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Nuclear structure at high excitation energies

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Abstract. The present paper deals with the investigation of hot GDR and quadrupole shapes of ^{106,120}Sn isotopes as a function of temperature and spin utilizing cranked quadrupole–quadrupole model interaction hamlitonian in the linear response theory and static path approximation to the grand canonical partition function.

Keywords. Average nuclear shape; giant dipole resonance; static path approximation; linear response theory.

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1. Introduction

Study of the structure of nuclei in extreme conditions of angular momentum, excitation energy (temperature) and isospin has recently become a very interesting and active area of research in nuclear physics. Experimentally compound nuclei can be formed at high excitation energies and in high angular momentum states through heavy ion fusion reactions. Also hot nuclei at low angular momenta can be produced in inelastic α-scattering type reactions. Since several years different isotopes of tin have been the subject matter of investigation of the change in quadrupole shape parameters β and γ as a function of excitation energy (temperature, T) and spin (J) [1–14]. From the many body mean field theories, like Hartree–Fock or Hartree–Fock–Bogoliubov, it is expected that with the increase of temperature even the deformed ground state ($T = 0$) shape would tend towards a spherical one [15]. On the other hand if the nucleus is already spherical in the ground state it would become oblate at high spins with the magnitude of β increasing with the increase of spin. Thus at a moderate high temperature of about $T = 2.0$ MeV one expects all nuclei to become oblate at high spins [15].

At a finite temperature the nuclear potential energy does not have a very well defined value of β, γ even for nuclei that are strongly deformed in the ground state. Therefore, at $T \geq 1.0$ MeV even for such nuclei one can talk only of an average shape ($\langle\beta\rangle, \langle\gamma\rangle$) at a given value of angular momentum. More than a decade ago, Gaardhoje *et al* [1] studied the shape of giant dipole resonance (GDR) built on excited states for ^{108}Sn and concluded that the data are consistent with a mostly spherical shape up to $T \approx 2.0$ MeV. A few years later Majka *et al* [2] reached a similar conclusion for ^{110}Sn in a study of $\alpha - \gamma$-ray angular correlation measurements. Recently Bracco *et al* [10] have studied the dependence of the GDR width and angular distribution of the γ-rays as a function of temperature and spin for ^{109,110}Sn and concluded that due to thermal fluctuation effects the average deformation becomes very large at $T \simeq 2.0$ MeV and $J \simeq 50\hbar$. In yet another study of the GDR properties of ^{106}Sn at $T \approx 2.0$ MeV, Bracco *et al* [11] and Mattiuzzi *et al* [12] infer that $\langle\beta\rangle \approx 0.3{-}0.6$ for $J = 0{-}60\ \hbar$. The value of γ is taken to be -60° (change in sign convention of γ) implying a non-collective rotation about the axis of symmetry. From these discussions it is clear that a detailed investigation of the shape evolution of hot, and highly spinning nuclei employing a theory beyond mean field which accounts for thermal fluctuations of the shape degrees of freedom is called for.

Now it is known that the effect of thermal shape fluctuations can best be incorporated in a microscopic theory through the so called static path approximation (SPA) to the path integral representation of the partition function [16–18]. Recently we have already applied this approach to study the structural properties as well as level densities of some pf-shell and medium mass nuclei [19–21] employing a quadrupole–quadrupole model interaction. The angular momentum effects can be included in the cranking approximation. In view of the available experimental data on GDR, it is desirable that simultaneously the GDR properties are also explained as a function of temperature and spin. Thus, we have also used this scheme in a linear response theory to calculate the GDR γ-absorbtion cross section at finite temperature and spin [22]. Here we shall discuss some of our recent results on GDR and quadrupole shape of ^{106,120}Sn isotopes where a cranked quadrupole–quadrupole model interaction hamiltonian is employed.

