

## Chiral symmetry and nuclear matter equation of state

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**Abstract.** We investigate the effect on the nuclear matter equation of state (EOS) due to modification of meson and nucleon parameters in nuclear medium as a consequence of partial restoration of chiral symmetry. To get the EOS, we have used Brueckner–Bethe–Golstone formalism with Bonn- $B$  potential as two-body interaction and QCD sum rule and Brown–Rho scaling prescriptions for modification of hadron parameters. We find that EOS is very much sensitive to the meson parameters. We can fit, with two body interaction alone, both the saturation density and the binding energy per nucleon.

**Keywords.** Equation of state; chiral symmetry.

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### 1. Introduction

The fundamental constituents of hadrons are quarks. Interaction of these quarks is described by quantum chromodynamics (QCD) in terms of exchange of gluons. QCD describes the hadronic physics very well at high energy; however, it fails to describe the hadronic physics at low energy domain because of failure of the perturbative treatment as a consequence of large coupling constant in that energy range. QCD Lagrangian in  $u$ ,  $d$  and to some extent  $s$  sector is invariant under chiral symmetry if we neglect the current masses ( $m_u \approx 2$  MeV,  $m_d \approx 5$  MeV,  $m_s \approx 150$  MeV) of these quarks in comparison to the QCD scale parameter  $\Lambda$  ( $\approx 250$  MeV). It is becoming more and more clear now that chiral symmetry of QCD Lagrangian and its spontaneous breaking [1] play a very important role in determining the structure of low mass hadrons which are comprised of  $u$ ,  $d$  and  $s$  quarks, and instantons play a crucial role in hadron correlators in mediating the spontaneous chiral symmetry breaking [2,3]. Physical confinement of quarks seems to play a lesser role. The spontaneous breaking of the chiral symmetry is signaled by the non-vanishing values in physical vacuum of the quark and gluon condensates [4–6]. Calculations based on chiral perturbation theory and QCD sum rule (QSR) indicate that values of these condensates are reduced when the hadrons are put in a medium, hence giving rise to partial restoration of chiral symmetry [7,8]. Thus, there is a lot of interest now a days to understand the mechanism of partial restoration in nuclear medium of chiral symmetry of QCD Lagrangian [9,10], and to isolate effects arising out of it, as it provides a handle to understand non-perturbative QCD phenomena. The most important consequence of chiral symmetry restoration has been identified as modification of hadron properties in nuclear matter [11,12]. Experimental evidence towards this is believed to be a large excess of  $e^+e^-$

pairs observed in the invariant mass region around 400 MeV in the 200A GeV central collisions of S on Au and W by CERES [13] and HELIOS [14] groups respectively. This has been explained [11,15] by arguing that rho meson mass in nuclear matter is reduced at the densities created in the collision.

Given the above scenario of modification of hadron parameters in nuclear medium due to non-perturbative QCD effects, we investigate its effect on the EOS of symmetric nuclear matter. In other words, we want to probe if, in the context of a two-body force model, the empirical EOS is enough to constraint the extent of medium modification of the hadron parameters due to partial restoration of chiral symmetry.

## 2. Brown–Rho scaling

Theoretical studies towards the modification of hadronic mass was triggered after pointing out by Brown and Rho [16], based on the restoration of scale invariance of QCD, that masses of hadrons would scale in nuclear medium as

$$\frac{m_N^*}{m_N} \approx \frac{m_V^*}{m_V} \approx \frac{m_\sigma^*}{m_\sigma} \approx \frac{f_\pi^*}{f_\pi}, \quad (1)$$

where the density dependent quantities are denoted by asterisks.  $m_N$ ,  $m_V$  and  $m_\sigma$  denote the masses of nucleon, vector mesons (rho, omega) and sigma mesons respectively.  $f_\pi^*$  is the in-medium pion decay constant, which is expected to vanish at high density when chiral symmetry is completely restored. Therefore, masses in eq. (1) should decrease with increasing density. Quantitative estimations of the dependence of masses of hadrons with respect to density have been made in quark meson coupling (QMC) model [17] and using QCD sum rule (QSR) approach [18]. In both of these two approaches the behavior of vector meson mass variation has been approximated by a term linear in density as

$$\frac{m_{\rho,\omega}^*}{m_{\rho,\omega}} = 1 - \alpha \tilde{\rho} = s_1, \quad (2)$$

where  $\tilde{\rho} = \left(\frac{\rho}{\rho_0}\right)$  with  $\rho_0$  as the nuclear matter saturation density. In both the approaches, value of  $\alpha$  has been found to lie around 0.17–0.18.

