

Supersymmetric unification in the light of neutrino mass

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Abstract. We argue that with the discovery of neutrino mass effects at super-Kamiokande there is a clear logical chain leading from the standard model through the MSSM and the recently developed minimal left right supersymmetric models with a renormalizable see-saw mechanism for neutrino mass to left right symmetric SUSY GUTS: in particular, SO(10) and $SU(2)_L \times SU(2)_R \times SU(4)_c$. The progress in constructing such GUTS explicitly is reviewed and their testability/falsifiability by proton decay measurements emphasized.

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1. Introduction

The recent observation by the super-Kamiokande detector of ‘intra-terrestrial’ flavour oscillations of muon neutrinos produced in the upper atmosphere is the first unambiguous experimental evidence of fundamental physics beyond the standard model of particle physics. It has injected a most welcome element of constraint into the fevered and ‘inspired’ speculative discourse that has characterized much of theoretical particle physics for over a decade and pointed in the direction in which the theory must be extended.

In this talk I shall argue, while adopting a minimalist scientific aesthetic (Occam’s razor, verifiability/falsifiability (Bacon/Popper) and ‘logical beauty of Nature’ (Einstein/Dirac) as the hallmarks of scientific discourse), that this very first experimental clue taken together with earlier compelling theoretical motivations points firmly to supersymmetric left right symmetric theories with a renormalizable see-saw mechanism for neutrino mass as the likely candidates for the next level of unification of force and matter.

The very first point to emphasize concerning the lessons of super-Kamiokande [1] is that it has underlined the ancient wisdom expressed inimitably in the Pratyabhinjyahridayam [2] of Kshemraja (Kashmir, *circa* 950 AC):

tannana anurupgraahayagraahakbhedaat

which translates as: ‘the universe becomes manifold by the differentiation of reciprocally adapted subjects and objects’. That is to say, like any physical theory, the standard model should not be regarded merely ‘mathematically’ as a formally consistent (‘axiomatic’) construct which incidentally codes all available data. Rather the SM should be regarded as an

effective field theory whose renormalizability and ‘unreasonable accuracy’ are signals that the current level of limitation ($E \leq 0.2$ TeV) on our experimental probes is far smaller than the scale of new physics Λ_N ; whose effects are therefore suppressed by this small ratio.

Recall that in the standard model the neutrino is an ‘odd-ball’ in the sense that besides being the only electrically neutral fermion it comes without a partner of opposite chirality. Therefore the lowest dimensional operator [3] that gives it a mass while respecting the symmetries of the SM is, schematically,

$$(H^\dagger L)^2/M, \quad (1)$$

where H is the scalar Higgs field, L the left handed lepton doublet and M an unknown mass scale characterizing this hitherto unknown phenomenon. When H develops a vev the neutrino acquire Majorana masses $\sim \langle H \rangle^2/M$. In the absence of new particles this description of neutrino mass is quite general. Applying Occam’s razor we shall adopt this minimal and general prescription for the incorporation of neutrino mass as the necessary extension of the SM indicated by the evidence for neutrino mass. From the favoured interpretation [1] of the super-Kamiokande data as evidence of tau neutrino mass $\sim 10^{-1.5}$ eV the approximate magnitude of the scale of the new physics leading to neutrino mass is thus $M \sim 10^{14 \pm 0.5}$ GeV.

The huge ratio M/M_W raises a fundamental difficulty in the viability of the SM as an effective field theory of fermions, gauge bosons and a scalar Higgs. As is well known, in QFT scalar masses (in contrast to fermions) receive radiative corrections which are quadratic in the scale at which the loop integrations are cut off. The existence of new physics at the high scale M to which the scalar Higgs is clearly well coupled thus implies that physical masses (such as that of the W and Z gauge bosons which incorporate the Higgs excitations) ~ 100 GeV receive radiative corrections $\sim M$ and thus require fine-tuning of bare parameters against these corrections order by order in loops. This is the crux of the so called ‘gauge-hierarchy’ problem. While fine-tuning is perfectly acceptable from a formal standpoint in the context of QFT, it is profoundly disquieting to our physical intuition to have to stabilize the low energy theory against corrections from far off scales in this ad-hoc manner. Thus much effort has been made to devise dynamical symmetry breaking schemes (technicolor, top condensation etc) that do not rely on fundamental scalars. However no satisfactory theory of this type has been found which can pass the various stringent constraints imposed by experiment. Therefore the alternative solution provided by (softly broken) supersymmetry has gained widespread acceptance in spite of the fact that, so far, the only ‘evidence’ in favour of this scheme is, at best, indirect and based on unproven hypotheses.

The supersymmetric resolution of the gauge hierarchy problem invokes the presence of superpartners of opposite statistics but otherwise mostly identical quantum numbers for each of the fields of the standard model. Thus chiral fermions are accompanied by complex scalars and vice versa while gauge bosons acquire Majorana fermion partners. Since Bose and Fermi loop corrections carry opposite signs and the relevant couplings are related by supersymmetry the troublesome quadratic dependence on the cut off cancels out leaving a correction of the form

$$\delta m_{H,W}^2 \sim \alpha_{EW} |m_{\text{Boson}}^2 - m_{\text{Fermion}}^2| \quad (2)$$

where the difference of mass between bosons and fermions is introduced by terms that

break supersymmetry only softly. Thus softly broken supersymmetric theories are insensitive to the details of the physics at much higher scales [4] and enable one to treat the supersymmetrized SM as a consistent and well defined effective field theory with local symmetry group G_{123} and a scalar doublet order parameter: as is consistent with the multitude of precision tests to which it has been subjected.

Given the indication of new physics at a high scale M and the need to supersymmetrize the theory in order that it be structurally stable against disruption by the unknown physics at some high scale it is natural to ask what might be the appearance of the theory at the large scale M . As is well known, in QFT the parameters of the lagrangian (couplings, masses etc.) should be taken to be scale dependent so as to control the effects of large logarithms of ratios of energy scales on the convergence of the perturbation series. One uses the renormalization group (RG) to run these parameters as a function of the energy scale (Q) to obtain the effective coupling at the scale of interest. Only fields which are light on the scale of interest contribute to the running up to that scale. Making the minimal assumption of no other new physics till the scale M , one can estimate the changes in the SM couplings (which are now known fairly accurately: better than 1% for the electroweak couplings and $\sim 5\%$ for the QCD coupling). One solves the RG equations for the gauge couplings $\alpha_i, i = 1, 2, 3$, of the SM gauge group G_{123} :

$$\frac{d\alpha_i}{dt} = \frac{1}{2\pi} b_i \alpha_i^2. \quad (3)$$

Here $t = \ln Q$, and we keep only terms to one loop order in gauge couplings and ignore the (small) effects of the Yukawa couplings of the matter fields which enter only at two loop order: For the standard model the one loop coefficients [5] are (N_g is the number of generations and N_H the number of Higgs doublets)

$$\vec{b} = \left(0, \frac{-22}{3}, -11\right) + \frac{4}{3} N_g (1, 1, 1) + N_H \left(\frac{1}{10}, \frac{1}{6}, 0\right). \quad (4)$$

At the time when the calculation of the running was first done (1975) till the mid 80's the EW couplings were known very approximately ($\sin^2 \theta_W = 0.215 \pm 0.014$ as of 1982 and the top quark mass was thought to be possibly as low as 20 GeV). The running showed that the three gauge couplings became equal to within the accuracy permitted by the low energy data for $N_g = 3$ and $N_H = 1$ at a scale $Q = M_{GUT} \sim 10^{14.5}$ GeV. Notice the coincidence of the unification scale with what we now strongly suspect to be the scale of the physics giving rise to neutrino mass. Much will be made of this in what follows. Furthermore, in 1982 Einhorn and Jones [6] and Marciano and Senjanovic carried out a detailed two loop analysis of the running for both supersymmetric and non-supersymmetric theories. They found that two loop corrections did not substantially alter the agreement with the then current value of $\sin^2 \theta_W$ (read together with the then prevalent lower limit on the top quark mass $m_t \geq 20$ GeV). However, the running of the couplings of the supersymmetric version of the SM clearly showed that the slower decrease of the QCD coupling due to the presence of additional color particles led to a unification scale $\sim 1.7 \times 10^{16}$ GeV while for $N_H = 2$ (the minimum value allowed by SUSY) the value of $\sin^2 \theta_W$ was as large as 0.233. This result was apparently incompatible with the data available at that time. With remarkable prescience they noted that the effect of the top quark mass on the ρ parameter of the standard model

$$\Delta\rho = \frac{3\alpha_{\text{em}}m_t^2}{16\pi\sin^2\theta_W m_W^2} + \dots \quad (5)$$

implied that the value of $\sin^2\theta_W$ would rise by ~ 0.017 if the top quark mass were ~ 200 GeV. Thus they *predicted* that in such a case the gauge coupling unification would occur *only* for the SUSY case while unification of couplings in the SM with one doublet would be ruled out. With the availability of precise data from LEP and the increasingly better lower limits for the top quark mass (culminating in its discovery at ~ 175 GeV) the conflict between coupling unification and the minimal supersymmetric standard model (MSSM) steadily ebbed while it developed and became sharper for the SM. In 1991 the analysis was redone [8] keeping the new data and its errors in view and the predictions of Einhorn, Jones, Marciano and Senjanovic were convincingly reaffirmed leading to a resurgence of interest in grand unified theories. The reason for their eclipse in the nonce having been that the lower scale of unification implied that the nucleon decay lifetime $\tau_N \sim (M_X/g_U)^4 \sim 10^{29\pm 1}$ yrs due to the exchange of SU(5) GUT gauge bosons was in conflict with the available experimental limits [9,10]. These also ruled out unification of SM couplings using larger N_H since those came with even lower values of M_X .

