

Stability and dynamics of fluid flow past deformable solid media

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Outline

- Introduction and motivation.
- Governing equations and stability theory.
- Two new high Reynolds number instabilities.
- Suppression of interfacial instabilities in two-layer flows by a soft solid layer.
- A new instability in viscoelastic flow past a deformable wall.
- Summary and outlook.

Flow past soft deformable solid surfaces

- Biological flows and Bio-technological applications.
- Aerospace and marine propulsion: drag reduction.
- Geophysical phenomena (volcanic tremors).
- Flow in microfluidic devices made of soft elastomers.
- Polymer processing by co-extrusion.

Fluid-dynamic aspects:

- Instabilities and Laminar–Turbulent transition.
- Laminar flow \Rightarrow lower drag, poor mixing.
- Turbulent flow \Rightarrow high mixing & drag.

Salient features of flow past soft solids

- Rigid walls: shear modulus of steel $\sim 10^{10}$ Pa
Soft materials: shear modulus $\sim 10^4 - 10^6$ Pa $\sim 10^{-5} G_{steel}$.
- Soft interfaces easily deformed by fluid stresses.
- **Elasto-hydrodynamic coupling** \Rightarrow Interfacial waves.
- **Wall dynamics could affect instabilities and transition.**
- Anomalous instabilities observed in experiments.
Rigid tube $Re \simeq 2100$; Deformable tubes: Re much lower.
- **Objective of our work:** To uncover qualitatively new instabilities in flow past deformable solid media.
(Will not consider tube collapse due to external pressure changes.)

Fluid Governing equations

- Incompressible; both Newtonian and Viscoelastic fluids.

$$\rho(\partial_t + v_j \partial_j)v_i = -\partial_i p + \partial_j \tau_{ij}$$

- **Newtonian fluid:** $\tau_{ij} = \eta(\partial_i v_j + \partial_j v_i)$
- **Viscoelastic fluid:** Upper-Convected Maxwell (UCM) model
 $\lambda[\partial_t \tau_{ij} + v_k \partial_k \tau_{ij} - \partial_k v_i \tau_{kj} - \partial_k v_j \tau_{ki}] + \tau_{ij} = \eta(\partial_i v_j + \partial_j v_i)$
- Material constants: Viscosity η and relaxation time λ .
- Nonzero first normal stress difference under simple shear.
- Applicable for polymer melts; solvent contribution easily added.

Model for deformable solid

- Incompressible linear viscoelastic solid

u_i : Displacement field in the solid.

$$\Sigma_{ij} = -p_w \delta_{ij} + \sigma_{ij}$$

$$\sigma_{ij} = G(\partial_i u_j + \partial_j u_i) + \eta_w \partial_t (\partial_i u_j + \partial_j u_i)$$

Governing Equations:

$$\partial_i u_i = 0$$

$$\rho \partial_t^2 u_i = \partial_j \Sigma_{ij}$$

- Material constants: Shear modulus G and Viscosity η_w .
- For simplicity, density of solid = density of fluid = ρ

Linear stability theory

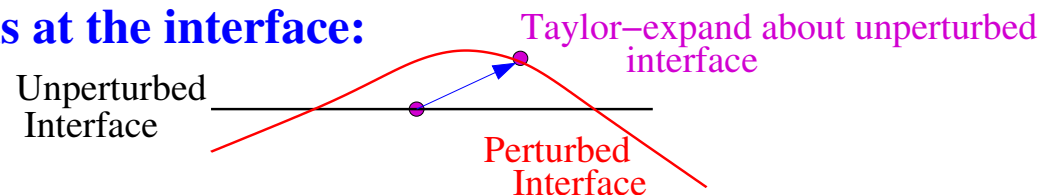
- Impose small fluctuations and linearize: $v_i = \bar{v}_i + v'_i$

$$v'_i(x, z, t) = \sum \tilde{v}_i(z) \exp[ikx + in\theta] \exp[-ikct]$$

- k : Wavenumber of fluctuations.
- Complex wave speed $c = c_r + i c_i$

$c_i > 0 \Rightarrow$ Unstable $c_i < 0 \Rightarrow$ Stable $c_i = 0 \Rightarrow$ Neutrally stable.

Conditions at the interface:



- Non-trivial couplings between the base flow and height fluctuations in the interface conditions.

High Reynolds number inviscid modes

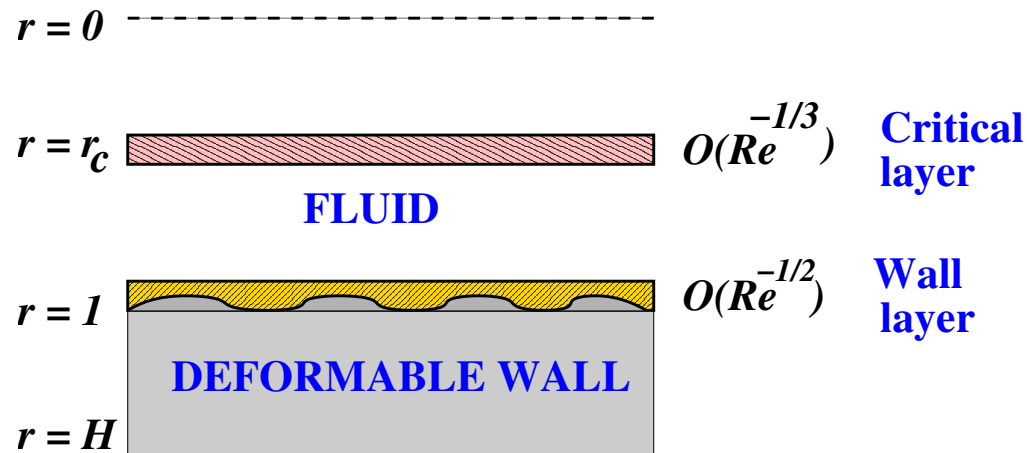
- $Re \gg 1$, Fluid inertial stresses \sim Wall elastic stresses $\rho V^2 \sim G$