## 3. Brueckner–Bethe–Goldstone formalism

The symmetric nuclear matter is defined as an infinite uniform system of nucleons with equal neutron and proton density interacting through the strong force alone. It is characterized by its EOS, that is energy per nucleon as a function of the nucleon density. We have empirical information about EOS through the saturation density,  $\rho_0$ , the energy per particle,  $\mathcal{E}(\rho_0)$ , at the saturation density and the incompressibility,  $K \left( = 9\rho_0^2 \left[ \frac{\partial^2[\mathcal{E}(\rho)]}{\partial \rho^2} \right]_{\rho_0} \right)$ , of saturated nuclear matter. Values of these quantities have been found to be:  $\rho_0 = 0.17 \pm 0.02 \text{ fm}^{-3}$ ,  $\mathcal{E}(\rho_0) = -16 \pm 1 \text{ MeV}$  and  $K = 210 \pm 30 \text{ MeV}$  [19]. There are several formalisms available now to compute nuclear matter EOS. In BBG formalism, one

first calculates the crucial quantity known as Brueckner reaction matrix,  $G(\rho, \omega)$ , at a density  $\rho$  related to Fermi momentum,  $k_F$ , as  $\rho = 2k_F^3/(3\pi^2)$ . The  $G$  matrix satisfies the Bethe–Goldstone equation,

$$G(\rho, \omega) = v_{NN} + v_{NN} \sum_{k_1 k_2} \frac{|k_1 k_2\rangle Q(k_1, k_2) \langle k_1 k_2|}{\omega - \epsilon(k_1) - \epsilon(k_2) + i\eta} G(\rho, \omega), \quad (3)$$

where  $v_{NN}$  is the two-body nucleon–nucleon ( $NN$ ) interaction potential and  $\omega$  is energy of the two scattering nucleons.  $Q(k_1, k_2)$  in eq. (3) is Pauli operator which prevents two nucleon in the intermediate states from scattering into the states in already occupied Fermi sea. The Pauli operator is written as

$$Q(k_1, k_2) = [1 - n(k_1)] [1 - n(k_2)], \quad (4)$$

with  $n(k)$  denoting the Fermi distribution function, which at zero temperature is given by the step function  $\theta(k - k_F)$ . The single particle energy,  $\epsilon(k)$ , in eq. (4) is written as

$$\epsilon(k) = \frac{\hbar^2 k^2}{2M} + U(k), \quad (5)$$

with  $U(k)$  as single particle potential, which is written (in the standard choice) as

$$U(k) = \sum_{k'} n(k') \langle k k' | G(\rho, \omega) | k k' - k' k \rangle. \quad (6)$$

By solving self-consistently the eqs (3), (5) and (6), for a given  $NN$  interaction potential,  $v_{NN}$ , one determines the  $G$ -matrix. The  $G$ -matrix can be considered as an in-medium effective interaction between two nucleons. The renormalization of bare two-body interaction occurs by the surrounding nucleons through Pauli blocking and the nuclear mean field. Nuclear matter equation of state is then obtained by computing, at a density  $\rho$ , the energy per nucleon,  $\mathcal{E}(\rho)$ , using the following expression

$$\mathcal{E}(\rho) = \frac{1}{A} \sum_{k \leq k_F} \frac{\hbar^2 k^2}{2M} + \frac{1}{2A} \sum_{k, k' \leq k_F} \langle k k' | G(\rho, \omega) | k k' - k' k \rangle. \quad (7)$$

Over the years, a large number of calculations of the nuclear matter EOS have been done by various groups using both phenomenological and microscopic two-body  $NN$  interaction potential [19–27]. It has been observed that calculations using non-relativistic approach and only two-body interaction potential fail to reproduce the saturation observables. To improve the situation, two paths have been followed, namely, (1) doing non-relativistic calculation with two-body and an additional three-body potential with a few adjustable parameters [28–30], or, (2) doing relativistic Dirac–Brueckner calculation with two-body potential alone [19,31–33]. Considerable success has been achieved in reproducing the saturation observables in both of these paths. Thus, the debate remains unresolved whether the improvement in reproducing the saturation properties is due to inclusion of three body force or it is a relativistic effect.