Given that the gauge couplings become equal at M_X it becomes natural to enquire whether the logic of EW unification via spontaneous breaking of gauge symmetry can be extended to further unify the three gauge groups of the SM and thereby explain peculiar clutch of fermion representations into which known matter is organized within the SM. The remarkable properties of the tracelessness of the electric charge operator $Q = T_{3L} + Y/2$, and the cancellation of gauge anomalies enjoyed by the chiral fermions of each generation of the SM, fairly cry out for the neat justification provided by embedding the SM in a GUT. As early as 1974 [11–13] it had been noted that the groups $G_{224} = G_{PS} = \text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(4)_c, \text{SU}(5), \text{SO}(10)$ were compellingly selected by the structure of standard model coupled with Occam's razor, while other possibilities were either more recondite or, since they included these groups as subgroups, more baroque. Each of the three minimal possibilities has certain special virtues and we shall conduct our discussion within the fairly general framework offered by them.

The first and earliest possibility, suggested by Pati and Salam [11], was the seminal idea of grand unification: consider the SM gauge generators to be linear combinations of the generators of a larger gauge group which leave the vacuum of the theory invariant. In other words the larger group is spontaneously broken at some large scale by the vevs of suitable Higgs fields. These vevs are left invariant by the SM generators so that the effective theory at lower energies had the symmetry of the SM till it in turn was broken by the SM Higgs vev. Pati and Salam noted that by promoting leptonicity to the status of a fourth colour, i) the SM gauge group could be embedded in $G_{224} = G_{PS} = \text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(4)_c$ with $\text{SU}(3)_c \in \text{SU}(4)_c$ and $Y/2 = T_{3R} + a\lambda_{15}$, ii) the fermion quantum number assignments of the SM then followed naturally from the identification of $\hat{Q}_L(2, 2, 4) \oplus \hat{Q}_L^c(2, 2, \bar{4})$ with the standard model fermions $(Q_L, L_L) \oplus (Q_L^c, L_L^c)$ where the antilepton doublet $L_L^c = (\nu^c, l^c)_L$. Notice the natural introduction of the chiral partner (ν_L^c) of the neutrino ν_L . As we shall see its mating with the neutrino, with the additional feature of majorana masses for each, in the context of the 'see-saw' mechanism [15], resolves her odd-ball and 'single-ular' status. The absence of ν_L^c from the observed low energy spectrum being attributed to both the fact that it is a SM singlet and so feels no EW gauge force and to the fact that being a SM singlet nothing protects it from obtaining a large mass.

iii) The symmetry breaking chain

$$\begin{aligned}
 G_{PS} &= \text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(4)_C \\
 &\xrightarrow{M_{P\bar{S}}} \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L} \times \text{SU}(3)_C \\
 &\xrightarrow{M_R} \text{SU}(2)_L \times \text{U}(1)_Y \times \text{SU}(3)_C
 \end{aligned} \tag{6}$$

can be realized by using the Higgs representations $A(1, 1, 15)$, $\delta^c(1, 3, 10)$, $\delta(3, 1, \bar{10})$ and $\phi(2, 2, 1)$ with

$$\langle A \rangle \sim M_{PS} \quad \langle \Delta^c \rangle \sim \langle \bar{\Delta}^c \rangle \sim M_R \quad \langle \phi \rangle \sim M_W. \tag{7}$$

Note the intermediate stage of symmetry breaking where G_{PS} is broken down to the so called ‘left-right symmetric’ gauge group $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L}$ which will be the focus of much of our discussion in what follows. Note also that the generator $\lambda_{15} \sim \text{Diag}(1, 1, 1, -3)$ of $\text{SU}(4)_c$ is, in appropriate normalization, nothing but the quantum number $B - L$ so that the electric charge in the LR model takes the appealing parity symmetric form [16]

$$Q = T_{3L} + T_{3R} + \frac{B - L}{2}. \tag{8}$$

By imposing a discrete symmetry (effectively parity) which interchanges the two $\text{SU}(2)$ factors one obtains a model in which the maximal P and C asymmetry of the SM is traced to the spontaneous breaking of the LR discrete symmetry. As we shall see this can be done while at the same time making ν_L^c heavy so that the neutrino’s ‘single-ularity’ is intimately tied up with source of the maximal parity violation inherent in the structure of the standard model.

The next possibility considered was $\text{SU}(5)$ [12] which has the appealing feature that it is a simple group. The SM gauge group is a maximal subgroup of $\text{SU}(5)$ and the SM families of fermions fit into a single (anomaly free!) pair of $\text{SU}(5)$ representations: $\bar{5}(d_L^c + L) \oplus 10(U^c, Q_L, e_L^c)$. The symmetry breaking from $\text{SU}(5)$ to G_{123} can be accomplished by the adjoint 24 of $\text{SU}(5)$ while the SM higgs doublet can be embedded in the fundamental 5. The principal point to note here is that the quantum numbers of the ν_L^c dictate that it is a $\text{SU}(5)$ singlet so that it is plausibly heavy at whatever scale $\text{SU}(5)$ is broken.

The final possibility Spin (10) (i.e $\text{SO}(10)$ plus fermions) is both inclusive (since it has G_{PS} and $\text{SU}(5)$ as subgroups) and aesthetically appealing since it is a simple group. Further a generation of fermions embeds neatly into a single irreducible representation of $\text{SO}(10)$: namely the irreducible spinor 16 which decomposes under G_{PS} as precisely $(2, 2, 4) \oplus (2, 2, \bar{4})$.

Various symmetry breaking chains such as

$$\begin{aligned}
 \text{SO}(10) &\xrightarrow{M_X} G_{PS} = \text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(4)_C \times D_{LR} \\
 &\xrightarrow{M_{P\bar{S}}} \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L} \times \text{SU}(3)_C \\
 &\xrightarrow{M_R} \text{SU}(2)_L \times \text{U}(1)_Y \times \text{SU}(3)_C,
 \end{aligned} \tag{9}$$

or

$$\begin{aligned}
\text{SO}(10) &\xrightarrow{M_X} G_{PS} = \text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(4)_C \\
&\xrightarrow{M_R} \text{SU}(2)_L \times \text{U}(1)_R \times \text{SU}(4)_C \\
&\xrightarrow{M_{PS}(M_{BL})} \text{SU}(2)_L \times \text{U}(1)_Y \times \text{SU}(3)_C
\end{aligned} \tag{10}$$

may be realized by using the Higgs multiplets 45, 54, 210 etc. While the Δ_s of the Pati–Salam model (whose vevs give neutrinos Majorana masses) embed neatly in the $126/126$.

Consider next the issue of neutrino mass. The crucial relevant facts about the neutrino in the SM are that:

(a) it is the only neutral fermion and can hence have a (Majorana) mass $M_{\nu\nu_L\nu_L}$ (in obvious two component notation) without breaking electromagnetic gauge invariance. However such a mass term is incompatible with the SM gauge invariance. A Majorana mass compatible with the SM can arise only via the $d = 5$ operator of eq. (1) or some even more suppressed operator

(b) it is unaccompanied by its chiral partner ν_L^c in strong contrast to the other fermions of the SM (we use the left handed anti-particle field rather than the right handed particle for uniformity of notation with the SUSY case)

(c) at the renormalizable level the SM enjoys an anomaly free global $\text{U}(1)_{B-L}$ symmetry which is violated by the operator of eq. (1). Thus the magnitude of neutrino mass is closely connected to the strength of $B - L$ violation.

Till recently all experimental data were compatible with all neutrinos having exactly zero mass. Observations at the 50 kT water Cerenkov detector super-Kamiokande in a deep mine in Japan during 1996–98 have shown that muon neutrinos produced in the upper atmosphere above the antipodes of super-K are depleted by the time they reach the detector while those from directly overhead show no deficit. The data are consistent with oscillations of the upcoming muon neutrinos into some other flavour of neutrino (ν_τ is favoured) whose mass differs from that of the muon neutrino by around $10^{-1.5}$ eV and which mixes with it the mass matrix mixing angle such that $\sin^2 \theta_M \geq 0.82$. There are also indications that electron neutrinos emitted by the sun are depleted due to flavour oscillations, into a neutrino with a mass squared difference of $\sim 10^{-5}-10^{-6}$ eV or a mass squared difference $\sim 10^{-10}$ eV and a large mixing angle. Finally the LSND collaboration reports that reactor muon neutrinos (from pion decays) possibly oscillate into ν_e with a mass difference as large as 1 eV. However this is in direct conflict with other reactor neutrino experiments such as KARMEN which show no such effect. The explanation of all three effects using neutrino flavour oscillations probably requires the invocation of a fourth ('sterile') flavour of neutrino which is neutral with respect to the standard model but very light. An experimental resolution of this controversy should be available in the next few years. Till that time it would be premature to assume the necessity of a sterile light neutrino.

