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Nuclear magnetic moment: Relativistic mean field description

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Abstract. Valence nucleon effective mass, which is almost constant, is proposed within the relativistic mean field theories of finite nuclei (closed shell $\pm$ one nucleon). It acquires a slight spin-orbit splitting due to relativistic effects. The relativistic Dirac magnetic moment operator $\vec{\mu}_{\rm op}$ is rewritten analytically in terms of angular momentum–Pauli spin coupled states and the effective mass. Introducing the nucleon effective charge, the iso-scalar and iso-vector corrections to the magnetic moment operator are extracted from the overall one parameter fit of the measured and the calculated values. The calculated values of magnetic moments are in overall fair agreement with the experiment as well as with the other detailed microscopic calculations.

Keywords. Relativistic mean field; nucleon effective mass; magnetic moment.

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1. Introduction

The nuclear magnetic moment μ has been a fascinating problem for nuclear structure physicists for more than four decades. Several different ideas and their implementations have emerged from the explanation of the deviations of measured μ from their Schmidt values. These include configuration mixing, meson exchange currents, electro-magnetic (EM) vertex correction, delta iso-bar effect, short range correlations, effect of core excited collective states and the relativistic effects (see for example [1–3]). A great deal of understanding of nuclear magnetic moments is achieved from this vast work. This activity led to the introduction of the general form of the effective magnetic moment operator $\vec{\mu}_{\rm op}$ as a linear combination of spin, angular momentum and tensor operators. The respective coefficients of these operators will depend explicitly on the detailed nuclear structure of the individual nucleus. The numerical study carried out so far is confined to light LS closed shell nuclei (mass number: 15, 17, 39 and 41) and the nuclei in the lead region. The study indicates a delicate cancelation of some of the corrections, specially those due to meson exchange currents while some others are small and further, the major corrections arise due to configuration mixing and core polarization. Barring a few investigations, all others are based on non-relativistic nuclear models. The relativistic models [1,2,4–6], in which core is treated in relativistic nuclear matter approximation, are also used in the calculation of μ. The relativistic mean field (RMF) theories [7–9] predicted an undesirable strong enhancement [1,2,4,6,10–14] for both the conventional iso-scalar and iso-vector orbital and spin magnetic moments. This discrepancy was remedied to some extent with the inclusion of ω–meson EM vertex correction (RPA in the nuclear matter approximation) [4,15,16] (see also [13]). We intend to study this problem here in more detail for better and clear understanding of this enhancement. During this process an effective mass of a single valence nucleon outside (missing from) a closed shell nucleus is defined and a simple, analytic relation between μ and this effective mass is derived. It provides a clear understanding of the overestimation of μ values in the RMF approximation. Using the theoretical values of the effective masses along with one parameter nucleon effective charge expression a global fit of the measured and calculated values of μ is carried out. The iso-scalar and iso-vector corrections to μ are extracted from the fit.

2. Formulation

The standard (nonlinear-σ, ω, ρ) interaction Lagrangian developed for nuclear physics applications is used with the RMF approximation. It is straight forward to write the coupled baryon spinor and the mesons mean field equations. The Lagrangian parameters, all the definitions and the notations are those from ref. [8]. The self consistent spherical solution for each nucleus under consideration is obtained using the usual numerical methods. This is indeed a good approximation for the nuclei selected for the present investigation. The outer most valence odd-nucleon Dirac orbital wave function (w.f.) satisfies

$$(-i\alpha\cdot\nabla + V(\vec{r}) + \beta M^*(\vec{r}))\psi(\vec{r}) \;=\; \varepsilon\psi(\vec{r}) \tag{1}$$

with the effective mass M^*

$$M^*(\vec{r}) = M + g_\sigma\sigma(\vec{r}) \quad \text{and}$$
$$V(\vec{r}) = g_\omega\omega^0(\vec{r}) + g_\rho\rho^0(\vec{r}).$$

The fourth components of the vector mean fields are $\omega^0(\vec{r})$ and $\rho^0(\vec{r})$ and M is the nucleon mass.