2. Formalism

Before we present the formalism used by us, we would like to comment briefly on the thermal fluctuation model of Ormand *et al* [13] which is quite successful as far the variation of the GDR width as a function of temperature is concerned. The GDR cross section is evaluated through

$$\sigma(E) = \mathcal{Z}_J^{-1} \int \frac{\mathcal{D}[\alpha]}{\mathcal{I}(\beta,\gamma,\theta,\psi)^{3/2}} \sigma(\alpha,\omega_J;E)\mathrm{e}^{-F(\alpha,J,T)/T}\,, \tag{1}$$

where E is the γ-ray energy, $\mathcal{D}[\alpha] = \beta^4\,\mathrm{d}\beta\,\sin(3\gamma)\,\mathrm{d}\gamma\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi\,\mathrm{d}\psi$ is the volume element with α standing for the quadrupole deformation parameters β and γ and the Euler angles θ, ϕ, ψ. The cranking frequency ω is so adjusted that on the average $\langle J\rangle = J + 1/2$. The partition function is given by

$$\mathcal{Z}_J = \int \mathcal{D}[\alpha]/\mathcal{I}^{3/2}\mathrm{e}^{-F/T}\,, \tag{2}$$

where, corresponding to the laboratory z-axis cranking, the moment of inertia factor is given by

$$\mathcal{I}(\beta,\gamma,\theta,\psi) = \mathcal{I}_1\sin^2\theta\cos^2\psi + \mathcal{I}_2\sin^2\theta\sin^2\psi + \mathcal{I}_3\cos^2\theta \tag{3}$$

with $\mathcal{I}_k$ representing the deformation dependent principal moments of inertia. The free energy is parametrized as

$$F(\alpha,J,T) = F(\alpha,\omega=0,T) + (J+1/2)^2/2\mathcal{I}(\beta,\gamma,\theta,\psi). \tag{4}$$

Various expansion coefficients are essentially fitted to the Strutinsky shell correction energy of the nucleus at a finite temperature and spin. The cross section under the integral in eq. (1) is modeled by a harmonic vibration along the three principal axes and then transformed approximately to the laboratory frame of reference [4,13].

It is to be noted that there is no many-body hamiltonian employed to describe all the relevant quantities. In our approach, presented below, we attempt to address to this question. Some results on the application of our approach will be discussed in the next section.

Details of the SPA approach can be found in several papers [16–18,20,23] for a quadrupole–quadrupole ($Q \cdot Q$) model interaction hamiltonian,

$$\hat{H} = \hat{H}_0 - \frac{1}{2}\chi_Q \sum_\mu (-1)^\mu \hat{Q}_{-\mu}\hat{Q}_\mu \; , \tag{5}$$

where $\hat{H}_0$ stands for the spherical part and $\hat{Q}_\mu = (r^2/b^2)Y_{2\mu}$ is the quadrupole operator with $b^2 = \hbar/m\omega_0$, m being the nucleon mass and $\hbar\omega_0 = 41A^{-1/3}$ MeV. The quadrupole interaction strength [24] is taken as

$$\chi_Q = 120A^{-5/3} f_c \text{ MeV}, \tag{6}$$

where f_c is a core polarization factor (≥ 1) and is unity if there is no assumption of an inert core. Corresponding to the hamiltonian (5) the mean field 1-body hamiltonian takes the form

$$\hat{h}(\beta,\gamma) = \hat{h}_0 - \hbar\omega_0\beta\frac{r^2}{b^2}\left[\cos\gamma Y_{20} + \frac{1}{\sqrt{2}}\sin\gamma(Y_{22} + Y_{2-2})\right]. \tag{7}$$

A normalized statistical density operator is defined as

$$\hat{D} = \mathrm{e}^{-\hat{H}/T}/\mathrm{Tr}(\mathrm{e}^{-\hat{H}/T}). \tag{8}$$