We take the paradigm that nucleons interact through exchange of mesons. The parameters of these mesons are modified inside a nuclear medium due to partial restoration of

chiral symmetry of QCD Lagrangian, which is a non-perturbative effect. As the amount of modification of meson parameters is density dependent, the two-body  $NN$  interaction potential inside the nuclear medium, thus, is expected to be density dependent and very much different to that when the nucleons are free. We give this modified two-body density dependent  $NN$  interaction potential in BBG formalism as input to obtain a density dependent  $G$ -matrix and compute the EOS with this  $G$ -matrix.

#### 4. Bonn-potential

The  $NN$  interaction potential which is based explicitly on meson exchange has been developed by Bonn group and is known as Bonn potential [19]. Therefore, this potential will be ideal to investigate the effect of in-medium meson parameter modification on the nuclear matter EOS. The phase shifts for free nucleon–nucleon scattering and deuteron ground state properties have been described very well by this potential. In this potential, exchange of pseudoscalar ( $0^-$ ) mesons, pion and eta, vector ( $1^-$ ) mesons, rho and omega and scalar mesons, ( $0^+$ ) sigma and delta have been considered. The meson parameters include, masses ( $m$ ), coupling constants ( $g$ ) and cut off masses ( $\Lambda$ ) of the form factor at the meson–nucleon interaction vertices. The masses and coupling constants of a meson determine respectively the range and strength of the potential contributed by exchange of that meson to the total potential. The cut off masses are phenomenological parameters in the form factors, which are generally of monopole or dipole form, introduced at the interaction vertices to take care of the off-shell nature of the exchanged mesons and the finite size of the interaction vertices. Representing the interaction vertices as uniform density spheres, the cut off masses are inversely related to the radius of the sphere for monopole form factors. The form factors, in a way, make the coupling constants dependent on the energy and momentum carried by the mesons at the vertices.

#### 5. Results and discussion

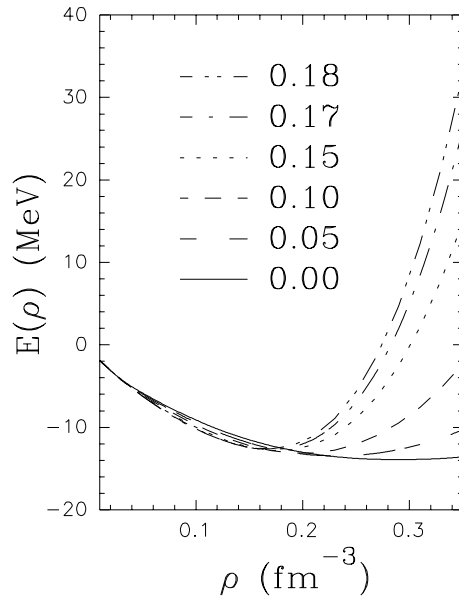
We modify the masses of the vector mesons using QMC and QSR prescriptions given in eq. (2) and use Brown–Rho scaling law given in eq. (1) to modify nucleon mass, masses of other mesons and the cut off masses as

$$\frac{m_N^*}{m_N} = \frac{m_\rho^*}{m_\rho} = \frac{m_\omega^*}{m_\omega} = \frac{m_\sigma^*}{m_\sigma} = \frac{m_\delta^*}{m_\delta} = \frac{\Lambda^*}{\Lambda} = s_1 . \quad (8)$$

We have not changed the masses of pseudoscalar mesons, pion and eta, as those are supposed to be Goldstone bosons of spontaneous chiral symmetry breaking and origin of masses of these mesons is related to explicit chiral symmetry breaking due to masses of  $u$ ,  $d$  and  $s$  quarks in QCD Lagrangian. We have modified cut off masses ( $\Lambda$ ) of all the interaction vertices. Reduction of the cut off masses inside nuclear medium means that the spatial extensions of the interaction vertices are increased, Also, this introduces some kind of dependence of coupling constant on medium density through the form factor, though the coupling constants have not been modified explicitly.