Let f and g be the radial wave functions of the upper and lower components of the Dirac spinor ψ. The matrix elements (ME) with respect to f and g are the radial integrals with the r^2 measure. From the Dirac radial equation for (f, g) it is easily seen that

$$\langle g|r|f\rangle \;=\; (K-1/2)/(2M) - \langle g|(M^*(\vec{r})/M-1)r|f\rangle - K/M\langle g|g\rangle. \tag{2}$$

Here $K = \pm(j+\frac{1}{2})$ for $j = l \mp 1/2$.

We now introduce the following approximation:

$$\langle g|(M^*(\vec{r})/M-1)r|f\rangle \;=\; \alpha\langle g|r|f\rangle \;=\; (\bar{M}^*/M-1)\langle g|r|f\rangle. \tag{3}$$

It is based on the observation that $M^*(r)$ is nearly a constant in the interior of a nucleus ($r \le R$) and that the product $rf(r)g(r)$ peaks in the nuclear surface region. This behavior is observed in all the RMF calculations carried out here. The value of α in eq. (3) may vary from nucleus to nucleus through its dependence on (f, g) and $M^*(r)$. In that case the relation (eq. (3)) loses its universality. The effective mass of a valence nucleon defined in eq. (3) is expected to be independent of its orbit and the nucleus. In order to verify this statement, we use the following relation obtained from eqs (2) and (3),

$$1/(2M(1+\alpha)) = \langle g|r|f\rangle/[(-(j+1),j) + p(lj)(2j+1)\langle g|g\rangle] = 1/[2\bar{M}^*], \quad (4)$$

where the phase $p(lj) = (-1)^{j+l-1/2}$, and the first choice in (j_1, j_2) is for $j = l + 1/2$ and the second is for $j = l - 1/2$.

The RMF coupled equations are solved self-consistently for the ground states of 11 odd-proton (outside (missing from) the closed core) and 12 odd neutron (outside (missing from) the closed core) nuclei using the Lagrangian parameter set NL1 [9]. In both the groups their ground state (gs) spins range from $1/2^\pi$ to $11/2^\pi$ for each parity $\pi = \pm 1$. In the single particle (sp) model of a nucleus treated here, these are the j^π quantum numbers of ψ. The numerical values of $\langle g|r|f\rangle$ and $\langle g|g\rangle$ for the odd valence nucleon orbitals are obtained from the RMF solutions of these nuclei. It is observed that the value of $\langle g|g\rangle$ is nearly the same, 0.025, for all the nuclei to a very good approximation. Using this, the values of $(M/\bar{M}^*)$ are calculated from eq. (4). They turn out to be almost independent of the radial, orbital and isotopic spin quantum numbers. Hence as expected, the valence nucleon effective mass in a finite nucleus is independent of a nucleus to a very good approximation. In our investigations we find the value of $(1/\bar{M}^*) \simeq 0.26$ (0.32) fm for the valence orbitals $j = l + 1/2$ ($j = l - 1/2$). The spin orbit splitting arises indirectly through the $\langle g|r|f\rangle$ dependence on the mean vector fields (ω^0, ρ^0) and the relativistic effect. The calculated values of effective masses are written in the following form:

