma - H}{\kappa(u+2v)} - \frac{\Xi(0)T^{3/2}}{\kappa}.
\end{aligned} \tag{16}$$

The last expression identifies the same boundary of the spin-glass phase (where $q_{\rm EA} = 0$) as that determined above. The free energy can also be computed as before, and we obtain

$$\mathscr{F}_{\rm sg} = \mathscr{F}_0 - \frac{(H_\sigma - H)^2}{\kappa^2(u+2v)} - \frac{4T^{5/2}\Gamma(5/2)\zeta(5/2)}{3\pi\kappa^2}, \tag{17}$$

and the magnetization follows as its H derivative. Now we find

$$M = M_0 - \frac{2(H_\sigma - H)}{\kappa^2(u+2v)} \quad \text{as } T \to 0 \text{ for } H < H_\sigma. \tag{18}$$

Comparing with (15) we see that there is a kink in the magnetization at the quantum critical point $H = H_\sigma$, as shown in figure 2. This singularity survives fluctuation corrections, even though the mean-field saturation of the magnetization in (15) does not.

This paper has outlined a scenario by which 2DEG can exhibit a quantum phase transition at a critical in-plane applied magnetic field $H = H_\sigma$: the transition is induced by the destruction of spin-glass order in the plane orthogonal to the applied field. We have argued that the instabilities of the spin exchange Hamiltonian for the ordered Wigner crystal to ferromagnetism and to spin-singlet states lend support to the possibility of spin-glass order in the disordered 2DEG. It would be interesting to compare H, T, and density dependent data to scaling forms like (4), (5). Theoretical analysis of the transport properties of the field-tuned transition at $H = H_\sigma$, and also of the density-tuned transition for the case $H_\sigma = 0$, will be of interest.

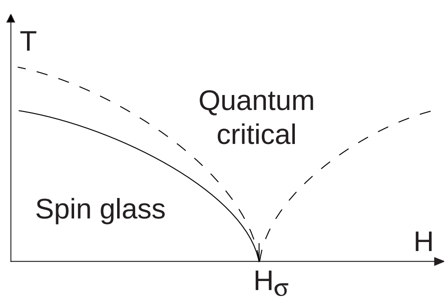

Figure 1. Phase diagram of the quantum spin-glass described by $\mathscr{S}_{\rm sg}$ in (2). The full line is a phase transition marking the mean-field boundary of the spin-glass phase – in $d = 2$ spin-glass order may only be present at $T = 0$. The dashed lines are crossovers into the quantum critical region, which survives fluctuations in $d = 2$. With hyperscaling, the characteristic energy in the quantum critical region $\sim T$; the mean-field theory has this energy $\sim T^{3/2}$.

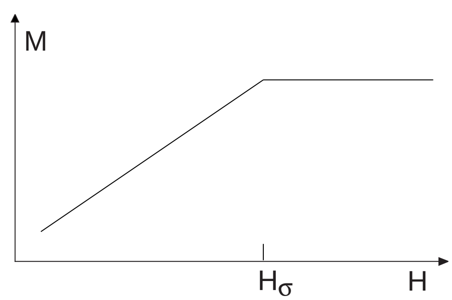

Figure 2. Mean field prediction for the magnetization, M, as a function of H at $T = 0$ in (15) and (18). Upon including fluctuations, the singularity at $H = H_\sigma$ will survive but the magnetization will be H dependent for $H > H_\sigma$. At $T > 0$ the singularity at $H = H_\sigma$ will be rounded out on a scale T if hyperscaling is obeyed by the critical point.

Acknowledgement

I thank Sergey Vitkalov, Myriam Sarachik and Sergey Kravchenko for useful discussions. This research was supported by US NSF Grant DMR 0098226.

References

[1] E Abrahams, S V Kravchenko and M P Sarachik, *Rev. Mod. Phys.* **73**, 251 (2001) and references therein
[2] S Ilani, A Yacoby, D Mahalu and H Shtrikman, *Phys. Rev. Lett.* **84**, 3133 (2000)
[3] B Bernu, L Candido and D M Ceperley, *Phys. Rev. Lett.* **86**, 870 (2001)
[4] S Chakravarty, S Kivelson, C Nayak and K Voelker, *Philos. Mag.* **B79**, 859 (1999)
[5] G Misguich, B Bernu, C Lhuillier and C Waldtmann, *Phys. Rev. Lett.* **81**, 1098 (1998)
[6] C Chamon, E R Mucciolo and A H Castro Neto, cond-mat/0107271
[7] B N Narozhny, I L Aleiner and A I Larkin, *Phys. Rev.* **B62**, 14898 (2000)
[8] S Sachdev, *Phil. Trans. Roy. Soc. London* **A356**, 173 (1998); cond-mat/9705074
[9] S A Vitkalov, H Zheng, K M Mertes, M P Sarachik and T M Klapwijk, *Phys. Rev. Lett.* **87**, 086401 (2001)
[10] A A Shashkin, S V Kravchenko, V T Dolgopolov and T M Klapwijk, *Phys. Rev. Lett.* **87**, 086801 (2001)
[11] V M Pudalov, M Gershenson, H Kojima, N Butch, E M Dizhur, G Brunthaler, A Prinz and G Bauer, cond-mat/0105081
[12] N Read, S Sachdev and J Ye, *Phys. Rev.* **B52**, 384 (1995)
[13] S Sachdev and T Senthil, *Ann. Phys.* **251**, 76 (1996)
[14] D Dalidovich and P Phillips, *Phys. Rev.* **B59**, 11925 (1999)
[15] S Sachdev, *Z. Phys.* **94**, 469 (1994)
[16] S Bogdanovich and D Popović, cond-mat/0106545
[17] A A Pastor and V Dobrosavljević, *Phys. Rev. Lett.* **83**, 4642 (1999)
[18] S Sachdev, *Quantum phase transitions* (Cambridge University Press, Cambridge, 1999)
[19] S Sachdev, T Senthil and R Shankar, *Phys. Rev.* **B50**, 258 (1994)
[20] S Sachdev, N Read and R Oppermann, *Phys. Rev.* **B52**, 10286 (1995)