Then the thermal average of an operator $\hat{O}$ in the SPA theory is given by

$$\langle\hat{O}\rangle = \frac{\int \mathrm{d}\mathcal{D}(\beta,\gamma,\Omega)\mathrm{e}^{(-\alpha\beta^2/2T)} z \; O(\beta,\gamma,\Omega,\omega,T)}{\int \mathrm{d}\mathcal{D}(\beta,\gamma,\Omega)\mathrm{e}^{(-\alpha\beta^2/2T)} z}, \tag{9}$$

where Ω stands for the Euler angles, whereas ω is the rotational or cranking frequency. The SPA partition function z is defined as

$$z(\beta,\gamma,\Omega,\omega,T) = \mathrm{Tr}\mathrm{e}^{-(\hat{H}^\omega - \mu_\mathrm{p}\hat{N}_\mathrm{p} - \mu_\mathrm{n}\hat{N}_\mathrm{n})/T} \tag{10}$$

with $\hat{H}^\omega = \sum_i \hat{h}^\omega(i)$ and $\alpha = (\hbar\omega_0)^2/\chi_Q$ and

$$\hat{h}^\omega = \hat{h} - \omega(\cos\theta\hat{j}_z - \sin\theta\cos\psi\hat{j}_x + \sin\theta\sin\psi\hat{j}_y) \tag{11}$$

with cranking axis as the z-axis in the laboratory frame.

The values of the chemical potentials $\mu_{\mathrm{p,n}}$ and the cranking frequency ω can be adjusted to satisfy the particle number and angular momentum constraints

$$N_{\mathrm{p,n}} = T(\partial lnz/\partial\mu_{\mathrm{p,n}}), \tag{12}$$

and

$$\langle\hat{J}_z\rangle = T(\partial lnz/\partial\omega). \tag{13}$$

2.1 GDR cross section

The GDR cross section in the intrinsic frame for a fixed shape is given by

$$\sigma_{\rm int}(E,\beta,\omega,T) = \sum_i \sigma_i(E,\beta,\omega,T), \tag{14}$$

with

$$\sigma_i(E,\beta,\omega,T) = -4\pi a E \,{\rm Im} R_{ii}(E,\beta,\omega,T), \tag{15}$$

β standing for all the deformation parameters including the Euler angles and $a = e^2/\hbar c$. The linearized Bethe–Salpeter equation [25] is to be solved to get the matrix

$$R(E) = (1 - R^0(E)\chi_D)^{-1} R^0(E) \tag{16}$$

with

$$(\chi_D)_i = \frac{3A}{NZ} m\omega_i^2 \,, \tag{17}$$

$$\omega_i \approx \omega_0 \left[1 - \sqrt{\frac{5}{4\pi}}\,\beta\cos\left(\gamma - \frac{2\pi}{3} i\right)\right] \,, \tag{18}$$

and

$$R^0 = R^0_{\rm p} + R^0_{\rm n} \,, \tag{19}$$

where

$$R^0_{ij} = \sum_{k,k'} \frac{\langle k \mid \hat{D}_i^\dagger \mid k'\rangle\langle k' \mid \hat{D}_j \mid k\rangle(f_{k'} - f_k)}{\epsilon_{k'} - \epsilon_k + E + i\eta}. \tag{20}$$

In the above $\mid k\rangle$ is the eigenstate of the cranked mean field Hamiltonian with an eigenvalue ϵ_k, and f_k is the Fermi distribution function. The dipole operators for protons and neutrons are defined as

$$(\hat{D}_i)_{\rm p} = \frac{N}{A}(x_i)_{\rm p} \,, (\hat{D}_i)_{\rm n} = -\frac{Z}{A}(x_i)_{\rm n} \,, \tag{21}$$

where $x_i = x, y, z$ for $i = 1, 2, 3$, respectively. Finally within the SPA the intrinsic GDR cross section is given by

$$\sigma_{\rm int}(E,\omega,T) = \frac{\int {\rm d}\mathcal{D}(\beta,\gamma,\Omega){\rm e}^{(-\alpha\beta^2/2T)} z(\beta,\gamma,\Omega,\omega,T)\sigma_{\rm int}}{\int {\rm d}\mathcal{D}(\beta,\gamma,\Omega){\rm e}^{(-\alpha\beta^2/2T)} z(\beta,\gamma,\Omega,\omega,T)}. \tag{22}$$