$$\bar{M}^* = [0.73 + 0.07\, p(lj)]M. \quad (5)$$

The small spin-orbit splitting of the effective mass was noticed earlier by Ichii *et al* [14] in their study of ^{15}N, ^{17}O, ^{39}K and ^{41}Ca nuclei. The absence of its iso-spin splitting may be the reflection of weak mean ρ-field in a nucleus as compared to the mean ω-field. From eq. (5), the average value of the effective mass, $\bar{M}_0^* = 0.73$ M. It is not difficult to explain the increase of effective mass of a valence nucleon in a finite nucleus as compared to its value in nuclear matter. It is expected that the value of $M^*(r)$ deep in the interior $(r = 0)$ is equal to its value in the nuclear matter $M^*(r=0) = M + g_\sigma\sigma(0) = M^*_{\mathrm{nm}}$. The nucleon effective mass in nuclear matter is $M^*_{\mathrm{nm}} = 0.57$ M for the strong interaction Lagrangian parameter set NL1 [9]. In the finite nucleus the valence nucleon effective mass is $M^*(R) = M + g_\sigma\sigma(R)$, where R is the valence nucleon orbit radius. The value of R can be taken to be half-σ radius, $\sigma(R) = \sigma(0)/2$. From these observations one has

$$\bar{M}_0^* = M^*(R) = M^*_{\mathrm{nm}} - g_\sigma\sigma(0)/2 = (M^*_{\mathrm{nm}} + M)/2 = 0.785\,\mathrm{M},$$

for NL1 interaction parameter set. The above estimated value of $\bar{M}_0^* = 0.785$ M is slightly on the higher side as compared to its average value 0.73 M for the NL1 interaction. However, it suffices to explain the increase of $\bar{M}_0^*$ in finite nuclei as compared to its nuclear matter value.

3. Nuclear magnetic moment and effective mass relation

The relativistic Dirac magnetic moment operator $\vec{\mu}$ is given by

$$\vec{\mu} = e/2(\vec{r} \times \vec{\alpha}) + (g_a e/4M)\, \beta\, \vec{\Sigma}, \tag{6}$$

where g_a is the nucleon anomalous g-factor: $g_a = -3.826$ for neutron and $g_a = 3.586$ for proton. In the spherical RMF approximation, the w.f. $\psi(\vec{r}) = \langle \vec{r}|(l1/2)jm\rangle = |ljm\rangle$. It is easy to obtain the following matrix element of $\vec{\mu}$ with respect to the Dirac w.f. $|ljm\rangle$,

$$\begin{aligned}\langle ljm'|\vec{\mu}|ljm\rangle = e\langle g|r|f\rangle(ljm'|\vec{\sigma}\cdot\hat{r}\hat{r} - \vec{\sigma}|ljm) \\ +(g_a e/4M)[\langle f|f\rangle(ljm'|\vec{\sigma}|ljm\rangle \\ -\langle g|g\rangle(ljm'|2\vec{\sigma}\cdot\hat{r}\hat{r} - \vec{\sigma}|ljm)].\end{aligned} \tag{7}$$

The round brackets used here in eq. (7) denote the ME's with respect to the Pauli spin coupled total angular momentum states. Using eqs (7), (2) and (3) the magnetic moment operator defined with respect to Pauli spin-angular momentum coupled states $|ljm)$, can be written as

$$\begin{aligned}\vec{\mu}_{\rm op} = e/(2M(1+\alpha))(\vec{l}+\vec{\sigma}) + e/(2M(1+\alpha))p(lj)(2j+1)\langle g|g\rangle(\vec{\sigma}\cdot\hat{r}\hat{r} - \vec{\sigma}) \\ +(g_a e/(4M))\left[\vec{\sigma}(1 - 2/3\langle g|g\rangle) - 2/3\langle g|g\rangle(3\vec{\sigma}\cdot\hat{r}\hat{r} - \vec{\sigma})\right].\end{aligned} \tag{8}$$

Here $\hat{r}$ is a unit vector. This relation (eq. (8)) expresses μ in terms of the effective mass $\bar{M}^* = M(1+\alpha)$, as seen from eq. (4).

The first term of eq. (8) explicitly shows that both the orbital and spin contributions coming from relativistic Dirac magnetic moment are renormalized due to scalar-iso-scalar mean nuclear σ-field in the RMF approximation. This observation has also been noticed in earlier relativistic numerical calculations of a few nuclei [12,13]. Since $\langle g|g\rangle$ $(= 0.025)$ is a constant, the magnetic moment operator in eq. (8) does not depend on the radial wave function of the valence nucleon as in the case of usual Schmidt line values.