Barring fine tunings, from mass differences in the range $10^{-2}-10^{-5}$ eV, we expect that the neutrino masses lie in the same range. For instance one could have m_{ν_τ} of the order of the super-K mass difference while $m_{\nu_\mu} \sim 10^{-3}$ eV $\gg m_{\nu_e}$ would account for the solar neutrino deficit via the MSW mechanism [14] given suitable values of the mixing angles. In any case it is clear that the presence of the neutrino masses militates for the presence of $B - L$ violating physics characterized by a scale $\sim 10^{13}-10^{16}$ GeV which is precisely in the range expected for the unification mass in GUT scenarios.

The next question is naturally as to what role the mass scale M plays in the new physics. The simplest and most natural explanation is provided by the ‘seesaw mechanism’ of [15]. If, like other SM fermions, the neutrino had a $SU(2)_L$ neutral chiral partner (ν_L^c), it, being color and charge neutral as well, would be a singlet with respect to the SM gauge group and might thus naturally take advantage of its ability to enjoy a Majorana mass term without breaking charge and pick up a mass characteristic of the high scale of $B - L$ breaking namely M . Moreover once it was present nothing would prevent the neutrino from pairing with it in a Dirac mass term so that while respecting the SM gauge symmetry and renormalizability one could add the following new terms to the Lagrange density:

$$(M_{\nu^c})_{ij}\nu_{L_i}^c\nu_{L_j}^c + h_{ij}H^\dagger L_i\nu_{L_j}^c. \quad (11)$$

Here the indices i, j run over the three flavours of neutrinos and M_{ν^c} is a matrix with eigenvalues $\sim M \sim M_{B-L}$. From the point of view of low energy physics at scales $E < 1$ TeV the observable physics will be coded in the (non-renormalizable) effective theory obtained by integrating out the superheavy neutrino states (and momentum components above that magnitude). This gives precisely the operator of eq. (1):

$$-h_{ij}(M_{\nu^c}^{-1})_{jk}h_{kl}H^\dagger L_i H^\dagger L_l. \quad (12)$$

The suppression of the left neutrino masses is directly related to the high masses of their chiral partners. This is called the Type I see-saw mechanism. In case the left neutrinos acquire (small!) Majorana masses M_{ν_L} from some other source then they will add to the above contribution giving the so called Type II see saw mechanism [17]. This happens quite inescapably in many of the natural unified models. We shall consider and drastically reduce their predictivity. For while the Yukawa couplings h_{ij} are related by the GUT to the masses of quarks the Type II additional contributions are not similarly constrained by the low energy data. Furthermore, one may extend the logic of the SM – where all masses arise via SSB – and introduce Higgs fields Δ^c capable of giving the ν^c fields Majorana masses when they acquire vevs $\sim M$. In that case, given mild conditions on the Higgs potential, the $B - L$ symmetry of the Lagrangian will be restored with $B - L(\nu^c) = +1, B - L(\Delta^c) = -2$. Thus the pattern of neutrino masses will arise from the spontaneous violation of $B - L$ in such a way that the low energy theory has a quasi exact $B - L$ symmetry violated only by the tiny left neutrino Majorana masses. This may nevertheless have important consequences for fundamental cosmological quantities such as the baryon to photon ratio since dressing of non-perturbative $B + L$ violating processes by the $B - L$ violating left neutrino masses could erase any $B - L$ (and therefore B) number created at early times [18].

2. The R -parity – LR symmetry connection

In the above discussion we have argued that the well verified standard model, together with data on neutrino oscillations and the contextual constraints of the formalism of effective QFT coupled with the minimalistic scientific aesthetic that has served us so well, motivate us to consider supersymmetric versions of the standard model in which the neutrino mass arises via a ‘seesaw’ mechanism. The technology of supersymmetrizing gauge theories is by now so well known that we shall not review it here but refer the reader to the excellent reviews available in the literature [19]. In the MSSM each of the SM fermion fields

$(Q, L, u^c, d^c, e^c)_L$ acquires a complex scalar partner with identical gauge quantum numbers while the gauge bosons acquire majorana fermion partners. The single Higgs doublet of the SM must however be replaced by at least a pair of doublets of opposite hypercharge together with their superpartners (Higgsinos) which are chiral fermions in order to cancel gauge hypercharge anomalies. Gauge invariant soft supersymmetry breaking terms (scalar masses $\phi^\dagger\phi$, trilinear scalar gauge invariants and gaugino masses) allow sufficient freedom to raise superpartner masses (thus respecting experimental constraints) and at the same time preserving the softening of loop divergences which motivated the introduction of SUSY.

However the presence of new scalar fields carrying the quantum numbers of matter fermions destroys one of the most appealing features of the SM namely the fact that gauge invariance and renormalizability ensure (perturbatively) exact B, L invariance of the Lagrangian. Since the quasi exactness of these symmetries is a fixed fact (indeed a prerequisite of our very existence!) this serendipitous corollary is not a minor gain in understanding. In the MSSM, on the other hand, there exist a number of new couplings which violate these crucial invariances. They may all be derived from the superpotential (we omit a possible LH term which may be removed by a redefinition of the Higgs and lepton fields, and have suppressed all gauge and generation indices)

$$W_{\mathcal{R}_p} = \lambda LLe^c + \lambda' LQd^c + \lambda'' u^c d^c d^c. \quad (13)$$

These new terms lead to catastrophic proton decay and a host of other exotic effects which are all known to be severely suppressed. For instance the product $\lambda\lambda'$ is thought to be constrained to be less than 10^{-25} by the absence of nucleon decay. Such small values of the couplings are very unnatural so that it is appealing to proceed on the minimal and natural assumption that these parameters are actually exactly zero (along with the corresponding scalar trilinear couplings in the soft SUSY breaking terms). In that case the Lagrangian requires $B - L$ symmetry albeit as an assumption rather than as an 'accidental' consequence of the model's symmetry and renormalizability.

At this point if we recall that neutrino masses are themselves signals of $B - L$ breaking, we may legitimately wonder if the MSSM plus neutrino mass would not reintroduce the terms arising from eq. (13) via radiative effects. However this is not so since the full power of $B - L$ invariance is not required for the purpose of forbidding these terms. It is sufficient to impose only a discrete Z_2 symmetry: the so called R -parity under which each 'new' type of field introduced by supersymmetry flips sign. It is a remarkable fact [20] that this symmetry is nothing but

$$R_p = (-1)^{3(B-L)+2S}, \quad (14)$$

where S is the field spin. Furthermore, completely equivalently, matter parity: $M_p = (-1)^{3(B-L)}$ accomplishes the same task without any loss of generality since the Lagrangian is bilinear in fermi fields and $(-1)^{2S} \in \text{spin}(1,3)$ (Lorentz group). Now note that the neutrino mass operator of eqs (1), (12) is in fact R -parity even and thus will not lead to catastrophic reintroduction of these terms by radiative corrections. This ad-hoc introduction of R -parity is, however, quite unsatisfactory, specially since global discrete symmetries are thought [21] to be violated by quantum gravitational effects. On the other hand the connection between R -parity and $B - L$ hints strongly at a deep connection between these symmetries. The overarching importance of $B - L$ in the standard model naturally motivates us to take this possibility seriously. It is then a pleasant surprise to realize that in

left–right symmetric theories $B-L$ is a gauge symmetry [16] and on supersymmetrizing R -parity is therefore a part of the gauge symmetry. Moreover these models accommodate the see-saw mechanism for neutrino mass in a very natural and convincing manner which singles them out as the prime candidates indicated by the discovery of neutrino mass.

In LR symmetric models [22] the gauge group of the SM is extended to $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$ while the SM fermions along with the chiral partner of the left neutrino group into the representations $L(2, 1, -1, 1) \oplus L^c(1, 2, 1, 1) \oplus Q(2, 1, 1/3, 3) \oplus Q^c(1, 2, -1/3, \bar{3})$. The Higgs sector consists of triplets $\Delta(3, 1, 2, 1) \oplus \Delta^c(1, \bar{3}, -2, 1)$ along with a bidoublet $\phi(2, 2, 0, 1)$ which contains a pair of SM doublets with $Y = \pm 1$. Mild conditions [19, 30] on the couplings are sufficient to implement a discrete symmetry which interchanges left with right chiral fermions $SU(2)_L$ with $SU(2)_R$, Δ with Δ^c etc. The system composed of the former can break $SU(2)_R \times U(1)_{B-L}$ down to $U(1)_Y$ at some scale larger than 1 TeV resulting in a model which is effectively the SM with the additional feature of a right handed neutrino. The see-saw mechanism can be implemented most naturally in this model since gauge invariance and renormalizability imply that the Yukawa couplings of the Higgs fields:

$$h_{ij}^a L_i \phi_a L_j^c + f_{ij} L_i \Delta L_j + f_{ij}^c L_i^c \Delta^c L_j^c + \text{h.c} \quad (15)$$

are precisely such as to give a large Majorana mass to the ν_L^c fields when the Δ^c field develops a large vev, while the vev of the ϕ field which is dominantly responsible for EW SSB gives rise to a Dirac mass term between ν_L and ν_L^c . Thus a see-saw mechanism occurs very naturally as a consequence of the hierarchy between $SU(2)_L \times U(1)_Y$ and $SU(2)_R \times U(1)_{B-L}$ breaking scales. Since the scalar potential in general allows couplings of the form

$$V = M^2 \Delta^2 + \Delta \phi^2 \Delta^c + \dots \quad (16)$$

It follows that once ϕ, Δ^c acquire vevs $\sim M_W, M$ respectively Δ acquires a vev due to the linear term generated:

$$\langle \Delta \rangle \sim M_W^2 / M \quad (17)$$

so that the seesaw is in general of Type II.

