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Renormalization of seesaw neutrino masses in the standard model with two-Higgs doublets

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Abstract. Using the theoretical ambiguities inherent in the seesaw mechanism, we derive the new analytic expressions for both quadratic and linear seesaw formulae for neutrino masses at low energies, with either up-type quark masses or charged lepton masses. This is possible through full radiative corrections arising out of the renormalizations of the Yukawa couplings, the coefficients of the neutrino-mass-operator in the standard model with two-Higgs doublets, and also the QCD–QED rescaling factors below the top-quark mass scale, at one-loop level. We also investigate numerically the unification of top-b-τ Yukawa couplings at the scale $M_I = 0.59 \times 10^8$GeV for a fixed value of $\tan\beta = 58.77$, and then evaluate the seesaw neutrino masses which are too large in magnitude to be compatible with the presently available solar and atmospheric neutrino oscillation data. However, if we consider a higher but arbitrary value of $M_I = 0.59 \times 10^{11}$ GeV, the predictions from linear seesaw formulae with charged lepton masses, can accommodate simultaneously both solar atmospheric neutrino oscillation data.

Keywords. Neutrino masses; radiative corrections; seesaw formula.

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1. Introduction

The implication of any observational evidence on the possible non-zero neutrino mass, and neutrino oscillations, would indicate a departure from the standard model. Recent exciting results from the super-Kamiokande experiments [1] have reconfirmed the initial data of atmospheric muon deficits, thereby suggesting a large mixing angle with $\delta m^2_{\mu x} \simeq$ (0.5–6)$\times 10^{-3}$ eV^2. The SOUDAN-2 results [2,3] also support these findings with their value of δm^2 being above 1×10^{-3} eV^2. Further positive evidences pour from MACRO data [4] on upward-going muons. Using the recent CHOOZ results [5] which exclude possibility of the oscillation $\nu_\mu \to \nu_e$, the super-Kamiokande result now just implies the oscillation $\nu_\mu \to \nu_\tau$ or a sterile neutrino ν_s (which is a singlet under the standard model). The collaboration using the liquid scintillator neutrino detector at Los Alamos

(LSND) has reported evidences for the appearance of $\bar{\nu}_\mu \to \bar{\nu}_e$ [6] and $\nu_\mu \to \nu_e$ [6] oscillations. Interpretation of the LSND data favours the choice $0.2\,\text{eV}^2 \leq \delta m^2 \leq 10\,\text{eV}^2$ for $0.002 \leq \sin^2 2\theta \leq 0.03$. However the KARMEN-2 experiment [7] which is also sensitive to this region of parameter space, restricts the allowed values to a relatively small subset of the above region. There are also indications of neutrino oscillations in the neutrino coming from the sun. The solar neutrino puzzle can be resolved [8] through matter enhance oscillation (MSW) with $\delta m^2_{\text{ex}} \simeq$ (0.8–2) $\times 10^{-5}\,\text{eV}^2$, $\sin^2 2\theta \sim 1$, or vacuum oscillation $\delta m^2_{\text{ex}} \simeq$ (0.5–6)$\times 10^{-10}\,\text{eV}^2$, $\sin^2 2\theta \sim 1$. The Sudbury Neutrino Observatory (SNO) [9] and BOREXINO [10] under construction, are expected to play major role in future. There are also other indications of the non-zero mass of the neutrinos. Neutrino mass in the range of (1–6) eV may provide viable candidate for the hot dark matter (HDM) component of the universe [11,12], and Majorana neutrino mass $m_e < 0.46\,\text{eV}$ may give positive evidence for the neutrinoless double beta decay [13–15].