The cross section in the laboratory frame for a fixed shape is given by [4,13]

$$\sigma_{\mathrm{lab}}(E,\omega,T,\beta,\gamma,\Omega) = -4\pi a E \sum_{\mu} \,\mathrm{Im}\, R_{D_{1\mu}D_{1\mu}}(E-\mu\omega,\beta,\gamma,\Omega,T). \tag{23}$$

where the subscript $D_{1\mu}$ indicates that the matrix R is expressed in the spherical tensor representation. Then performing integration similar to that in eq. (22) one would obtain the cross section in the laboratory frame of reference.

It should be noted that if the orientation angles are set to zero, the limit of γ integration should be such that it allows for full quadrupole shape evolution with the possibility of rotation about and perpendicular to the prolate as well as the oblate symmetry axes.

3. Results and discussion

First we briefly discuss about some of the results on the application of the thermal fluctuation model of Ormand *et al* [13] outlined in the previous section. Mattiuzzi *et al* [12] reproduce the GDR parameters well and find that for ^{106}Sn the average value of the quadrupole deformation parameter $\langle\beta\rangle \approx 0.3$ at $T = 1.8$ MeV and $J = 0$, and it becomes of the order of 0.55 if J is increased to 60$\hbar$. For ^{120}Sn the increase in the average value $\langle\beta\rangle$ at $J = 60$ is about 0.15 compared to its $J = 0$ value. So, the effect of thermal fluctuation correction on the shape is found to be very large. In view of this it is rather surprising that there is no such discussion about the average value $\langle\gamma\rangle$. They seem to believe that the shape becomes oblate at $T = 1.8$ MeV and $J = 60$.

In [13] calculations are performed for ^{120}Sn and ^{208}Pb. The increase of the GDR width Γ with the increase of T is more or less in agreement with experimental data at $J = 0$. It is also found that up to about $J = 40$ there is no dependence on spin. But for the range $J = 40{-}60$ at $T = 1.6$ MeV the calculation shows that Γ increases by about 1.5 MeV for ^{120}Sn and about 1 MeV for ^{208}Pb. The increase for Pb seems rather large [30].

Next we would like to present some of our results. As in our recent paper [26], the dipole interaction strength χ_D is reduced by 25% as compared to the one given by eq. (17) above. This is required to get the GDR energy corresponding to the peak value of the cross section, E_0, close to the experimental value of about 16 MeV. Here we present our results for a model space consisting of 54 negative parity and 86 positive parity orbitals (total 140) extending the basis space up to the principal quantum number $N = 6$ major shells with ^{56}Ni as the inert core. The value of the core polarization factor f_c = 1.60 for ^{106}Sn and 1.75 for ^{120}Sn so that the equilibrium value of the quadrupole deformation parameter β_0 in the ground state is about 0.1. The spherical single particle energies are the spherical Nilsson model energies with A-dependent parameters [27]. Also the radial matrix elements of the quadrupole operator are reduced by factors $(N_0 + 3/2)/(N + 3/2)$, where $N_0 = 3$ and $N > 3$ are the principal quantum numbers. For a discussion on such factors see [28]. Following the z-axis cranking approach, as discussed in the previous section, to generate very high angular momentum states, recently [29] we have found that the effect of orientation fluctuation corrections on the GDR cross section is quite insignificant, at least for ^{106}Sn. Therefore, here we have computed the GDR cross sections at high spins following the standard principal axis (x-axis) cranking approach without the inclusion of the orientation fluctuation effects.