Finally it bears mention that doublets $\chi(2, 1, 1, 1) \oplus \chi^c(1, 2, -1, 1)$ may be used instead of triplets to break the left–right symmetry. The price one must pay is that the triplets with $B-L = \pm 2$ required to implement the see-saw mechanism must now be composites of the χ, χ^c fields so that the required terms are of dimension 5 and therefore non-renormalizable. Such models develop other ugly features once supersymmetry is introduced. In particular they violate R -parity maximally thus destroying part of the motivation for studying LR supersymmetric models. We shall not consider these models further in this talk and refer the reader to the literature where they have been discussed exhaustingly [23].

3. Minimal SUSY LR models

Recapitulating: the SM needs SUSY and the MSSM needs R/M -parity to be consistent with experiment. R/M -parity is essentially the $(-)^{3(B-L)}$ global subgroup of $U(1)_{B-L}$ symmetry, which is the only anomaly free continuous global symmetry of the SM and

remains a symmetry of the MSSM if R -parity is imposed. Thus in SUSY theories which gauge $B-L$ – among which the most natural and appealing are LR symmetric theories – R/M -parity is automatic at the level of the Lagrangian. Moreover these theories naturally incorporate the see-saw mechanism which explains the smallness of left neutrino masses as a consequence of the decoupling of their heavy (SM singlet) chiral partners. In such theories, when the seesaw is renormalizable (i.e the large majorana masses of the ν_L^c fields come from the vevs of fields Δ^c which carry $B-L = -2$) the breaking of $B-L$ symmetry by the $\langle \Delta^c \rangle$ still preserves R -parity. Thus the only possible source of R -parity violation in the low energy theory is a vev for the scalar partner of the ν_L, ν_L^c fields [24].

A significant advance [25] has been the realization that when – as is generically the case and as is now experimentally indicated – the scale of $B-L$ violation is $\gg M_W$, phenomenological constraints and the structure of the SUSY vacuum ensure that R -parity is preserved. Viable minimal LR supersymmetric models (MSLRM) have been constructed in detail [25–27] and embedded in GUTS while retaining these appealing properties.

Since the argument for R -parity exactness is so simple and general we present it first in isolation before going on to the details of the MSLRMs. Given $M_{B-L} \gg M_W \sim M_S$ (the scale of SUSY breaking) it immediately follows that the scalar partners of the ν_L^c fields also have positive mass squares $\sim M_{B-L}^2$ and hence are protected from getting any vevs modulo effects suppressed by these large masses. Thus when the ν_L^c superfield is integrated out the effective theory is the MSSM with R -parity and with $B-L$ violated only by (the SUSY version of) the highly suppressed operator of eq. (1). As a result if, for any reason, the scalar $\tilde{\nu}_L$ were to obtain a vev [24] the low energy theory would contain an almost massless scalar (i.e a ‘doublet’ (pseudo) Majoron) in its spectrum. However since such a pseudo-Majoron couples to the SM gauge fields like its superpartner the neutrino [24] it would make an appreciable contribution to the width of the Z gauge boson and is therefore conclusively ruled out by the very precise measurements of this quantity at LEP (for essentially the same reasons that the Z width limits the number of light neutrinos to be 3). It is worth emphasizing that these arguments, being based on decoupling, are very robust. In a particular note that the majoron/longitudinal $B-L$ mode expected from the spontaneous violation of a global/local $B-L$ symmetry at the large scale M_{B-L} will always decouple. The pseudo-majoron referred to would arise because *as a consequence of the R -parity of the model* the low energy theory has a quasi exact $B-L$ symmetry which becomes exact in the limit $m_{\nu_L} \rightarrow 0$ or equivalently $M_{B-L} \rightarrow \infty$. In fact it is easy to check that the leading contribution to the mass of this pseudo-majoron is $M_J^2 \sim M_S m_\nu$ which is far smaller than M_Z . Thus it is not possible to evade this argument by making the doublet majoron heavy as has been suggested in the literature [28]. It is also easy to check that the vev for $\tilde{\nu}_L^c$ induced via trilinear scalars in the soft SUSY breaking potential ($M_S \langle LH \rangle L^c + \dots$), after both $H, \tilde{\nu}_L$ obtain vevs, is so small ($\langle \tilde{\nu}_L^c \rangle \sim M_S M_W \langle \tilde{\nu}_L \rangle / M_{\tilde{\nu}^c}^2$) that it gives a contribution to the pseudo-Majoron mass

$$m_J \sim \frac{M_S m_\nu}{M_W} \quad (18)$$

which is even less effective in evading the Z -width bound. To sum up, quite generally [27]: The low energy effective theory of MSLRMs with a renormalizable is the MSSM with exact R -parity so the lightest supersymmetric particle (LSP) is stable.

Consider next the detailed structure of MSLRMs. Firstly, just as for the MSSM, the non-zero $B-L$ charges (+2, -2) of the Δ, Δ^c force one into introducing partners $\bar{\Delta}, \bar{\Delta}^c$

for both with opposite $B - L$ to cancel gauge anomalies due to their fermionic components. Then if one attempts to build a minimal renormalizable model with just these fields one finds that one cannot introduce Δ interactions in the superpotential with renormalizable terms only. Thus LR SSB is impossible. Historically the way around this was thought [29] to be the introduction of a parity odd gauge singlet field. In a surprising reanalysis [30] it was pointed out that in that model, in the generic case where $M_R \gg M_S$, there was a circular flat direction passing through the LR breaking SUSY vacuum and that once soft SUSY breaking corrections were switched on the vacuum inevitably preferred to settle in a charge breaking direction. Thus it was concluded that either one must give up renormalizability or one must settle for the restriction to the corner of parameter space where $M_R \sim M_{B-L} \sim M_W$, in which case it also followed that R -parity must be spontaneously broken. The latter alternative is now clearly quite unacceptable. In a series of papers [25–27] the generic case of large M_R was reanalyzed with particular attention to the structure of the SUSY vacuum using the powerful theorems available to characterize SSB in SUSY models. This has established the fields of generic MSLRMs (in addition to the supersymmetrized anomaly free set of fields of the LR model $(Q, Q^c, L, L^c, \phi, \Delta, \bar{\Delta}, \Delta^c \bar{\Delta}^c)$) as being one of the following:

- (a) Introduce a pair $\Omega(3, 1, 0, 1) \oplus \Omega^c(1, 3, 0, 1)$ of $SU(2)_{L/R}$ triplet fields. Then one can achieve SSB of the LR symmetry via the Ω s and separately the SSB of the $B - L$ symmetry, at an independent scale M_{B-L} by the $\Delta^c, \bar{\Delta}^c$ fields.
- (b) Stay with the minimal set of fields, but (reasoning that small non-renormalizable corrections must be counted when the leading effects are degenerate) include the next order $d = 4$ operators allowed by gauge invariance in the superpotential. These operators are of course suppressed by some large scale M and may be thought to arise either from Planck scale physics ($M \sim M_{\text{Planck}}$) or when one integrates out heavy fields in some GUT in which the LR model is embedded ($M \sim M_X$). The principal effect of allowing such terms is that the charge breaking flat direction is lifted and one obtains a phenomenologically viable low energy effective theory (with characteristic additional fields at the scale M_{B-L}^2/M_R : see below).
- (c) Finally one may introduce a parity odd singlet in either of cases (a), (b) which we shall for convenience refer to as cases (a') and (b'). This case is not quite academic or non-minimal since such parity odd singlets arise very naturally when one embeds these models in $SO(10)$.

Although these models contain a very large number of scalar fields the rigorous study of the structure of their vacuum is facilitated by a powerful theorem [31] applicable to SUSY vacua. Recall that the potential of a supersymmetric gauge theory has the positive definite form:

$$\begin{aligned}
 V_{\text{SUSY}} &= \sum_{\text{Chiral}} |F_i|^2 + \sum_{\text{Gauge}} D_\alpha^2 \\
 &= \sum_i \left| \frac{\partial W(\phi_i)}{\partial \phi_i} \right|^2 + \frac{1}{2} \sum_\alpha (g_\alpha \phi^\dagger T_\alpha \phi)^2.
 \end{aligned} \tag{19}$$

Thus the minimization involves finding the set of field values for which the D and F terms for the different gauge generators α and complex scalar fields ϕ_i vanish. One then has the following remarkable result:

Theorem. (a) *The set of all independent gauge invariant holomorphic invariants*

$x(\phi)_a, \bar{x}(\phi^\dagger)^a$ formed from the chiral fields furnish (complex) coordinates for the manifold of D -flat vacua.