The anomalous part of $\vec{\mu}_{\rm op}$ is not normalized by the σ-field. It has been pointed out in the published literature that this part of $\vec{\mu}_{\rm op}$ arises due to the nucleon structure whose length scale is too small compared to the mean σ-field correlation length in a nucleus, hence it should not be affected by nuclear interactions.

All the relativistic corrections to the magnetic moment are proportional to $\langle g|g\rangle = 0.025$, which is rather small. Thus eq. (8) explicitly shows how and why the relativistic magnetic moments of all the finite nuclei are enhanced compared to their respective Schmidt values. It is to be noted that some of the relativistic corrections to $\vec{\mu}_{\rm op}$ involve the factor $(2j+1)$ which become substantial for high spin heavy nuclei.

4. Phenomenology of magnetic moments

The magnetic moment operator in eq. (8) is written for an odd-proton nucleus in its extreme spherical sp model. The major contributions those modify the values of the coefficients of different terms in eq. (8) come from core polarization and configuration mixing corrections, specially when the spin-flip partner states are excited. Since such calculations in finite nuclei are extremely difficult specially for high spin ground states of heavier nuclei, even in non-relativistic theories, a very few such studies have been reported [17] for high spin ground states of heavier nuclei within the relativistic nuclear models. Here, we resort to a phenomenological approach based on the expectation that the anomalous part of $\vec{\mu}$

operator is not changed much in the nuclear medium. Thus $\vec{\mu}$ (in units of $(e/(2M))$ is now written in iso-spin formalism as

$$\vec{\mu} = q(\vec{r} \times \vec{\alpha}) + (g/2)\beta\vec{\Sigma}, \tag{9}$$

with the nucleon effective charge

$$q = M[(e_{\rm n} + e_{\rm p}) + \tau_3(e_{\rm n} - e_{\rm p})]/2 \quad \text{and} \tag{10}$$

$$\begin{aligned} g &= [(g_{an} + g_{ap}) + \tau_3(g_{an} - g_{ap})]/2 \\ &= g_0 + \tau_3 g_1; \end{aligned} \tag{11}$$

$e_{\rm n}$ and $e_{\rm p}$ are the neutron $(\tau_3 = 1)$ and proton $(\tau_3 = -1)$ effective charges respectively. In writing eq. (9) we tacitly assumed that the corrections due to the tensor and other forms, would be small. The values of e_i $(i$ = n, p) are determined from the average fit of the calculated magnetic moments of all the nuclei under consideration to their experimental values. Such a procedure is expected to include at least phenomenologically some of the correlations and meson exchange effects in finite nuclei. One has then to add only the configuration mixing corrections to μ. Using the result in eq. (8) and collecting the terms together, the magnetic moment operator now becomes

$$\begin{aligned} \vec{\mu}_{\rm op} = &(q/(1+\alpha))\vec{l} \\ &+[2q/(1+\alpha) + g(1 - 2/3\langle g|g\rangle) - p(lj)(4/3)q(2j+1)\langle g|g\rangle/(1+\alpha)]\vec{s} \\ &-(2/3)[g\langle g|g\rangle - p(lj)q\,(2j+1)\,\langle g|g\rangle/(1+\alpha)](3\vec{s}\cdot\hat{r}\hat{r} - \vec{s}). \end{aligned} \tag{12}$$