Theoretically, Gell-Mann, Ramond and Slansky [16], and Yanagida [17] were the first authors to point out that small left-handed Majorana neutrino masses can naturally arise through seesaw mechanism in which a large mass of the right-handed Majorana neutrino is associated with the spontaneous symmetry breaking of the left-right symmetric grand unified theories such as SO(10) GUT where the gauge couplings of the standard model are unified (i.e. grand desert model). However, the grand desert models are ruled out by experimental constraints on electroweak mixing angle and proton lifetime. But predictions from SO(10) GUT are consistent with experimental data provided intermediate symmetries such as SU(2)$_L\times$ SU(2)$_R\times$ SU(4)$_C$ $(= G_{224})$, exist in the model as in case of two-step breaking of SO(10) [18–20]. In order to determine the allowed value of the intermediate symmetry breaking scale M_I at which $\text{SU}(4)_C$ is broken in a class of SO(10) models, we usually have two independent methods. One method consists of running the gauge couplings and using the matching conditions at M_I. The other is by demanding that the Yukawa couplings of the third generation get unified at M_I. Since SO(10) contains the maximal subgroup G_{224}, the simplest and most attractive assumption about the scale at which SU(2)$_R$ breaks is that it is the same scale at which SU(4)$_C$ breaks. This would be the case if the intermediate symmetry breaking were done by the right-handed neutrino masses $M_N \leq M_I$. Such a two-step SO(10) model is particularly promising in the sense that it includes the seesaw mechanism in a natural way and predict small neutrino masses. The above consideration is generally valid to other groups than SO(10) where SU(4)$_C$ and SU(2)$_R$ break at the same scale [18].

In this paper we follow the second option in which the top-b-τ Yukawa couplings unify at M_I, and this will be a consequence of the low-energy data and two-Higgs doublet standard model (SM) which may emerge from SO(10) GUT. Freire [18] had investigated the unification of three Yukawa couplings of the third generation in SO(10) with two-Higgs-doublets SM and observed the intermediate scale at $M_I \approx (10^{12.2} - 10^{13.6})$ GeV for $\tan\beta \approx$ 35–45, yielding the bounds, 80 GeV $\leq m_{\text{top}} \leq 180$ GeV. Subsequently Parida and Usmani [20] investigated quark-lepton unification scale at $M_I \approx (10^{8.5}$–$10^{9.5})$ GeV for m_{top} = (160–190) GeV with $\tan\beta \approx$ 52.80–261.94, respectively. In the light of the precise value of top-quark mass from TEVATRON, the question of quark-lepton unification with the accurate value of M_I, and generation of small neutrino masses through seesaw mechanism valid at the intermediate scale, can be reexamined, and compared with the recent observations on solar and atmospheric neutrino oscillations.

We organize the paper in the following way. Section 2 is devoted to the derivation of the low-energy radiative corrections to both quadratic and linear seesaw formulae for Majorana neutrino masses in the standard model with two-Higgs doublets, which might emerge from SO(10) GUT. In §3 we outline the procedure for the numerical analysis and present the main results. The last section is devoted to summary and conclusion.

2. Derivation of analytic expressions for neutrino masses

The theoretical ambiguities occurring in the seesaw mechanism lead to four types of linear and quadratic seesaw models [21] with Dirac-type fermion masses m_i are either up-quark masses ($m_{\rm top}, m_c, m_u$) or charged lepton masses (m_τ, m_μ, m_e). In case of the familiar quadratic seesaw formula, the right-handed neutrino masses are assumed to be degenerate $M_{N_i} = M_N$, and the left-handed Majorana neutrino masses m_{ν_i} vary as m_i^2 (and hence quadratic),

$$m_{\nu_i}(M_I) = \frac{m_i^2(M_I)}{M_N},\ M_N \leq M_I,\ i = 1,2,3. \tag{1}$$

Alternatively, the eigenvalues of M_{N_i} may follow the same hierarchy as m_i in case of non-degenerate right-handed neutrino mass (i.e. M_{N_i} is approximately proportional to m_i). This leads to linear seesaw formula where m_{ν_i} vary as m_i [21],

$$m_{\nu_i}(M_I) = m_i(M_I)\,\frac{m_i(M_I)}{M_{N_i}}. \tag{2}$$

Here $m_i = h_i V_{u,d}$, where $V_u = V\sin\beta$ for up-type quarks and $V_d = V\cos\beta$ for charged leptons; and $M_{N_i} = f_i V_R$. If $f_i \simeq h_i$ at the intermediate scale M_I, then the linear seesaw formula (2) becomes

$$m_{\nu_i}(M_I) \simeq m_i(M_I)\,\frac{V_{u,d}(M_I)}{V_R}, \tag{3}$$

where the vacuum expectation values are taken as $V = 174\,\text{GeV}$ at the top-quark mass scale, and $V_R \approx M_N = M_I$. The left-handed Majorana neutrino mass matrix can have the form $m_{\nu_i} = -\frac{1}{4}K_{ii}V^2$ [22], where K_{ii} ($i = 1,2,3$) is the coefficient of the dimension five neutrino-mass-operator, $\mathcal{L}_{\nu\nu} \sim \frac{1}{4}K_{ii}\nu\nu\phi\phi$, and has a dimension of [mass]$^{-1}$. The neutrino mass ratio due to the renormalization effect, can be estimated as