(b) The invariants x_a left undetermined by the conditions $F(\phi)_i = 0$ are coordinates for the space of vacua that are both F and D flat i.e for the space of SUSY vacua.

A phenomenologically acceptable SUSY vacuum must be isolated from other vacua by barriers which prevent its decay into those vacua. Thus the existence of flat directions in field space (undetermined holomorphic invariant) connecting it with unacceptable vacua is not acceptable. On the other hand since the vacuum cannot break colour and electric charge it follows that the soft SUSY breaking terms must provide positive masses to scalars so that this disaster does not occur.

With the theorem one can characterize the flat directions out of the LR symmetry breaking vacuum [27] and show that they violate R -parity only if they also break charge and hence must be prevented from doing so by the soft SUSY breaking terms. Then it follows that the supersymmetric LR asymmetric vacua are isolated and the argument given above for the preservation of R -parity at all scales goes through without any difficulty. The detailed analysis of symmetry breaking also allows one to calculate the mass spectrum of the theory. Besides, the usual particles of the SM and their superpartners at M_S one finds that certain superfields associated with $SU(2)_R \times U(1)_{B-L}$ breaking remain relatively light and for favourable values of the parameters may even be detectable at current or planned accelerators. Thus in cases (a) and (a') one finds that a complete supermultiplet with the quantum numbers of $\Omega(3, 1, 0, 1)$ has a mass M_{B-L}^2/M_R . If $M_{B-L} \ll M_R$ then these particles could be detectable. However given the expectation of $M_{B-L} > 10^{14}$ GeV from neutrino mass this does not appear to be a likely possibility. In cases (b) and (b'), which one may consider as the truly minimal alternative one finds instead that the entire slew of fields $\Delta, \bar{\Delta}, \delta_{--}^c, \bar{\delta}_{++}^c, \delta_0^c + \bar{\delta}_0^c, H'_u, H'_d$ have masses $\sim M_R^2/M$. If, for instance, $M \sim 10^{19}$ GeV and $M_R \sim 10^{11}$ GeV then these particles could conceivably be detectable specially because they include exotic particles with charge 2 which are coupled to the usual light fermions of the model.

In the above H'_u, H'_d are a pair Higgs doublets left over after a fine-tuning to keep one pair of doublet superfields light out of the four (i.e two bidoublets) that must be introduced to allow sufficient freedom in the Yukawa couplings. In other words, with a single bidoublet field the restrictive form of the superpotential ensures that the up and down quark mass matrices are proportional, which is not acceptable. However a non trivial feature of these models is that the symmetry breaking at the right handed scales furnishes vevs that can discriminate between $SU(2)_L$ doublets with $T_{3R} = \pm 1/2$. With two bidoublets one can then ensure that the Yukawa coupling matrices couplings of one pair of light doublets are not mutually proportional.

In an interesting pair of papers following [26] (which originally pointed out the possibility of light doubly charged particles in the MSLRM), Mohapatra and collaborators [33,32] analyzed the phenomenology of light doubly charged particles and also extended the argument to include the lepto-quark Higgs in the Pati-Salam GUT. They find that if doubly charged lepto-quark scalars with masses ~ 100 GeV exist they will mediate exotic scattering processes such as $\mu^+ e^- \rightarrow \mu^- e^+$ with cross sections in the picobarn range and hence may be detectable at upcoming detectors. In the Pati-Salam case they find that entire lepto-quark $(3, 1, 10)$ multiplets can remain light and thus give rise to measurable rates for neutron-antineutron oscillations described by the effective operator $u_c^2 d_c^4$ which arises via the exchange of Δ s. This yields a value for the oscillation time

$$\tau_{N\bar{N}} \sim \frac{\lambda \langle \Delta^c \rangle f^3}{M_{d^c \bar{d}^c} M_{u^c \bar{u}^c}} \sim 10^9 - 10^{12} \text{ sec} \quad (20)$$

which may be detectable by upcoming experiments at Oakridge [34].

4. LR SUSY GUTS

As we have seen, *LR* SUSY models are natural candidates for SUSY unification which accommodates neutrino mass. Thus it is natural to consider further unification in which the various factors of the *LR* symmetry group are unified with each other. The two most appealing possibilities are unification within the Pati–Salam group $SU(2)_L \times SU(2)_R \times SU(4)_C$ and $SO(10)$. The multiplets 45, 210 of $SO(10)$ contain parity odd singlets [35] and the Pati–Salam gauge group is a subgroup of $SO(10)$. Thus the study of $SO(10)$ unification teaches one much about the Pati–Salam case as well. Therefore we [36] have re-examined $SO(10)$ SUSY unification [37–39] keeping in view the progress in understanding of *LR* SUSY models detailed above and developed a minimal $SO(10)$ theory of *R*-parity and neutrino mass with the appealing features of automatic *R*-parity conservation. A detailed and explicit study of the SSB at the GUT scale and various possible intermediate scales was performed. The mass spectra in various cases could be explicitly computed. In particular the pseudo-goldstone supermultiplets with possibly low intermediate scale masses ($\sim M_R^2/M_{PS}, M_{PS}^2/M_X$ etc.) that often arise in SUSY GUTS (see [38] for an early example involving $SO(10)$) were determined. With these computed (rather than assumed spectra) a preliminary one-loop RG survey of coupling constant unification in such models was carried out.

In $SO(10)$ matter parity *M* is a finite gauge symmetry, since under *M*

$$16 \xrightarrow{M} -16 \quad 10 \xrightarrow{M} 10 \quad (21)$$

and all other representations built out of the fundamental 10, such as 45, 54, 126, etc. are even. The symmetry in (21) is simply C^2 , where *C* is the center of $SO(10)$, so that under it $16 \rightarrow i16, 10 \rightarrow -10$. This points strongly towards using a 126-dimensional Higgs for the breaking of *B – L* and the see-saw mechanism.

We wish to construct a renormalizable $SO(10)$ theory with a see-saw, and this requires the minimum set of Higgs representations which break $SO(10)$ down to the MSSM:

$$S = 54 \quad A = 45, \quad \Sigma = 126, \quad \bar{\Sigma} = \bar{126}. \quad (22)$$

Although $SO(10)$ is anomaly-free one has to use both Σ and $\bar{\Sigma}$ in order to ensure the flatness of the *D*-piece of the potential at large scales $\gg M_W$.

The $SU(2)_L \times SU(2)_R \times SU(4)_C$ decompositions of the $SO(10)$ multiplets we use are as follows:

$$\begin{aligned} \psi = 16 &= Q(2, 1, 4) + Q^c(1, 2, \bar{4}), \\ S = 54 &= (1, 1, 1) + (1, 1, 20) + (3, 3, 1) + (2, 2, 6), \\ A = 45 &= \sigma(1, 1, 15) + \Omega(3, 1, 1) + \Omega^c(1, 3, 1) + (2, 2, 6), \\ \Sigma = 126 &= \Delta(3, 1, \bar{10}) + \Delta^c(1, 3, 10) + \phi'(2, 2, 15) + H'_C(1, 1, 6), \\ \bar{\Sigma} = \bar{126} &= \bar{\Delta}(3, 1, 10) + \bar{\Delta}^c(1, 3, \bar{10}) + \phi'(2, 2, 15) + \bar{H}'_C(1, 1, 6), \\ H = 10 &= \phi(2, 2, 1) + H_C(1, 1, 6). \end{aligned} \quad (23)$$

The most general superpotential one may build from the fields $S, A, \Sigma, \bar{\Sigma}$ involved in high scale gauge symmetry breaking is:

$$W = \frac{m_S}{2} \text{Tr} S^2 + \frac{\lambda_S}{3} \text{Tr} S^3 + \frac{m_A}{2} \text{Tr} A^2 + \lambda \text{Tr} A^2 S + m_\Sigma \Sigma \bar{\Sigma} + \eta_S \Sigma^2 S + \bar{\eta}_s \bar{\Sigma}^2 S + \eta_A \Sigma \bar{\Sigma} A. \quad (24)$$

Assigning vevs to suitable submultiplets as:

$$\begin{aligned} s &= \langle (1, 1, 1)_S \rangle \\ a &= \langle (1, 1, 15)_A \rangle \\ b &= \langle (1, 3, 1)_A \rangle \\ \sigma &= \langle (1, 3, 10)_\Sigma \rangle \\ \bar{\sigma} &= \langle (1, 3, \bar{10})_{\bar{\Sigma}} \rangle \end{aligned} \quad (25)$$

one can achieve two representative and interesting symmetry breaking chains which moreover present structural features in counterpoint. They are

(a) The case where $SU(2)_R$ is broken before the $SU(4)_c$:

$$\begin{aligned} \text{SO}(10) \quad \langle (1, 1, 1)_S \rangle = M_X \quad & G_{PS} = SU(2)_L \times SU(2)_R \times SU(4)_C \times D_{LR} \\ \langle (1, 3, 1)_A \rangle = M_R \quad & SU(2)_L \times U(1)_R \times SU(4)_C \\ \langle (1, 3, 10)_\Sigma \rangle = \langle (1, 3, \bar{10})_{\bar{\Sigma}} \rangle = M_{PS} (M_{BL}) \quad & SU(2)_L \times U(1)_Y \times SU(3)_C. \end{aligned} \quad (26)$$

In this case the renormalizable LR model without a parity singlet (case (a)) is embedded in the Pati–Salam model and that in $SO(10)$.