The $\vec{\mu}_{\rm op}$ in eq. (12) has the general form proposed earlier [1,18]. The magnetic moment operator in eq. (12) has two parameters $e_{\rm p}$ and $e_{\rm n}$ to be determined phenomenologically. The nuclei, whose measured values μ of $\vec{\mu}_{\rm op}$ are considered (for the fit) fall in four different groups depending on whether the odd valence nucleon is a proton or a neutron in the orbits $j = l \pm 1/2$. Therefore the average fitting procedure with such a data would require different values of $e_{\rm p}(j)$ and $e_{\rm n}(j)$. Obviously such a procedure is not much useful. Therefore, some restrictions are to be imposed in order to reduce the number of free parameters to only one. The past investigations have shown that the iso-scalar magnetic moments of nuclei are not renormalized in nuclei to a very good approximation. From eq. (11) it is observed that g_0 is much less than g_1 indicating the consistency of the anomalous magnetic moment contribution with the observation. Therefore, we impose the required constraint that the iso-scalar induce charge is zero, [for $e_{\rm p} = 1 + \delta e_{\rm p}$ and $e_{\rm n} = \delta e_{\rm n}(\delta e_i, i = {\rm p}, {\rm n}$ are the proton and neutron induced charges)] this constraint amounts to $\delta e_{\rm p} = -\delta e_{\rm n}$. We note that the nucleon effective mass expression (eq. (7)) points out that the deviation of $\bar{M}^*$ from its average value is proportional to the phase $p(lj)$. Therefore, we constrained the induced charges in eq. (12) so that $\delta e_{\rm n}/(1+\alpha) = xp(lj)$. The value obtained from the average fit to the measured magnetic moments [19] of nuclei under consideration is $x = 0.2$. Using this the values of $e_{\rm p}$ and $e_{\rm n}$ the nucleon charge operator in eq. (10) can be written as

$$q \;=\; [0.365 + 0.035p(lj)] + \tau_3[-0.351 + 0.111p(lj)]. \tag{13}$$

Similarly, after some straightforward calculations, the $\vec{\mu}_{\rm op}$ in eq. (12) becomes

$$\vec{\mu}_{\rm op} \;=\; \sum_{i=0}^{1}(g_{li}\vec{l}+g_{si}\vec{s}+g_{pi}[3\vec{s}\cdot\hat{r}\hat{r}-\vec{s}])\tau_3^i. \tag{14}$$

The sum $i\;=\;0,1$ corresponds to iso-scalar and iso-vector respectively. In deriving the expressions of the g_i-coefficients the eqs (11)–(13) are used. The effective coefficients $g_{li}=g_{li}^0+\delta g_{li}$ and $g_{si}=g_{si}^0+\delta g_{si}$ are written into their bare (not renormalized) values g_{li}^0 and g_{si}^0 plus the empirically extracted corrections δg_{li} and δg_{si}. The empirical values thus derived are

$$\begin{aligned}
\delta g_{l0} &= 0.0;\; \delta g_{s0} \;=\; [0.0020-0.0167(2j+1)p(lj)]\,,\\
\delta g_{p0} &= [0.0020+0.0084(2j+1)p(lj)];\;\; \delta g_{l1} \;=\; 0.1955p(lj),\\
\delta g_{s1} &= [0.0619-0.0065(2j+1)]+[0.4000+0.0167(2j+1)]p(lj),\\
\delta g_{p1} &= [0.0618+0.0033(2j+1)-0.0083(2j+1)p(lj)].
\end{aligned} \tag{15}$$

From eq. (15) it is observed that the values of δg_{l0}, δg_{s0} and δg_{p0} are very small (of the order of relativistic corrections) as compared to their iso-vector values $[\delta g_{l1}(\delta g_{s1})=[x(2x)]\,p(lj)]$.