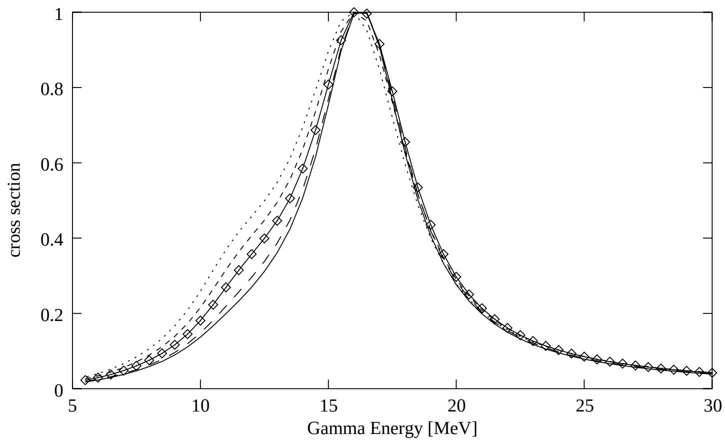

Figure 1. Normalized γ-absorption cross section $\sigma(E)$ of the GDR in ^{106}Sn at various temperatures and $J = 0$. The solid, long dashed, solid with diamond, short dashed and dotted curves correspond to $T = 0.5, 1, 2, 3, 4$ MeV, respectively.

Figure 1 shows the plot of $\sigma(E)$ at $\omega = 0$ for ^{106}Sn. The solid, long dashed, solid with diamond, short dashed, and dotted curves are the results at $T = 0.5, 1, 2, 3$ and 4 MeV, respectively. The diamond points indicate the value of E at which actual numerical calculations have been performed with $\delta E = 0.5$ MeV. The average values of the deformation parameters at the respective temperatures are as $\langle\beta\rangle = 0.120, 0.166, 0.226, 0.243$, and 0.249, and $\langle\gamma\rangle = 24.2°, 24.3°, 25.6°$ and $26.7°$. As the figure shows, the value of E_0 is about 16 MeV, with exact value as 16.3 MeV at $T = 0.5$ MeV and 15.9 MeV at $T = 4$ MeV. We obtain $\Gamma = 4.47, 4.67, 5.15, 5.54$ and 5.93 MeV at $T = 0.5$ to 4 MeV showing a net increase of only about 1.5 MeV at such a high temperature. This increase is by about 1.2 MeV in case of ^{120}Sn in our calculation. This is quite small compared to the experimental values [12,13].

Figure 2 depicts the variation of the GDR shape of ^{106}Sn as a function of J at $T = 2$ MeV. The value of the GDR energy is $E_0 = 16.2$ MeV at $J = 0$ and 16.5 MeV at $J = 69$. The five curves: solid, long dashed, solid with diamond, short dashed and dotted ones correspond to $J = 0, 26, 40, 57$ and 69, respectively. The corresponding values of the width Γ are $5.15, 5.83, 6.51, 7.48$ and 8.07 MeV. The width increases by about 3 MeV at the highest spin compared to its value at $J = 0$. The corresponding value for ^{120}Sn is found to be about 2 MeV. The average values of the shape parameters are as $\langle\beta\rangle = 0.226, 0.225, 0.224, 0.225$ and 0.228, and $\langle\gamma\rangle = 24.2°, 9.1°, -0.5°, -11.8°$ and $-18.8°$ at $T = 2$ MeV and $J = 0, 26, 40, 57$ and 69, respectively.

Thus, the average value of β remains around 0.23 whereas γ shows a change from positive triaxial to negative triaxial shape for increase of J from 0 to 69. In the mean field approximation at very high spins this nucleus is expected to exhibit a non-collective rotation about the oblate symmetry axis which corresponds to $\gamma = -60°$ in our convention. In this sense, the thermal average of about $-19°$ at $J = 69$ and $T = 2$ MeV should be quite reasonable. It may be emphasized that besides the constancy of $\langle\beta\rangle$, the width shows quite reasonable increase with the increase of J [12].