(b) In the other case $SU(4)_c$ is broken simultaneously with the breaking of the LR discrete symmetry while preserving the rest of the LR gauge group:

$$\begin{aligned} \text{SO}(10) \quad \langle (1, 1, 1)_S \rangle = M_X \quad & G_{PS} = SU(2)_L \times SU(2)_R \times SU(4)_C \times D_{LR} \\ \langle (1, 1, 15)_A \rangle = M_{PS} \quad & SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \\ \langle (1, 3, 10)_\Sigma \rangle = \langle (1, 3, \bar{10})_{\bar{\Sigma}} \rangle = M_R \quad & SU(2)_L \times U(1)_Y \times SU(3)_C. \end{aligned} \quad (27)$$

In $SO(10)$ GUTS the discrete LR symmetry may be naturally embedded in the gauge group. In fact $D_{LR} = \sigma_{23} \sigma_{67}$. Then it follows that the $\sigma(1, 1, 15)$ submultiplet of the 45 multiplet contains a SM singlet which is parity odd. Thus when this singlet is used for the second stage of SSB, LR symmetry is broken even though the gauge group is still $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$. So this case corresponds to embedding SUSY LR with a POS into $SU(2)_L \times SU(2)_R \times SU(4)_C$ and that into $SO(10)$. Integrating out the fields left heavy after GUT scale symmetry breaking down to the PS gauge group gives a non-renormalizable superpotential involving only the $(1, 1, 15)_A, (1, 3, 10 + \bar{10}), (3, 1, 10 + \bar{10})$ which is the PS generalization of the non renormalizable model with a parity odd singlet (Case (b')).

Notice that one needs both the multiplets $A(45)$ and the multiplet $S(54)$. For if one drops the symmetric multiplet (54) one finds that the vevs necessarily preserve $SU(5)$. While if

one drops the antisymmetric multiplet $A(45)$ one can break $SO(10)$ to G_{PS} but the vev of the 126 and $\bar{126}$ multiplets vanishes.

The non-trivial F equations are

$$\begin{aligned}
 F_{(1,1,1)_S} &= m_s s - \frac{1}{2}\lambda_S s^2 + \frac{2}{5}\lambda(a^2 - b^2) = 0, \\
 F_{(1,1,15)_A} &= m_A a + 2\lambda_A a s + \frac{1}{2}\eta_A \sigma \bar{\sigma} = 0, \\
 F_{(1,3,1)_A} &= m_A b - 3\lambda_A b s + \frac{1}{2}\eta_A \sigma \bar{\sigma} = 0, \\
 F_{(1,3,10)_\Sigma} &= \sigma [m_\Sigma + \eta_A(3a + 2b)] = 0, \\
 F_{(1,3,10)_{\bar{\Sigma}}} &= \bar{\sigma} [m_\Sigma + \eta_A(3a + 2b)] = 0,
 \end{aligned} \tag{28}$$

while the D equations demand only $\sigma = \bar{\sigma}$.

The choice of the ratio a/b determines the chain of breaking. Note that a, b must both be non-zero for the $\sigma\bar{\sigma}$ to be non-zero. Thus there is no Dimopoulos Wilczek mechanism here unless $M_R^2/M_X \sim M_W$ or $M_{PS}^2/M_X \sim M_W$.

- (a) $s \sim M_X \gg b \sim M_R \gg \sigma = \bar{\sigma} \sim M_{PS} \gg a \simeq M_{PS}^2/M_X$ corresponding to the chain of eq. (26) is achieved by fine-tuning

$$m_A - 3\lambda_A s \simeq \frac{\sigma^2}{b} \ll s \tag{29}$$

which then ensures

$$a \simeq \frac{\sigma^2}{s} \ll \sigma \tag{30}$$

$$s \sim M_X \gg b \sim M_R \gg \sigma = \bar{\sigma} \sim M_{PS} = M_{B-L} \gg a \simeq M_{PS}^2/M_X.$$

- (b) Similarly for case (b) one interchanges the roles of a and b .

$$s \sim M_X \gg a \sim M_{PS} \gg \sigma = \bar{\sigma} \sim M_R \gg b \simeq M_R^2/M_X$$

corresponding to the chain of eq. (27) is achieved by fine-tuning

$$m_A + 2\lambda s \simeq \frac{\sigma^2}{a} \ll s \tag{31}$$

which then ensures

$$b \simeq \frac{\sigma^2}{s} \ll \sigma. \tag{32}$$

Note that we have ignored the vevs of the superpartners of the matter fields in the 16 along with the vevs $\langle 10_H \rangle$ since these would break the SM symmetries which we know to be preserved to be far below the scales under discussion. Moreover the couplings

Table 1. Mass spectrum for the symmetry breaking chain $SO(10) \xrightarrow{M_X} SU(2)_L \times SU(2)_R \times SU(4)_C \xrightarrow{M_R} SU(2)_L \times U(1)_R \times SU(4)_C \xrightarrow{M_{PS}} SU(2)_L \times U(1)_Y \times SU(3)_C$. The states in $(1, 3, 10)$ and $(1, 3, \overline{10})$ were decomposed according to their T_{3R} number, for example $(1, +, 10)$ denotes the component of $(1, 3, 10)$ with $T_{3R} = +1$, etc.

State	Mass
all of S in 54	
all of A in 45, except $(3, 1, 1)_A + (1, 3, 1)_A$	$\sim M_X$
all of Σ in $126 + \overline{\Sigma}$ in $\overline{126}$, except $SU(4)_C$ decuplets	
$(3, 1, \overline{10})_\Sigma + (3, 1, 10)_{\overline{\Sigma}}$	
$(1, \overset{0}{-}, \overline{10})_\Sigma$ and $(1, \overset{+}{0}, 10)_{\overline{\Sigma}}$	$\sim M_R$
ω_c^\pm from $(1, 3, 1)_A$	
color triplets and singlets from $(1, +, \overline{10})_\Sigma$ and $(1, -, 10)_{\overline{\Sigma}}$	$\sim M_{PS}$
$(3, 1, 1)_A$	$\sim \text{Max} \left[\frac{M_R^2}{M_X}, \frac{M_{PS}^2}{M_R} \right]$
color sextets from $(1, +, \overline{10})_\Sigma$ and $(1, -, 10)_{\overline{\Sigma}}$	$\sim M_{PS}^2/M_X$

$W = \psi\psi H + \psi\psi\overline{\Sigma} \dots$ in the superpotential imply that if 10 vev is negligible then the 16 vev (in the $\tilde{\nu}^c$ direction; the other vevs break SM symmetries) is zero if $\overline{126}$ vev is not. So that the protection of $\langle \tilde{\nu}^c \rangle = 0$ carries over from the LR symmetric case.

With these solutions of the SUSY potential minimization equations in hand we can calculate the mass spectrum of the theory. One obtains [36] in case (a) as shown in table 1.

Similarly for case (b) we get as shown in table 2.

Notice particularly how the left handed Δ_s become superheavy ($M \sim M_X$) while the right handed ones do not become superheavy due to breaking of D_{LR} . Given these mass spectra one may carry out the RG analysis of the gauge coupling evolution. As data one has the values $\alpha_i(M_Z), i = 1, 2, 3$ while the mass scales M_S, M_{PS}, M_R, M_X in case (a) and $M_S, M_{B-L}, M_{PS}, M_X$ in case (b) together with the value of the coupling constant at unification α_U are unknowns. Since the deviation of $\log M_S$ from $\log M_Z$ cannot be very large it may be ignored in an approximate one-loop analysis. Then one obtains a one parameter family of solutions for the mass scales of interest. For instance in case (b) one obtains for the case of two light Higgs doublets ($t = \log_{10} (M/\text{GeV})$)

$$t_X = 16.4t_{PS} = 14.7 + 0.07/\alpha_U, t_R = 13.9 + \frac{0.1}{\alpha_U}, \quad (33)$$

while consistency of the assumptions made regarding the relative magnitudes of the intermediate geometric scales ($M_{PS}^2/M_X, M_R^2/M_{PS}$) requires that $\alpha_U^{-1} < 23.6$.

If one assumes that an additional pair of Higgs doublets begins to contribute above the scale M_R^2/M_X (as indicated by the symmetry breaking in case (b) of the LR SUSY model) then one finds (here $\bar{t} = \log_{10} Q$):

Table 2. Mass spectrum for the symmetry breaking chain $SO(10) \xrightarrow{M_X} SU(2)_L \times SU(2)_R \times SU(4)_C \xrightarrow{M_{PS}} SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \xrightarrow{M_R} SU(2)_L \times U(1)_Y \times SU(3)_C$.