The earlier studies in the lead region, yielded the values (see for example [1]: $\delta g_l^{\pi}\;=\;0.13$, $\delta g_l^{\nu}\;=\;-0.08$, $\delta g_s^{\pi}\;=\;-2.0$ and $\delta g_s^{\nu}\;=\;2.0$. The iso-scalar contribution of these to δg_i's in lead (Pb) region are $\delta g_{l0}\;=\;0.025$, $\delta g_{s0}\;=\;0.0$, while in the lighter nuclei (e.g. in sd, pf shell) $\delta g_{l0}\;=\;0.02$ to 0.03, $\delta g_{s0}\;=\;-0.11$ to -0.14 and $\delta g_{p0}\;=\;0.001$ to 0.016. As compared to these, the values from eq. (15) for ^{209}Pb, ^{41}Ca and ^{17}O are respectively: $\delta g_{l0}=0.0$ for all, $\delta_{s0}=-0.165$, -0.132, -0.098 and $\delta g_{p0}=0.086$, 0.069, 0.052. Thus positive iso-scalar parts of δg_i's are consistent with the earlier investigations. However, in the case of iso-vector contributions the situation is different. The iso-vector correction to δg_i in the lead region determined from the above proton (π) and neutron (ν) values are $\delta g_{l1}\;=\;-0.105$, $\delta g_{s1}\;=\;2.0$ and the same in lighter nuclei are $\delta g_{l1}\;=\;0.02$ to 0.07, $\delta g_{s1}\;=\;-0.13$ to -0.44 and $\delta g_{p1}\;=\;0.68$ to 0.98. The signs of all the δg_i's should be reversed before comparing with those in eq. (15) because of the opposite iso-spin convention used in this reference. The corresponding values obtained from eq. (15) for ^{209}Pb, ^{41}Ca and ^{17}O are respectively, $\delta g_{l1}\;=\;0.2$ for all, $\delta g_{s1}\;=\;0.564$, 0.544, 0.523, $\delta g_{p1}=0.011$, 0.021, 0.032. The δg_{l1} and δg_{s1} values of the lighter nuclei and eq. (15) are of comparable magnitude while our value of δg_{p1} is too small. The calculated values of δg_{s1} in lead region is too large.

The calculated value of μ obtained from eq. (14) along with the experiment, the earlier calculations and the Schmidt values are listed in table 1. Clearly the global values extracted from eq. (14) agree well with the experiment as well as with the earlier calculations [13,17].

The core polarization contribution to μ can be obtained by subtracting the calculated values of the valence contributions (shown in parenthesis in the table 1) from the valence plus core contributions under the ref. [17]. Similarly the valence configuration mixing contributions is extracted by subtracting the Schmidt values from the corresponding contributions of the valence contributions (shown in parenthesis). The results reveal the following:

1. Near equal contributions for $h_{\rm n}$, $h_{\rm p}$, $p_{\rm n}$ and $p_{\rm p}$ ($h(p)$ represents the hole (particle) and the subscripts n and p refer to neutron and proton respectively) and their magnitude is rather small ranging between -0.13 to -0.40.

2. The valance configuration mixing contribution to μ is small for $h_{\rm n}$ and $p_{\rm n}$ while it is large for $h_{\rm p}$ and $p_{\rm p}$ up to $^{40}_{20}$Ca. This trend is reversed for heavier nuclei like $^{209}_{83}$Bi.

The present one parameter fit can be used to estimate the valence contributions by subtracting the average value -0.3 (roughly constant) of the core polarization.

The iso-scalar magnetic moment obtained by using eq. (14) are compared with the corresponding experimental and Schmidt values in table 2. The present values are indeed very

Table 1. Magnetic moment of odd-A ((p) odd-proton, (n) odd-neutron) nuclei. The entries in parenthesis corresponds to the valence contributions only of ref. [17].