At a given density  $\rho$  and value of  $\alpha$ , we get modified meson parameters from eq. (8), which generate a modified  $NN$  interaction potential with which nucleons are interacting

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**Figure 1.** Dependence of equation of state for symmetric nuclear matter on the parameter  $\alpha$ . Values of  $\alpha$  are as shown in the figure ( $\beta = 1.0$ ). Calculations have been done with Bonn- $B$  potential as two body interaction.

inside nuclear medium. We then compute the nuclear matter EOS solving the set of eqs (3)–(7) and keeping both  $\alpha$  and  $\rho_0$  of eq. (2) as parameters. The results are shown in figure 1 for  $\rho_0 = 0.17 \text{ fm}^{-3}$ . The curve for  $\alpha = 0$  represent the nuclear matter EOS when the meson parameters have not been modified. In this case, the saturation density is about  $0.29 \text{ fm}^{-3}$  which is quite away from the empirical value. We see in figure 1 that the saturation density decreases with the increase of the value of  $\alpha$ , with  $\alpha = 0.17$  prescribed by QMC and QSR calculations, the saturation density is  $0.17 \text{ fm}^{-3}$  which is the empirical saturation density. However, the energy per particle increases with the increase of  $\alpha$ . We see the energy per particle from the EOS with  $\alpha = 0$  to be  $-13.5 \text{ MeV}$ . This value is higher than the empirical value. For  $\alpha = 0.17$ , the energy per particle is  $-12.72 \text{ MeV}$  which is even higher. For other values of  $\rho_0$  in eq. (2), trend of the result is similar i.e. for increasing values of  $\alpha$  minima of the EOS curves shift towards lower values of density, however, for no value of  $\rho_0$  and  $\alpha$  the binding energy could be brought down to its empirical value. The feature that binding energy is more at higher values of  $\alpha$  is due to increase of kinetic energy term in eq. (7) as a consequence of decrease of nucleon mass governed by eq. (2). So, we find that with the prescription given in eq. (8) and with Bonn- $B$  potential, it is not possible to bring both the saturation density and energy per particle to their respective empirical values.

To solve this problem we take two different medium dependence for usual masses and cut off masses. This means that radii of the interaction vertices do not scale in the same way as the masses do. There are indications towards this from the strict limits on the possible variation of the size of the nucleon in nuclear matter [34]. We take the density dependence of usual masses and cut off masses as

$$\frac{m_N^*}{m_N} = \frac{m_\rho^*}{m_\rho} = \frac{m_\omega^*}{m_\omega} = \frac{m_\sigma^*}{m_\sigma} = \frac{m_\delta^*}{m_\delta} = s_1 ,$$

$$\frac{\Lambda^*}{\Lambda} = s_1 - e^{-\rho^2} [s_1 - s_1^\beta] = s_2. \quad (9)$$

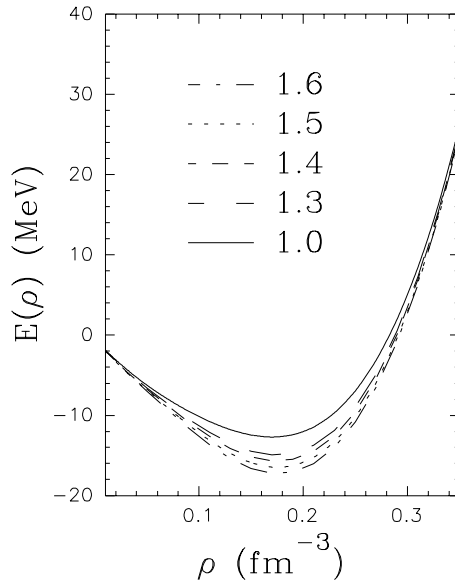
We have chosen the form of  $s_2$  in eq. (9) with the following considerations. Looking at the form for  $s_1$  in eq. (2), we find that masses of the hadrons go to zero at density  $\frac{\rho_0}{\alpha}$ . It seems, at that density there could be a phase transition from nuclear matter to quark matter. So, at that density radii of hadrons and the spatial extensions of the interaction vertices should be very large, infinite so to say. As,  $\Lambda$  is inversely related to the spatial extensions of the interaction vertices, its dependence with respect to density should be similar to the dependence of masses with respect to density at high density domain. We have fixed the form of  $s_2$  at high density region from the above considerations. At  $\rho = 0$ , the constraint is  $\Lambda^* = \Lambda$ . With these two constraints and taking only one parameter  $\beta$ , we have chosen the form for  $s_2$  as given in eq. (9). By construction  $s_2$  should not disturb the EOS curve at high as well as low density region, the job it has to do is only to bring down the binding energy at saturation density to the experimental value. The dependence,  $s_2$ , is mildly nonlinear below and near the saturation density and takes the same form as  $s_1$  at higher densities. Value of the parameter  $\beta$  and the Gaussian factor in front of the parenthesis in eq. (9) determine extent of nonlinearity of  $s_2$ . For  $\beta = 1$ , we see from eq. (9) that  $s_2$  goes over to  $s_1$ . The ratio of in-medium radius of the interaction vertices,  $\langle r \rangle^*$ , to that in free space,  $\langle r \rangle$ , is related to the cut off parameters as  $\langle r \rangle^* / \langle r \rangle = \Lambda / \Lambda^*$ . For nonlinear parametrization the radius of the interaction vertices increases faster and tend to have a plateau near the saturation density in comparison to monotonic increase in case of linear parametrization.