State	Mass
all of S in 54 all of A in 45, except $(1, 1, 15)_A$ all of Σ in $126 + \bar{126}$ in $\bar{126}$, except $SU(4)_C$ decuplets Color triplets $H_C(1, 1, 6)$ in 10.	$\sim M_X \sim \langle S \rangle$
$(3, 1, \bar{10})_\Sigma + (3, 1, 10)_{\bar{\Sigma}}$ By D_{LR} breaking color triplets and sextets of $(1, 3, \bar{10})_\Sigma$ and $(1, 3, 10)_{\bar{\Sigma}}$ color triplets of $(1, 1, 15)_A$ (Higgs)	$\sim M_{PS} \sim \langle A \rangle$
ν^c $\delta_c^0 + \bar{\delta}_c^0, \delta_c^\pm, \bar{\delta}_c^\pm$ from the color singlets of $(1, 3, \bar{10})_\Sigma$ and $(1, 3, 10)_{\bar{\Sigma}}$	$\sim M_R \sim \langle \Sigma \rangle$
Color octet and singlet in $(1, 1, 15)$	$\sim \text{Max} \left[\frac{M_R^2}{M_{PS}}, \frac{M_{PS}^2}{M_X} \right]$
$\delta_c^-, \bar{\delta}_c^{++}, \delta_c^0 - \bar{\delta}_c^0$ from the color singlets of $(1, 3, \bar{10})_\Sigma$ and $(1, 3, 10)_{\bar{\Sigma}}$	$\sim M_R^2/M_X$

$$t_X = 14.2 + 0.09/\alpha_U; \quad t_{PS} = 12.7 + 0.15/\alpha_U, \quad t_R = 9.9 + \frac{0.28}{\alpha_U}. \quad (34)$$

Here consistency requires that $\alpha_U^{-1} < 22.7$ while the current bounds on proton decay mediated by gauge particles imply that $t_X > 15.5$ so that $t_P > 14.7, t_R > 13.5$. Similar results obtain in case (a). Generically requiring $t_X \geq 15.5$ while $M_R, M_{PS}, M_{B-L} \geq 10^{13}$ GeV or so. Even given the uncertainties of the analysis in which we have ignored threshold and 2-loop effects etc. The qualitative conclusion is clear: these models are victims of a sort of ‘SU(5) conspiracy’ as far as the gauge breaking chain is concerned since the intermediate scales are so close to the GUT scale.

5. Fermion masses and proton decay

The complicated question of realistic mass spectra in the class of SO(10) SUSY GUTS we have focused on has been considered by several sets of authors following the work of Babu and Mohapatra [39,40]. As usual in GUTS the fermion masses derived from the GUT dictated relations between Yukawas must be RG improved by running down to EW scales. Making the assumption that the light bidoublets that give masses to the charged fermions are in fact a mixture of the bidoublets contained in the 10 plet and $\bar{126}$ Higgs which have suitable Yukawa couplings it has been found that even without the freedom allowed by the Type II seesaw mechanism for neutrino mass it is possible to fit the observed fermion

masses if one takes the light bidoublets to be a mixture of those contained in two 10 plets and a single $\overline{126}$ plet provided the LSND data are discounted. However the precise way in which this light mixture arises (while also making all colored fields superheavy) has not been worked out.

Since the generation-wise freedom to choose the values of the Yukawa couplings continues to be present, SO(10) alone does not shed much light on the pattern of fermion masses. However recent work [41] has pointed out interesting connections between seesaw neutrino masses and proton decay via $d = 5$ operators in SUSY SO(10) GUTS. Recall that in traditional GUTS the exchange of superheavy gauge bosons mediates proton decay leading to a 4 fermion $\Delta B = \Delta L = 1$ operator ($qqql$) with coefficient $\sim g^2/M_X^2$. This implies a nucleon decay dominantly in the πe channel with a lifetime $\sim 10^{28\pm 1} (M_X/10^{14.5}\text{GeV})^4$ yrs. Since the current limits on nucleon decay lifetime τ_N via these channels are $> 10^{33}$ yrs [1] it follows that the non-supersymmetric case is contraindicated. On the other hand since $M_X \sim 10^{16}$ in the minimal SUSY SU(5) GUT, it is compatible with this limit, as is any theory which reduces to it at a sufficiently high scale (like the examples above). It is perhaps worth remarking that with the lower values of M_X possibly in the SO(10) case the $d = 6$ nucleon decay operators with their characteristic flavour diagonal decay modes may be observable in contrast to the minimal SUSY SU(5) case.

However in SUSY theories there is a much faster source of nucleon decay via $d = 5$ operators [42–44] that arise from the exchange of the color triplet partners of the EW Higgsinos between two fermions and two sfermions. After dressing by gaugino exchange to convert the scalars into fermions one obtains the effective four fermi operator but with a coefficient which is uncertain due to the uncertainty in SUSY breaking parameters and in $\tan\beta$. For low to moderate $\tan\beta$ wino dressed diagrams are dominant over those with dressing by other gauginos. Since constraints of Bose symmetry and color antisymmetry imply that the 4 fermi operators must be flavor non-diagonal the dominant decay of nucleons is ($N \rightarrow K\nu_\mu$) with life times [44]:

$$\begin{aligned}\tau(p \rightarrow K^+ \bar{\nu}_\mu) &\sim \left| \frac{M_{H_C}}{10^{17}\text{GeV}} \frac{M_S}{\text{GeV}} \right|^2 10^{32}\text{yrs}, \\ \tau(p \rightarrow K^+ \bar{\nu}_e) &\sim \left| \frac{M_{H_C}}{10^{17}\text{GeV}} \frac{M_S}{\text{GeV}} \right|^2 10^{33}\text{yrs}, \\ \tau(n \rightarrow K^0 \bar{\nu}_\mu) &\sim \left| \frac{M_{H_C}}{10^{17}\text{GeV}} \frac{M_S}{\text{GeV}} \right|^2 10^{31.5}\text{yrs}.\end{aligned}\tag{35}$$

Charged lepton modes are suppressed relative to these neutrino modes by 10^4 or more. Current experimental limits are in the region of 10^{33} yrs and improving. Thus $d = 5$ decay modes represent a strong constraint on SUSY GUT models which are already (at least for minimal models) on the verge of being ruled out. For large $\tan\beta$ [45] gluino and Higgsino dressed diagrams which are flavor suppressed can become important leading to much larger charged lepton decay rates making them comparable with the dominant modes in the low $\tan\beta$ case. Thus, if observed, nucleon decay can provide insight into the EW symmetry breaking in SUSY theories and in particular into the value of $\tan\beta$.

Finally in a notable pair of papers Babu, Pati and Wilczek [41] have recently pointed out that in SO(10) theories with either a renormalizable (mediated by $W = \overline{126}_H 16_M 16_M + \dots$) or a non renormalizable (mediated by $W = (\overline{16}_H)^2 16_M^2/M + \dots$) seesaw mechanism for

neutrino mass there exist $d = 5$ nucleon decay operators that arise via exchange of color triplet Higgsinos from the $\bar{1}26$ or $\bar{1}6$ respectively. The strength of these operators is thus directly linked to the Yukawa coupling ($f \sim m_{\text{Dirac}}^2 / (m_{\nu_L} \langle \Delta^c \rangle)$) between the seesaw Higgs and the matter fields. Assuming the neutrino Dirac masses at the GUT scale take the values suggested by SO(10) ($\sim 1\text{--}10$ MeV, $1\text{--}4$ GeV, $100\text{--}120$ GeV) and a suitable right handed Majorana neutrino mass matrix to ensure compatibility of the see-saw masses with the values suggested by the atmospheric and solar neutrino data gives $f_{33} \sim 5 \times 10^{-2}$. A detailed analysis then shows that even at low $\tan \beta$ the charged lepton $d = 5$ decay modes can become comparable to the neutrino final state modes. Moreover the lifetimes suggested are on the borderline of conflict with the current experimental limits. Thus this work has opened an interesting and amazing direct connection between the seesaw mechanism for neutrino mass and nucleon decay in GUTS: two topics that earlier were thought to be quite separate.

It is worth noting here that the classic mechanisms for cosmic baryogenesis via GUT scale baryon number violation have fallen into disfavor since it was realized that the standard model leads to unsuppressed $B + L$ (but not $B - L$) violation at high temperatures due to non-perturbative processes. However these very processes can be enlisted [18] to provide a very robust mechanism for leptogenesis at high scales based on the lepton number and CP violating out of equilibrium decay of right handed neutrinos at temperatures $> 10^{10}$ GeV. This lepton asymmetry is then equilibrated by non-perturbative processes into a baryon and lepton asymmetry.

To summarize

- There is a clear logical chain leading from the SM with neutrino mass to the minimal supersymmetric LR models with renormalizable seesaw mechanisms developed in detail recently.
- These MSLRMs have the MSSM with R -parity and seesaw neutrino masses and have quasi exact $B - L$ as their effective low energy theory. They can also have light Higgs triplet supermultiplets in their low energy spectra leading to very distinctive experimental signatures.
- They can be embedded in GUTS based on the PS group or SO(10). The former case may be more suitable in stringy scenarios which so far disfavor light (on stringy scales) SO(10) GUT Higgs of dimension > 54 .
- The SSB in the SO(10) SUSY GUT has been worked out explicitly and the mass spectra calculated. This allowed us to perform a RG analysis based on calculated spectra leading to the conclusion that $M_X \geq 10^{15.5}$ GeV while $M_R, M_{PS} \geq 10^{13}$ GeV. With M_X at its lower limit the $d = 6$ operators for nucleon decay can become competitive with the $d = 5$ operators raising the possibility that observation of $p \rightarrow \pi^0 e^+$ need not rule out SUSY GUTS after all.
- Fermion mass spectra can be compatible with charged fermion mass data and neutrino mass values suggested by neutrino oscillation data from super-Kamiokande and solar neutrino oscillation experiments.
- Dimension five operators in theories with seesaw lead to a remarkable connection between neutrino masses and nucleon decay which constrains these models fairly tightly and makes them testable by upcoming nucleon stability measurements.