$$\frac{m_{\nu_i}(t_I)}{m_{\nu_i}(t_0)} = \left[\frac{K_{ii}(t_I)}{K_{ii}(t_0)}\right] \times \left[\frac{V(t_I)}{V(t_0)}\right]^2,$$

where $t_0 = \ln(m_{\rm top}/1\,\text{GeV})$ and $t_I = \ln(M_I/1\,\text{GeV})$.

Similarly, the Dirac-type fermion mass ratios are also estimated as

$$\frac{m_i(t_I)}{m_i(t_0)} = \left[\frac{h_i(t_I)}{h_i(t_0)}\right] \times \left[\frac{V_{u,d}(t_I)}{V_{u,d}(t_0)}\right].$$

With the above radiative corrections to m_{ν_i} and m_i, the quadratic and linear seesaw formulae in eqs (1), (3) can be expressed at the top-quark mass scale (t_0)

$$m_{\nu_i}(t_0) = C_{\nu_i}(t_0)\,\frac{m_i^2(t_0)}{M_N}, \tag{4}$$
$$C_{\nu_i}(t_0) = \left[\frac{h_i(t_I)}{h_i(t_0)}\right]^2 \times \left[\frac{K_{ii}(t_0)}{K_{ii}(t_I)}\right]$$

for the quadratic seesaw formulae, and

$$m_{\nu_i}(t_0) = C_{\nu_i}(t_0)\,m_i(t_0)\,\frac{V_{u,d}(t_0)}{V_R}, \tag{5}$$
$$C_{\nu_i}(t_0) = \left[\frac{h_i(t_I)}{h_i(t_0)}\right] \times \left[\frac{K_{ii}(t_0)}{K_{ii}(t_I)}\right]$$

for the linear seesaw formulae, respectively. In eqs (4), (5), the effect of radiative corrections arising from the renormalization of the vacuum expectation value (V) cancels on both sides of the equations.

Since the radiative correction coefficients C_{ν_i} involve the ratios of the Yukawa couplings, and also the coefficient of neutrino-mass-operator, we first collect all the relevant renormalization group equations (RGEs) for these quantities in two-Higgs doublet standard model, in the energy range $m_{\text{top}} \le \mu \le M_I$. One-loop RGEs for Yukawa couplings of quarks and leptons are given by [23–25]

$$16\pi^2 \frac{\mathrm{d}h_{\text{top}}}{\mathrm{d}t} = h_{\text{top}}\left[\frac{9}{2}h_{\text{top}}^2 + \frac{1}{2}h_b^2 - \sum_i C_i g_i^2\right],$$
$$16\pi^2 \frac{\mathrm{d}h_b}{\mathrm{d}t} = h_b\left[\frac{9}{2}h_b^2 + \frac{1}{2}h_{\text{top}}^2 + h_\tau^2 - \sum_i C_i' g_i^2\right],$$
$$16\pi^2 \frac{\mathrm{d}h_\tau}{\mathrm{d}t} = h_\tau\left[\frac{5}{2}h_\tau^2 + 3h_b^2 - \sum_i C_i'' g_i^2\right],$$
$$16\pi^2 \frac{\mathrm{d}h_{u,c}}{\mathrm{d}t} = h_{u,c}\left[3h_{\text{top}}^2 - \sum_i C_i g_i^2\right],$$
$$16\pi^2 \frac{\mathrm{d}h_{\mu,e}}{\mathrm{d}t} = h_{\mu,e}\left[3h_b^2 + h_\tau^2 - \sum_i C_i'' g_i^2\right], \tag{6}$$

where $t = \ln(\mu/1\,\text{GeV})$, $h_j =$ Yukawa coupling of the jth fermion, and

$$\begin{pmatrix} C_i \\ C_i' \\ C_i'' \end{pmatrix} = \begin{pmatrix} 17/20 & 9/4 & 8 \\ 1/4 & 9/4 & 8 \\ 9/4 & 9/4 & 0 \end{pmatrix},\ i = Y, 2L, 3C.$$