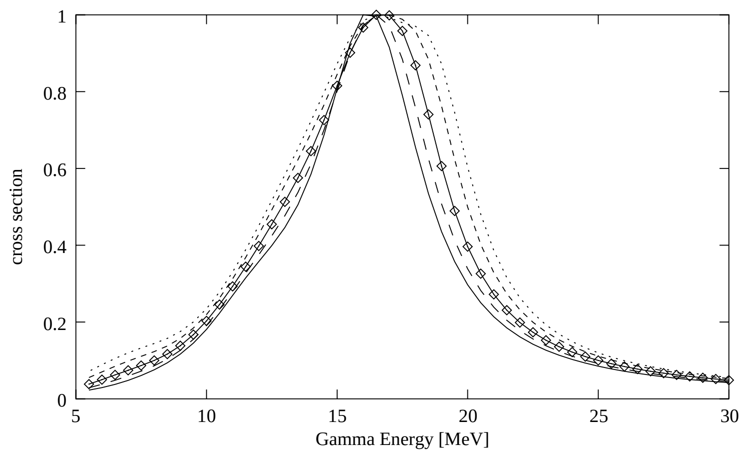

Figure 2. Same as in figrue 1 at $T = 2$ MeV and at various J values. The five curves: solid, long dashed, solid with diamond, short dashed and dotted curves are for $J = 0$, 26, 40, 57 and 69, respectively.

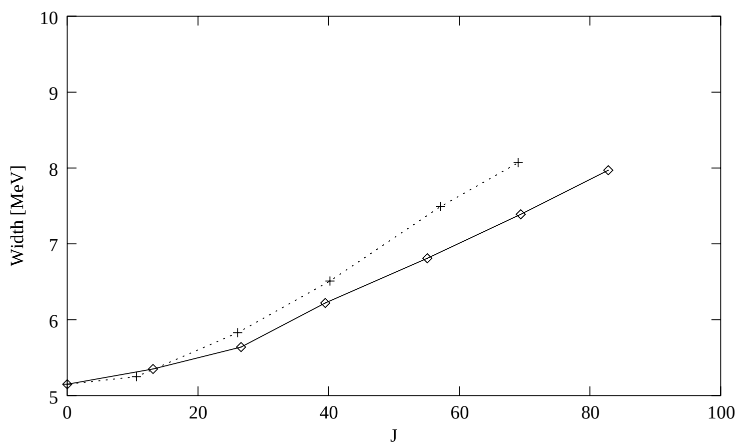

Figure 3. GDR width Γ as a function of J at $T = 2$ MeV for ^{106}Sn (dotted curve with crosses) and ^{120}Sn (solid curve with diamonds) following x-axis cranking. The points correspond to the actual values of J for which calculation is done.

Finally, in figure 3 we show the relative increase of Γ as a function of J at $T = 2$ MeV for ^{106}Sn and ^{120}Sn within the x-axis cranking approach. As expected [12,30], the increase of Γ is more rapid for the lighter isotope. The main problem in the present approach is that the increase of the width with temperature is not enough.

4. Conclusions

From our present study we would like to draw the following conclusions.

1. Given a many-body hamiltonian, a grand canonical partition function can be calculated within the SPA at a given value of temperature and spin (in cranking approximation). Then, including effects of thermal fluctuation corrections, an average energy and shape parameters can be computed.

We find that at the highest values of T and J considered here for Sn isotopes the average $\langle\beta\rangle$ comes out to be ≤ 0.25. On the other hand at $T = 2.0$ MeV the average value $\langle\gamma\rangle$ goes from a positive triaxial value to a negative triaxial value with the increase of J. This is not reported by others in the context of the GDR shape studies.