Orbital	Nucleus	Cal.			Expt.	Schmidt
		Ref. [17]	Ref. [13]	Present		
$\frac{1}{2}^+$ p	$^{19}_{9}$F			2.536	2.629	2.793
$\frac{1}{2}^+$ n	$^{131}_{56}$Ba			-1.688	-0.709	-1.913
$\frac{1}{2}^+$ n	$^{115}_{50}$Sn			-1.688	-0.919	-1.913
$\frac{3}{2}^-$ p	$^{71}_{31}$Ga			3.311	2.562	3.793
$\frac{3}{2}^-$ n	$^{53}_{24}$Cr			-1.510	-0.475	-1.913
$\frac{3}{2}^-$ n	$^{61}_{28}$Ni			-1.510	-0.750	-1.913
$\frac{5}{2}^+$ p	$^{17}_{9}$F	4.901 (5.045)	4.99	4.077	4.722	4.793
$\frac{5}{2}^+$ p	$^{141}_{59}$Pr			4.077	4.275	4.793
$\frac{5}{2}^+$ n	$^{17}_{8}$O	-2.031 (-1.905)	-2.03	-1.325	-1.894	-1.913
$\frac{5}{2}^+$ n	$^{91}_{40}$Zr	-2.129 (-1.900)		-1.325	-1.304	-1.913
$\frac{7}{2}^-$ p	$^{41}_{21}$Sc	6.068 (6.412)	6.07	4.840	5.535	5.793
$\frac{7}{2}^-$ p	$^{53}_{25}$Mn			4.840	5.024	5.793
$\frac{7}{2}^-$ n	$^{41}_{20}$Ca	-2.197 (-1.905)	-2.13	-1.138	-1.595	-1.913
$\frac{7}{2}^-$ n	$^{49}_{22}$Ti			-1.138	-1.104	-1.913
$\frac{9}{2}^+$ p	$^{93}_{41}$Nb			5.603	6.170	6.793
$\frac{9}{2}^+$ p	$^{107}_{49}$In			5.603	5.585	6.793
$\frac{9}{2}^+$ n	$^{85}_{36}$Kr			-0.950	-1.005	-1.913
$\frac{9}{2}^+$ n	$^{87}_{38}$Sr			-0.950	-1.094	-1.913
$\frac{11}{2}^-$ p	$^{133}_{57}$La			6.364	7.500	7.793
$\frac{11}{2}^-$ n	$^{133}_{56}$Ba			-0.761	-0.910	-1.913
$\frac{1}{2}^-$ p	$^{15}_{7}$N	-0.149 (0.020)	-0.29	-0.267	-0.283	-0.264
$\frac{1}{2}^-$ p	$^{89}_{39}$Y	-0.122 (0.081)		-0.267	-0.137	-0.264
$\frac{1}{2}^-$ n	$^{15}_{8}$O	0.535 (0.667)	0.65	0.610	0.719	0.638
$\frac{1}{2}^-$ n	$^{207}_{82}$Pb	0.544 (0.673)		0.610	0.593	0.638
$\frac{3}{2}^+$ p	$^{39}_{19}$K	0.448 (0.855)	0.33	0.251	0.391	0.124
$\frac{3}{2}^+$ n	$^{39}_{20}$Ca	0.848 (1.170)	0.96	0.943	1.022	1.148
$\frac{5}{2}^-$ p	$^{85}_{37}$Rb			1.124	1.353	0.863
$\frac{5}{2}^-$ n	$^{63}_{28}$Ni			0.977	0.752	1.366
$\frac{5}{2}^-$ n	$^{205}_{82}$Pb			0.977	0.712	1.366
$\frac{7}{2}^+$ p	$^{133}_{55}$Cs			2.116	2.582	1.717
$\frac{7}{2}^+$ n	$^{111}_{50}$Sn			0.912	0.608	1.488
$\frac{9}{2}^-$ p	$^{209}_{83}$Bi	3.731 (5.066)		3.162	4.111	2.624
$\frac{9}{2}^-$ n	$^{135}_{60}$Nd			0.801	0.783	1.565

close to the respective values reported earlier [13,17] also listed in the same table as well as with the experiment.

The global values of μ calculated from eq. (14) are shown in figures 1 and 2 along with the measured magnetic moments [19]. There is a slight improvement in the agreement between the calculated values and the data points except for $l = 0$ as compared with the Schmidt values. Since other corrections discussed in the introduction are included, only on the average phenomenologically, better agreement over the whole atomic mass number, is not expected.

The calculations have been repeated with the recently introduced Lagrangian parameter set NLSH [20] and the results are very similar as expected. Therefore, the observations made here also equally hold for this set of Lagrangian parameters.

Table 2. Iso-scalar magnetic moments.