The two-loop RGEs for the evolution of gauge couplings $(g_i^2/4\pi = \alpha_i)$ are given by [23]

$$(16\pi^2)^2 \frac{\mathrm{d}g_i}{\mathrm{d}t} = g_i\left[16\pi^2 b_i g_i^2 + \sum_{j=1}^{3} b_{ij} g_i^2 g_j^2\right], \tag{7}$$

where

$$b_i = (21/5, 3, -7),$$
$$b_{ij} = \begin{pmatrix} 104/25 & 18/5 & 144/5 \\ 8/5 & 8 & 12 \\ 11/10 & 9/2 & -26 \end{pmatrix}. \tag{8}$$

In order to get a closed form analytic solution, the dominant part of RGE for the coefficient of the neutrino-mass-operator can be approximated as [22,26]

$$16\pi^2 \frac{\mathrm{d}K_{33}}{\mathrm{d}t} \simeq K_{33}\left[6h_{\mathrm{top}}^2 + h_\tau^2 + 2\lambda_2 - 3g_2^2\right],$$
$$16\pi^2 \frac{\mathrm{d}K_{22,11}}{\mathrm{d}t} \simeq K_{22,11}\left[6h_{\mathrm{top}}^2 + 2\lambda_2 - 3g_2^2\right], \tag{9}$$

where ($ii = 11, 22, 33$) refer to the family indices, and the evolution equations for the five quartic-Higgs scalar coefficients $(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)$ are given in paper by Hill *et al* [27]. Following the standard method of integration in the energy range $t_0 \leq t \leq t_I$ where $t_0 = \ln(m_{\mathrm{top}}/1\,\mathrm{GeV})$, $t_I = \ln(M_I/1\,\mathrm{GeV})$, and $t = \ln(\mu/1\,\mathrm{GeV})$, eqs (6), (9) reduce to $[h_i(t_I)/h_i(t_0)]$ and $[K_{ii}(t_0)/K_{ii}(t_I)]$ which are expressible in terms of integrals I_i, I_{λ_i}, and ratios $[\alpha_i(t_I)/\alpha_i(t_0)]$ where

$$I_i = \int_{t_0}^{t_I} \frac{h_i^2(t)}{16\pi^2}\,\mathrm{d}t,\ i = \tau, b, \mathrm{top},$$
$$I_{\lambda_i} = \int_{t_0}^{t_I} \frac{\lambda_i(t)}{16\pi^2}\,\mathrm{d}t,\ i = 1,2,3,4,5. \tag{10}$$

The QCD–QED rescaling factors for fermion masses, defined by [23] $\eta_i = (m_i(m_i))/(m_i(m_{\mathrm{top}}))$, are estimated following the standard techniques at three-loop order in QCD. These factors take care of the renormalization of fermion masses from top-mass scale down to the respective low-energy scale of each Dirac-fermion mass in question. Collecting all pieces, the standard seesaw formulae (4), (5) can be presented in four different types.

Type I: Quadratic seesaw formula (QSF) with up-type quark masses

The low energy neutrino masses are given by

$$m_{\nu_\tau}(t_0) = C_{\nu_\tau}(t_0)\frac{m_{\mathrm{top}}^2}{M_N},$$
$$m_{\nu_\mu}(t_c) = C_{\nu_\mu}(t_c)\frac{m_c^2}{M_N},$$
$$m_{\nu_e}(0) = C_{\nu_e}(0)\frac{m_u^2}{M_N}, \tag{11}$$

where $t_c = \ln(m_c/1\,\mathrm{GeV})$, $t_1 = 0 = \ln(1\,\mathrm{GeV}/1\,\mathrm{GeV})$, and the coefficients C_{ν_i} are calculated as

$$
\begin{aligned}
C_{\nu_\tau}(t_0) &= R_{\rm I}(t_0)\ \exp\left[3I_{\rm top}+I_b-I_\tau-2I_{\lambda_2}\right],\\
C_{\nu_\mu}(t_c) &= \frac{R_{\rm I}(t_0)}{\eta_c^2}\ \exp\left[-2I_{\lambda_2}\right],\\
C_{\nu_e}(0) &= \frac{R_{\rm I}(t_0)}{\eta_u^2}\ \exp\left[-2I_{\lambda_2}\right],
\end{aligned} \tag{12}
$$

with

$$
R_{\rm I}(t_0)=\prod_{i=1}^{3}\left[\frac{\alpha_i(t_I)}{\alpha_i(t_0)}\right]^{p_{1i}},\ p_{1i}=(-17/84,-1/4,8/7).
$$