2. In view of the experimental data on GDR and their empirical connections with the average quadrupole shape parameters, it is desirable that within SPA the GDR energy and width are also calculated. We do this following the linear response theory at a finite temperature. However, when it is applied to the study of Sn isotopes using a simple $\hat{Q}\cdot\hat{Q}$ interaction hamiltonian, our results on Γ as a function of T are not satisfactory, though the variation as a function of J at a fixed T is quite good.

3. It seems that within SPA the thermal fluctuation corrections are not adequately incorporated, or more likely the use of a simple $\hat{Q}\cdot\hat{Q}$ hamiltonian is unrealistic.

4. Though the approach presented here is the best practicable approach available so far, it is still phenomenological in nature, the way the SPA theory is used in conjunction with the linear response theory. Thus, there is still a need of a profound theory.

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References

[1] J J Gaardhoje *et al*, *Phys. Rev. Lett.* **53**, 148 (1984)
[2] Z Majka *et al*, *Phys. Rev. Lett.* **58**, 322 (1987)
[3] J J Gaardhoje, *Phys. Rev. Lett.* **56**, 1783 (19986)
[4] M Gallardo, M Diebel, T Dossing and R A Broglia, *Nucl. Phys.* **A443**, 415 (1985)
[5] J J Gaardhoje, *Nucl. Phys.* **A488**, 261c (1988)
[6] A Bracco *et al*, *Phys. Rev. Lett.* **62**, 2080 (1989)
[7] W E Ormand, P F Bortignon and R A Broglia, *Nucl. Phys.* **A599**, 57c (1996)
[8] J J Gaardhoje, *Ann. Rev. Nucl. Part. Sci.* **42**, 483 (1992)
[9] A Bracco *et al*, *Nucl. Phys.* **A569**, 51c (1994)
[10] A Bracco *et al*, *Phys. Rev. Lett.* **74**, 3748 (1995)
[11] A Bracco *et al*, *Nucl. Phys.* **A599**, 83c (1996)
[12] M Mattiuzzi *et al*, *Nucl. Phys.* **A612**, 262 (1997)
[13] W E Ormand, P F Bortignon, R A Broglia and A Fracco, *Nucl. Phys.* **A614**, 217 (1997)
[14] J H Gundlach *et al*, *Phys. Rev. Lett.* **65**, 2523 (1990)
[15] A L Goodman, *Phys. Rev.* **C39**, 2008 (1989)
[16] B Lauritzen and G Bertsch, *Phys. Rev.* **C39**, 2412 (1989)

[17] G H Lang, C W Johnson, S E Koonin and W E Ormand, *Phys. Rev.* **C48**, 1518 (1993)
[18] A Ansari in *Advances in theoretical physics* edited by A P Pathak (Narossa Publishing House, New Delhi, 1996) p. 55
[19] B K Agrawal and A Ansari, *Phys. Rev.* **C46**, 2319 (1992)
[20] B K Agrawal and A Ansari, *Nucl. Phys.* **A567**, 1 (1994)
[21] B K Agrawal and A Ansari, *Phys. Lett.* **B421**, 13 (1998)
[22] B K Agrawal, A Ansari and P Ring, *Nucl. Phys.* **A615**, 183 (1997)
[23] R Rossignoli, A Ansari and P Ring, *Phys. Rev. Lett.* **70**, 1061 (1993)
[24] S Åberg, *Phys. Lett.* **B157**, 9 (1985)
[25] R Ring *et al*, *Nucl. Phys.* **A419**, 261 (1984)
[26] A Ansari, N D Dang and A Arima, *Phys. Rev.* **C62**, 011302(R) (2000)
[27] S G Nilsson *et al*, *Nucl. Phys.* **A131**, 1 (1969)
[28] M Baranger and K Kumar, *Nucl. Phys.* **A110**, 490 (1968)
[29] A Ansari, N D Dang and A Arima, *Phys. Rev.* **C63**, 24301 (2001)
[30] D Kusnezov, Y Alhassid and K A Snover, *Phys. Rev. Lett.* **81**, 542 (1998)