Orbital	Nucleus	Cal.			Expt.	Schmidt
		Ref. [17]	Ref. [13]	Present		
$1p_{3/2}$	${}^{11}_{5}B + {}^{11}_{6}C$	1.897 (2.251)		1.801	1.725	1.880
$1p_{1/2}$	${}^{13}_{6}C + {}^{13}_{7}N$	0.386 (0.615)		0.342	0.380	0.374
$1p_{1/2}$	${}^{15}_{7}N + {}^{15}_{8}O$	0.387 (0.687)	0.36	0.342	0.436	0.374
$1d_{5/2}$	${}^{17}_{8}O + {}^{17}_{9}F$	2.870 (3.140)	2.96	2.952	2.828	2.880
$1d_{5/2}$	${}^{27}_{13}Al + {}^{27}_{14}Si$	2.903 (3.565)		2.952	2.787	2.880
$1d_{3/2}$	${}^{39}_{19}K + {}^{39}_{20}Ca$	1.288 (2.025)	1.28	1.194	1.413	1.272
$1f_{7/2}$	${}^{41}_{20}Ca + {}^{41}_{21}Sc$	3.871 (4.507)	3.94	3.502	3.835	3.880

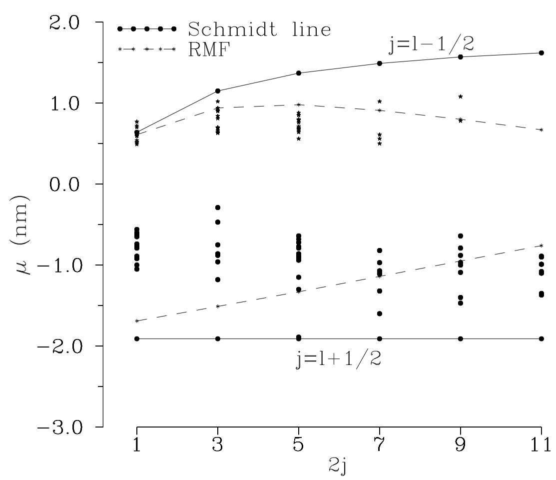

Figure 1. The magnetic moments of odd-neutron nuclei are shown as a function of $2j$. The upper and lower curves are for $j = l \pm 1/2$ respectively. The filled circles and stars are the measured magnetic moments of $j = l+1/2$ and $j = l-1/2$ nuclei respectively. The Schmidt lines and RMF lines are self explanatory.

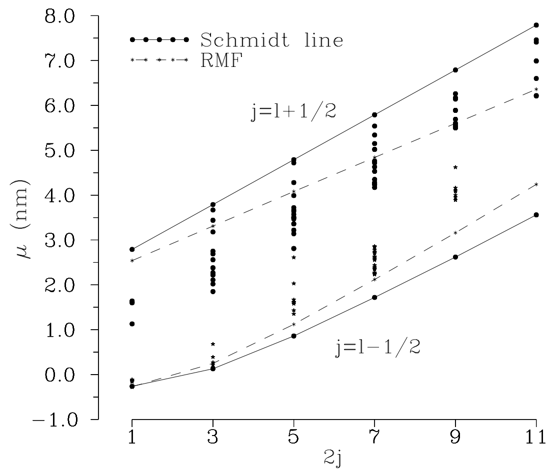

Figure 2. The magnetic moments of the odd-proton nuclei are shown as a function of $2j$. For the rest see caption of figure 1.

5. Conclusion

In this work, for the first time, the relativistic effective mass of a valence odd-nucleon in a nucleus is defined. The single particle nuclear magnetic moment in the RMF approximation is shown to have the same form as that of $\vec{\mu}_{\rm eff}$ emerged from the non-relativistic investigations. It is analytically demonstrated that both the spin and the angular momentum contributions to μ are enhanced in the RMF calculations due to the smaller effective mass. The one parameter fit of the calculated values of μ and the experimental data is performed.

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