Type II: Quadratic seesaw formula (QSF) with charged lepton masses

In this case, the quark masses in formulae (12) are replaced by corresponding charged leptons masses

$$
\begin{aligned}
m_{\nu_\tau}(t_\tau) &= C_{\nu_\tau}(t_\tau)\frac{m_\tau^2}{M_N},\\
m_{\nu_\mu}(t_\mu) &= C_{\nu_\mu}(t_\mu)\frac{m_\mu^2}{M_N},\\
m_{\nu_e}(t_e) &= C_{\nu_e}(t_e)\frac{m_e^2}{M_N}.
\end{aligned} \tag{13}
$$

The analogous expressions for C_{ν_i} defined at the respective lepton mass scales, are worked out as

$$
\begin{aligned}
C_{\nu_\tau}(t_\tau) &= \frac{R_{\rm II}(t_0)}{\eta_\tau^2}\ \exp\left[I_{\rm top}+6I_b-I_\tau-2I_{\lambda_2}\right],\\
C_{\nu_\mu}(t_\mu) &= \frac{R_{\rm II}(t_0)}{\eta_\mu^2}\ \exp\left[-6I_{\rm top}+6I_b+2I_\tau-2I_{\lambda_2}\right],\\
C_{\nu_e}(t_e) &= \frac{R_{\rm II}(t_0)}{\eta_e^2}\ \exp\left[-6I_{\rm top}+6I_b+2I_\tau-2I_{\lambda_2}\right],\\
R_{\rm II}(t_0) &= \prod_{i=1}^{3}\left[\frac{\alpha_i(t_I)}{\alpha_i t_0)}\right]^{p_{2i}},\ p_{2i}=(-15/28,-1/4,0),
\end{aligned} \tag{14}
$$

where $t_i = \ln(m_i/1\,\text{GeV}), i=\tau,\mu,e.$

Type III: Linear seesaw formula (LSF) with up-type quark masses

The neutrino masses at low energies defined at the respective fermion mass scales are now given by

$$m_{\nu_\tau}(t_0) = C_{\nu_\tau}(t_0)\frac{m_{\rm top}V_u}{V_R},$$
$$m_{\nu_\mu}(t_c) = C_{\nu_\mu}(t_c)\frac{m_c V_u}{V_R},$$
$$m_{\nu_e}(0) = C_{\nu_e}(0)\frac{m_u V_u}{V_R}, \tag{15}$$

with the expressions for C_{ν_i} at low energies,

$$C_{\nu_\tau}(t_0) = R_{\rm III}(t_0)\,\exp\left[-\frac{3}{2}I_{\rm top} + \frac{1}{2}I_b - I_\tau - 2I_{\lambda_2}\right],$$
$$C_{\nu_\mu}(t_c) = \frac{R_{\rm III}(t_0)}{\eta_c}\,\exp\left[-3I_{\rm top} - 2I_{\lambda_2}\right],$$
$$C_{\nu_e}(0) = \frac{R_{\rm III}(t_0)}{\eta_u}\,\exp\left[-3I_{\rm top} - 2I_{\lambda_2}\right], \tag{16}$$

where

$$R_{\rm III}(t_0) = \prod_{i=1}^{3}\left[\frac{\alpha_i(t_I)}{\alpha_i(t_0)}\right]^{p_{3i}},\ p_{3i} = (-17/168, 1/8, 8/14).$$

Type IV: Linear seesaw formula (LSF) with charged lepton masses

Defining neutrino masses at the respective mass scales of the charged leptons,

$$m_{\nu_\tau}(t_\tau) = C_{\nu_\tau}(t_\tau)\frac{m_\tau V_d}{V_R},$$
$$m_{\nu_\mu}(t_\mu) = C_{\nu_\mu}(t_\mu)\frac{m_\mu V_d}{V_R},$$
$$m_{\nu_e}(t_e) = C_{\nu_e}(t_e)\frac{m_e V_d}{V_R}, \tag{17}$$