- Further work on the doublet triplet splitting problem, the question of fermion mass spectra, two loop RG analysis etc. is required.
- Thus LR SUSY seesaw models and their GUT generalizations look good. The show has just begun but it aint over till the fat neutrino sings!

References

- [1] The super-Kamiokande collaboration: Evidence for oscillation of atmospheric neutrinos, hep-ex/9807003
Search for proton decay via $p \rightarrow e^+ \pi^0$ in a large water Cerenkov detector, hep-ex/9806014
- [2] Pratyabhijnahridayam of Ksemraja (Motilal Banarsidass, New Delhi, 1975)
- [3] S Weinberg, *Phys. Rev. Lett.* **43**, 1566 (1979)
E Akhmedov, Z Berezhiani and G Senjanovic, *Phys. Rev. Lett.* **69**, 3013 (1992)
- [4] R Kaul, *Phys. Lett.* **B109**, 19 (1982)
- [5] H Georgi, H Quinn and S Weinberg, *Phys. Rev. Lett.* **33**, 451 (1974)
- [6] M B Einhorn and D R T Jones, *Nucl. Phys.* **B196**, 475 (1982)
- [7] W Marciano and G Senjanovic, *Phys. Rev.* **D25**, 3092 (1982)
- [8] U Amaldi, W de Boer and H Furstenau, *Phys. Lett.* **B260**, 447 (1991)
P Langacker and M Luo, *Phys. Rev.* **D44**, 817 (1991)
- [9] M Krishnaswamy, M G K Menon, N Mandal, V Narasimhan, B V Sreekantan, S Ito and S Miyake, *Phys. Lett.* **B106**, 339 (1981); **B115**, 349 (1982)
- [10] K S Hirata *et al*, *Phys. Lett.* **B220**, 308 (1989)
- [11] J Pati and A Salam, *Phys. Rev.* **D10**, 275 (1974)
- [12] H Georgi and S L Glashow, *Phys. Rev. Lett.* **32**, 438 (1974)
- [13] H Fritzsch and P Minkowski, *Ann. Phys.* **93**, 193 (1975)
H Georgi, in *Particles and Fields* edited by C E Carlson (AIP, New York, 1975) p. 575
- [14] L Wolfenstein, *Phys. Rev.* **D17**, 2369 (1978); *Phys. Rev.* **D20**, 2364 (1979)
S Mikheyev and A Yu Smirnov, *Nuovo Cimento* **9C**, 17 (1986)
- [15] M Gell-Mann, P Ramond and R Slansky, in: *Supergravity* edited by F van Nieuwenhuizen and D Freedman (Amsterdam, North Holland, 1979) p. 315
T Yanagida, in: *Workshop on the unified theory and baryon number in the Universe* edited by O Sawada and A Sugamoto (KEK, Tsukuba, 1979) 95
R N Mohapatra and G Senjanovic, *Phys. Rev. Lett.* **44**, 912 (1980)
- [16] R N Mohapatra and R E Marshak, *Phys. Rev. Lett.* **44**, 1316 (1980)
- [17] R N Mohapatra and G Senjanovic, *Phys. Rev.* **D23**, (1981) 165
- [18] M Fukugita and T Yanagida, *Phys. Lett.* **174**, 45
- [19] J Bagger and J Wess, *Supersymmetry and Supergravity* (Princeton University Press, 1983)
R N Mohapatra, *Unification and Supersymmetry* (Springer Verlag, 1991)
H P Nilles, *Phys. Rep.* **110**, 1 (1984)
- [20] R N Mohapatra, *Phys. Rev.* **D34**, 3457 (1986)
A Font, L Ibanez and F Quevedo, *Phys. Lett.* **B228**, 79 (1989)
- [21] S Giddings and A Strominger, *Nucl. Phys.* **B307**, 854 (1988)
- [22] J C Pati and A Salam, *Phys. Rev.* **D10**, 275 (1974)
R N Mohapatra and J C Pati, *Phys. Rev.* **D11**, 566, 2558 (1975)
G Senjanovic and R N Mohapatra, *Phys. Rev.* **D12**, 1502 (1975)
- [23] K S Babu and S M Barr, *Phys. Rev.* **D48**, 5354 (1993)
K S Babu and S M Barr, *Phys. Rev.* **D50**, 3529 (1994)

- [24] C S Aulakh and R N Mohapatra, *Phys. Lett.* **119B**, 136 (1982)
- [25] C S Aulakh, K Benakli and G Senjanovic, *Phys. Rev. Lett.* **79**, 2188 (1997)
- [26] C S Aulakh, A Melfo, and G Senjanovic, *Phys. Rev.* **D57**, 4174 (1998)
- [27] C S Aulakh, A Melfo, A Rasin and G Senjanovic, *Phys. Rev.* **D58**, 115007 (1998)
- [28] D Comelli, A Masiero, M Pietroni and A Riotto, *Spontaneous breaking of R-parity in the MSSM revisited: Phys. Lett.* (1993)
- [29] M Cvetič, *Phys. Lett.* **B164**, 55 (1985)
- [30] R Kuchimanchi and R N Mohapatra, *Phys. Rev.* **D48**, 4352 (1993); *Phys. Rev. Lett.* **75**, 3989 (1995)
- [31] F Buccella, J P Derendinger, S Ferrara and C A Savoy, *Phys. Lett.* **B115**, 375 (1982)
I Affleck, M Dine and N Seiberg, *Nucl. Phys.* **B241**, 493 (1984)
I Affleck, M Dine and N Seiberg, *Nucl. Phys.* **B256**, 557 (1985); For a comprehensive treatment of the subject, see M Luty and W Taylor, *Phys. Rev.* **D53**, 3399 (1996)
- [32] Z Chacko and R N Mohapatra, Supersymmetric $SU(2)_L \times SU(2)_R \times SU(4)_c$ and observable neutron–antineutron oscillations, hep-ph/9802388
- [33] B Datta and R N Mohapatra, Phenomenology of light remnant doubly charged Higgs fields in the supersymmetric left–right model, hep-ph/9804277
- [34] Y Kamyshkov, Proceedings of the workshop on Future prospects of baryon instability search in p -decay and $N - \bar{N}$ oscillation edited by S J Ball and Y Kamyshkov, ORNL-6910, p.281. For present experimental limit see, M Baldo-ceolin *et al*, *Phys. Lett.* **B236**, 95 (1990); *Zeit. Phys.* **C63**, 409 (1994)
- [35] T Kibble, G Lazarides and Q Shafi, *Phys. Rev.* **D26**, 435 (1982)
D Chang, R N Mohapatra and M K Parida, *Phys. Rev. Lett.* **52**, 1072 (1984)
- [36] Charanjit S Aulakh, Borut Bajc, Alejandra Melfo, Andrija Rasin and Goran Senjanovic, SO(10) Theory of R -parity and neutrino mass (to appear)
- [37] S Dimopoulos and F Wilczek, report No. NSF-ITP-82-07, August 198 (unpublished)
- [38] C S Aulakh and R N Mohapatra, *Phys. Rev.* **D28**, 217 (1983)
- [39] K S Babu and R N Mohapatra, *Phys. Rev. Lett.* **70**, 2845 (1993)
D-G Lee and R N Mohapatra, *Phys. Rev.* **D51**, 1353 (1995)
B Brahmachari and R N Mohapatra, *Phys. Rev.* **D58**, 015003 (1998)
- [40] Hin-ya Oda, Eitchi Takasugi, Minoru Tanaka and Masaki Yoshimura, *Unified Explanantion of Quark and Lepton Masses and Mixings in the Supersymmetric SO(10) Model*, hep-ph/9808241
- [41] K S Babu, Jogesh C Pati and Frank Wilczek, *Suggested New Modes in Supersymmetric Proton Decay*, hep-ph/9712307; *Fermion Masses, Neutrino Oscillations and Proton Decay in the Light of super-Kamiokande*, hep-ph 9812538
- [42] S Weinberg, *Phys. Rev.* **D26**, 287 (1982)
N Sakai and T Yanagida, *Nucl. Phys.* **B197**, 533 (1982)
- [43] S Dimopoulos, S Raby and F Wilczek, *Phys. Lett.* **B112**, 133 (1982)
J Ellis, D V Nanopoulos and S Rudaz, *Nucl. Phys.* **B202**, 43 (1982)
- [44] J Hisano, H Murayama and T Yanagida, *Nucl. Phys.* **B402**, 46 (1993)
- [45] K S Babu and S M Barr, *Proton Decay and Realistic Models of Quark and Lepton Masses*, hep-ph/9506261 and references therein