We now use eqs (8) and (9) to find the hadron parameters at a given density and values of  $\alpha$  and  $\beta$ . With these modified hadron parameters we calculate the interaction potential and then compute once again the EOS solving the set of eqs (3)–(7) keeping  $\alpha$ ,  $\beta$  and  $\rho_0$  as parameters. We find that binding energy can not be brought down to its experimental value except for  $\rho_0 = 0.17 \text{ fm}^{-3}$  in eqs (2) and (9). The results are shown in figure 2 for  $\alpha = 0.17$  and various values of  $\beta$ . We see that there is very little effect of  $\beta$  on the saturation density. However, the energy per particle is sensitive to the value of  $\beta$  and reduced with the increase of  $\beta$ . We see that for  $\beta = 1.5$  we get  $-16.5 \text{ MeV}$  as energy per particle and the saturation density has changed to  $0.175 \text{ fm}^{-3}$  from  $0.17 \text{ fm}^{-3}$ . Thus, we find that with the prescription given in eq. (9) and with Bonn- $B$  potential, it is possible to bring both the saturation density and energy per particle to their respective empirical values with  $\alpha = 0.17$  and  $\beta = 1.5$ .

For increasing values of  $\alpha$  and  $\beta$ , the EOS becomes more and more stiff. We find the incompressibility,  $K$ , of the EOS corresponding to  $\alpha = 0.17$  and  $\beta = 1.5$  to be  $530 \text{ MeV}$  in comparison to the incompressibility  $K = 194 \text{ MeV}$  of the curve with  $\alpha = 0.0$  and  $\beta = 1.0$  (in which case the meson parameters have not been modified). We get the value of incompressibility,  $K$ , much higher than the values quoted before, obtained [35] in the framework of non-relativistic Hartree–Fock RPA models with effective interaction.

The high value of  $K$  in our calculation is because of two reasons. It is due to, firstly, low value of nucleon mass at non zero density given by eq. (2); and, secondly, the interaction potential becoming more attractive only around the saturation density through eq. (9) to reproduce the experimental binding energy. So, it is not possible in this framework to

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**Figure 2.** Dependence of equation of state for symmetric nuclear matter on the parameter  $\beta$ . Values of  $\beta$  are as shown in the figure. Calculations have been done with Bonn- $B$  potential as two body interaction and taking  $\alpha = 0.17$ .

get a softer EOS. We feel, however, that inclusion of three body force may decrease the value of the incompressibility.

At the end, we mention that there are several versions of Bonn potential and using those in BBG formalism with the continuous choices for single particle potential one gets different values of saturation density and energy per particle. Therefore, it is expected that one would require different set of parameters  $\alpha$  and  $\beta$  to bring the saturation density and energy per particle to their respective empirical values for EOS's obtained using those potentials. That will provide some kind of a limit on these parameters. Moreover, one would like to know the effect of meson parameter modification on the EOS obtained using relativistic Dirac-Brueckner formalism or non-relativistic BBG formalism with Bonn potential as two body force and microscopic three-body force based on meson exchange formalism [36]. We are presently investigating these issues.

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