the coefficients C_{ν_i} are now expressed by

$$C_{\nu_\tau}(t_\tau) = \frac{R_{\rm IV}(t_0)}{\eta_\tau}\,\exp\left[-\frac{7}{2}I_{\rm top} + 3I_b + I_\tau - 2I_{\lambda_2}\right],$$
$$C_{\nu_\mu}(t_\mu) = \frac{R_{\rm IV}(t_0)}{\eta_\mu}\,\exp\left[-6I_{\rm top} + 3I_b + I_\tau - 2I_{\lambda_2}\right],$$
$$C_{\nu_e}(t_e) = \frac{R_{\rm IV}(t_0)}{\eta_e}\,\exp\left[-6I_{\rm top} + 3I_b + I_\tau - 2I_{\lambda_2}\right], \tag{18}$$
$$R_{\rm IV}(t_0) = \prod_{i=1}^{3}\left[\frac{\alpha_i(t_I)}{\alpha_i(t_0)}\right]^{p_{4i}},\ p_{4i} = (-45/165, 1/8, 0).$$

While deriving the expressions for C_{ν_i} in all four types (I–IV) of seesaw formulae, family mixing based on certain texture is not considered for simplicity. The right-handed

Majorana neutrino mass scale is taken as $M_I \simeq V_R = M_N$. One can also express the ratios $C_{\nu_\tau}/C_{\nu_{\mu,e}}$ and $m_{\nu_\tau}/m_{\nu_{\mu,e}}$ in all four types of seesaw formulae in order to show the hierarchical structures of the three neutrino species.

3. Numerical solution and results

As necessary ingredients for the numerical estimation of C_{ν_i} and m_{ν_i} in eqs (11)–(18), our next step is to solve the RGEs in eqs (6), (7), (9) for g_i and h_i at different points of energy scale, $m_{\rm top} \leq \mu \leq M_I$, using the standard Runge-Kutta method. The integrals I_i defined in eq. (10) are then evaluated for the fixed M_I which shall be determined by the unifications of Yukawa couplings, $(h_{\rm top}, h_b, h_\tau)$ for a certain fixed value of $\tan\beta$.

The input values of the Yukawa couplings at the top-quark mass scale, are given by [28,29]

$$\begin{aligned} h_{\rm top}(m_{\rm top}) &= m_{\rm top}(m_{\rm top})/(174\ \sin\beta), \\ h_b(m_{\rm top}) &= m_b(m_b)/(174\eta_b\ \cos\beta), \\ h_\tau(m_{\rm top}) &= m_\tau(m_\tau)/(174\eta_\tau\ \cos\beta), \end{aligned} \tag{19}$$

where $m_{\rm top}(m_{\rm top})$, $m_b(m_b)$, and $m_\tau(m_\tau)$ are the running masses. For heavy flavours of quark, the difference between pole (expt.) and running masses are quite significant. The most recent determination of the input parameters are [30,31]

$$\begin{aligned} m_{\rm top}(m_{\rm top}) &= 166.5\,\text{GeV}, & m_\tau &= 1.785\,\text{GeV}, \\ m_b(m_b) &= 4.2\,\text{GeV}, & m_\mu &= 0.105\,\text{GeV}, \\ m_c &= 1.25\,\text{GeV}, & m_e &= 0.0005\,\text{GeV}, \\ m_u &= 0.005\,\text{GeV}, \end{aligned}$$

where the running masses of top and bottom quarks are obtained from the respective pole masses $m_{\rm top}^{\rm pole} = 175.6\,\text{GeV}$ and $m_b^{\rm pole} = 4.7\,\text{GeV}$, respectively, at two-loop level. The CERN-LEP measurements are given by

$$\begin{aligned} \sin^2\theta_\omega(M_Z) &= 0.2316 \pm 0.0003, \\ \alpha_s(M_Z) &= 0.118 \pm 0.004, \\ \alpha^{-1}(M_Z) &= 127.9 \pm 0.1, \end{aligned}$$

whereas the gauge couplings $(\alpha_i = g_i^2/4\pi)$ at the top-quark mass scale are evaluated using two-loop RGE,

$$\begin{aligned} \alpha_1^{-1}(m_{\rm top}) &= 58.51 \pm 0.1, \\ \alpha_2^{-1}(m_{\rm top}) &= 30.15 \pm 0.06, \\ \alpha_3^{-1}(m_{\rm top}) &= 9.30\,^{+0.297}_{-0.278}. \end{aligned} \tag{20}$$

The QCD–QED rescaling factors for the fermion masses are estimated up to 3-loop level following the procedure of Barger *et al* [23],

$$\begin{aligned} \eta_b &= 1.540\,^{+0.095}_{-0.087}, & \eta_\tau &= 1.017 \pm 0.001, \\ \eta_c &= 2.184\,^{+0.829}_{-0.439}, & \eta_\mu &= 1.028 \pm 0.004, \\ \eta_u &= 2.488\,^{+1.852}_{-1.859}, & \eta_e &= 1.046 \pm 0.011, \end{aligned}$$

where the uncertainties which are calculated from input values of $\alpha_3(M_Z)$ and $\alpha^{-1}(M_Z)$, are higher than the conventional values [24]. The choice of the initial values of the quartic-Higgs scalar coefficients are taken from the paper by Hill *et al* [27] as $\lambda_1 = 0.69$, $\lambda_2 = 0.79$, $\lambda_3 = 0.46$, $\lambda_4 = -0.60$, $\lambda_5 = 0.01$. As a result of the numerical solution of the differential equations (6), (7), we have shown in figure 1 the evolution of the three Yukawa couplings $(h_{\rm top}, h_b, h_\tau)$ as a function of energy scale $t = \ln(\mu/1\,\text{GeV})$, and observed the lepto-quark unification scale at $M_I = 0.59 \times 10^8$ GeV for the value of $\tan\beta = 58.77$. The values of integrals I_i and the three gauge couplings α_i at the scale M_I, are numerically evaluated as

$$\alpha_i(t_I) = (0.0201, 0.0418, 0.0422) \quad \text{for} \quad i = 1, 2, 3,$$
$$I_i = (0.048, 0.046, 0.031, 0.062) \quad \text{for} \quad i = \text{top}, b, \tau, \lambda_2.$$

Making use of the above values, we present, in table 1, the low-energy values of the coefficients $C_{\nu_{\tau,\mu,e}}$ for all four types (I–IV), while the predictions for the neutrino masses are given in table 2. The uncertainties are calculated only from input values of $\alpha_3(M_Z)$ and $\alpha^{-1}(M_Z)$, whereas the uncertainties in M_I due to threshold effects and top-quark mass, are not considered here.

It is important to note that our low value of $M_I = 0.59 \times 10^8$ GeV, compared to those of Freire [18] as well as Parida *et al* [32], can be understood from the low input value of running top-quark mass, $m_{\rm top} = 166.5$ GeV which corresponds to pole mass of 175.6 GeV [30]. In table 1, there is enhancement in $C_{\nu_{\mu,e}}$ values for types (II–IV) compared to those of type I, and this will show the low value of hierarchical mass ratios predictions. The present numerical predictions from types I–IV seesaw formulae on neutrino masses shown in table 2, could not accommodate both solar and atmospheric neutrino oscillation data, while LSND data can be fitted with the predictions from type I. Attempt

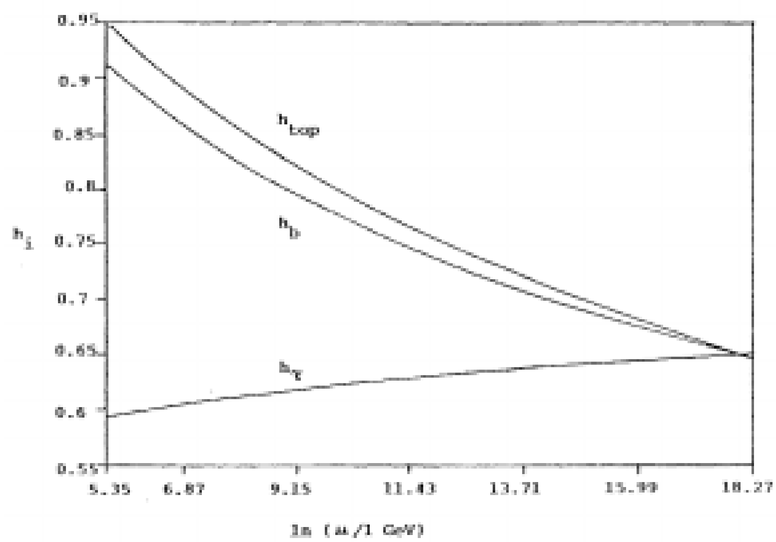

Figure 1. The evolution of the three Yukawa couplings of the third generation as a function of energy scale, $t = \ln(\mu/1\,\text{GeV})$ for fixed value of tan $\beta = 58.77$.

Table 1. Numerical predictions of the low-energy values of the coefficients C_{ν_i} defined at the respective fermion mass scale $m_i(m_i)$ for all four types of seesaw formulae as explained in the text.

Type	C_{ν_τ}	C_{ν_μ}	C_{ν_e}
I	$0.324\,^{+0.017}_{-0.016}$	$0.058\,^{+0.038}_{-0.029}$	$0.045\,^{+0.065}_{-0.031}$
II	0.899 ± 0.005	0.734 ± 0.009	0.709 ± 0.018
III	$0.483\,^{+0.013}_{-0.012}$	0.207 ± 0.060	$0.182\,^{+0.103}_{-0.080}$
IV	0.857 ± 0.002	$0.348\,^{+0.089}_{-0.096}$	$0.305\,^{+0.162}_{-0.131}$

Table 2. Numerical predictions of neutrino masses of three families using C_{ν_i} values given in table 1.

Type	m_{ν_τ}(eV)	m_{ν_μ}(eV)	m_{ν_e}(eV)
I	$(1.52 \pm 0.08) \times 10^5$	$1.54\,^{+0.99}_{-0.77}$	$\left(1.89\,^{+2.75}_{-1.30}\right) \times 10^{-5}$
II	$(4.86 \pm 0.03) \times 10^1$	$(1.37 \pm 0.02) \times 10^{-1}$	$(3.01 \pm 0.01) \times 10^{-6}$
III	$(2.37 \pm 0.06) \times 10^5$	$\left(7.64\,^{+2.17}_{-2.24}\right) \times 10^2$	$2.68\,^{+1.52}_{-1.18}$
IV	$(0.077 \pm 0.0001) \times 10^3$	$(0.018 \pm 0.005) \times 10^2$	$0.008\,^{+0.004}_{-0.003}$

to explain all the three data sources (solar, atmospheric, and LSND), or at least both solar and atmospheric neutrino oscillation data, seems to be difficult with our low value of M_I. In order to obtain correct magnitude of neutrino masses, our predictions in table 2 can be approximately normalized for higher values of M_I [18,20], by a multiplicative factor $[0.59 \times 10^8\,\text{GeV}/M_I]$. For an arbitrary value of $M_I = 0.59 \times 10^{11}$ GeV, there is enough scope for accommodating experimental data, and in particular, type IV would give $m_{\nu_i} = (0.077, 0.002, 0.0076 \times 10^{-3})$ eV for $i = \tau, \mu, e$, respectively. These can accommodate both atmospheric and solar neutrino oscillation data if the oscillation channels are interpreted as $\nu_\mu \to \nu_\tau$ and $\nu_e \to \nu_\mu$, respectively. In short, the Yukawa couplings unification in the standard model with two-Higgs doublets, implies an intermediate scale which is too low to be compatible with any type of the seesaw mechanism. However, with an ad-hoc or otherwise motivated use of the higher values of M_I, linear seesaw formula with charged lepton masses (type IV) gives correct predictions of the three neutrino masses, and their hierarchical mass ratios compared with recent observational data on neutrino oscillations.

4. Summary and conclusion

We first investigate in this work the unification of three Yukawa couplings of the third generation in two-Higgs doublet standard model which might emerge from SO(10) model with a single intermediate symmetry. For a fixed value of $\tan\beta = 58.77$, we obtain the unification scale at $M_I = 0.59 \times 10^8$ GeV. Then the low-energy seesaw formulae with radiative corrections arising from fermion mass renormalization and neutrino-mass-operator coefficients, are obtained. We study both quadratic and linear seesaw formulae with either up-quark or charged lepton masses, leading to four types (I–IV) of seesaw formulae. Using our low value of M_I we then evaluate the neutrino masses which are too large in magnitude to be compatible with solar and atmospheric oscillation data. However, our numerical predictions for type IV seesaw formula with an arbitrary value $M_I = 0.59 \times 10^{11}$ GeV, show good overlapping with experimental data of solar and atmospheric neutrino oscillations. Though we concentrate on single value of $\tan\beta$ and M_I in this work, their variations would give wider spectrum of interesting results.

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