$$\Rightarrow V \sim (G/\rho)^{1/2}$$

- Inviscid fluid governing equations.
- Classical theorems of stability for flow in a deformable tube \Rightarrow Can a given flow be unstable in the inviscid limit ?
- Developing flow in the entrance region, flow in a converging tube, fully-developed flow subjected to non-axisymmetric perturbations.
- Singular perturbation in the inviscid limit: Viscous effects important in two thin layers even at high Re .

Inviscid instability

- Inviscid limit: Logarithmic singularity at $r = r_c$ where $U(r_c) = c_r$
- **Critical and wall layers:**

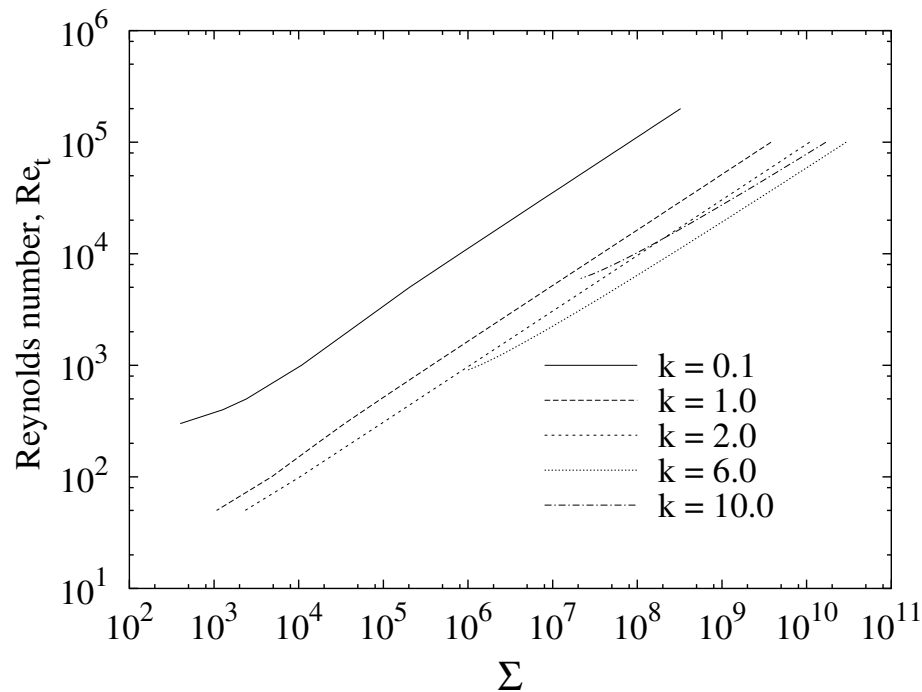


- Multiple solutions at leading order for $c^{(0)}$.
- **Flow unstable** in the inviscid limit when $\rho V^2 / G$ increases.
- Inviscid results corroborated by numerical solution at large Re .

Reynolds number vs. Nondimensional elasticity Σ

- Flow in entrance region: $X = 0.050, H = 2$;

$$\Sigma = \rho G R^2 / \eta^2$$



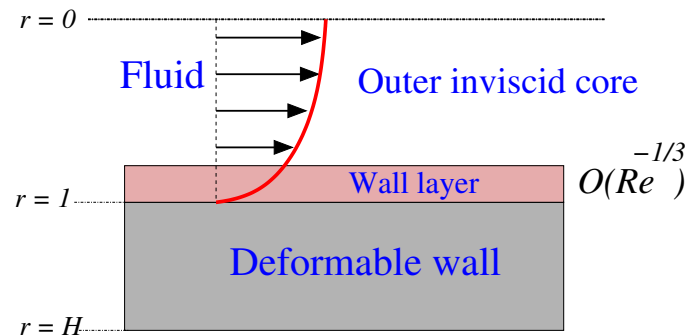
- Instability extends to moderate Reynolds number $Re \sim \Sigma^{1/2}$

Summary of Inviscid instability

- Instability requires the presence of the critical layer singularity.
- **Absent** in fully developed flow in a deformable tube subjected to axisymmetric disturbances and in plane Couette flow in a channel.
- Instability predicted for developing flow in a deformable tube, flow in a converging deformable tube, and for fully-developed flow subjected to axisymmetric disturbances.
- **Absent** in rigid tubes.
- Instability driven by convective transport of energy in the critical layer.
- Critical Reynolds number $Re \propto \Sigma^{1/2}$; continues to moderate Re .

High- Re Wall mode instability

- $Re \gg 1$; wavespeed of fluctuations $O(Re^{-1/3})$ smaller than the maximum velocity of the base flow.
- Viscous stresses confined to a layer of thickness $O(Re^{-1/3})$ smaller than the tube radius *near the fluid-solid boundary*.
- Viscous shear stresses in the wall layer \sim Elastic stresses in the solid.
- $\frac{\eta V}{R Re^{-1/3}} \sim G \Rightarrow \boxed{Re \propto \Sigma^{3/4}}; \quad \boxed{V \gg (G/\rho)^{1/2}}$



Wall mode instability in flow past deformable solids

- **Rigid tubes:** Wall modes are stable (Gill, 1965)

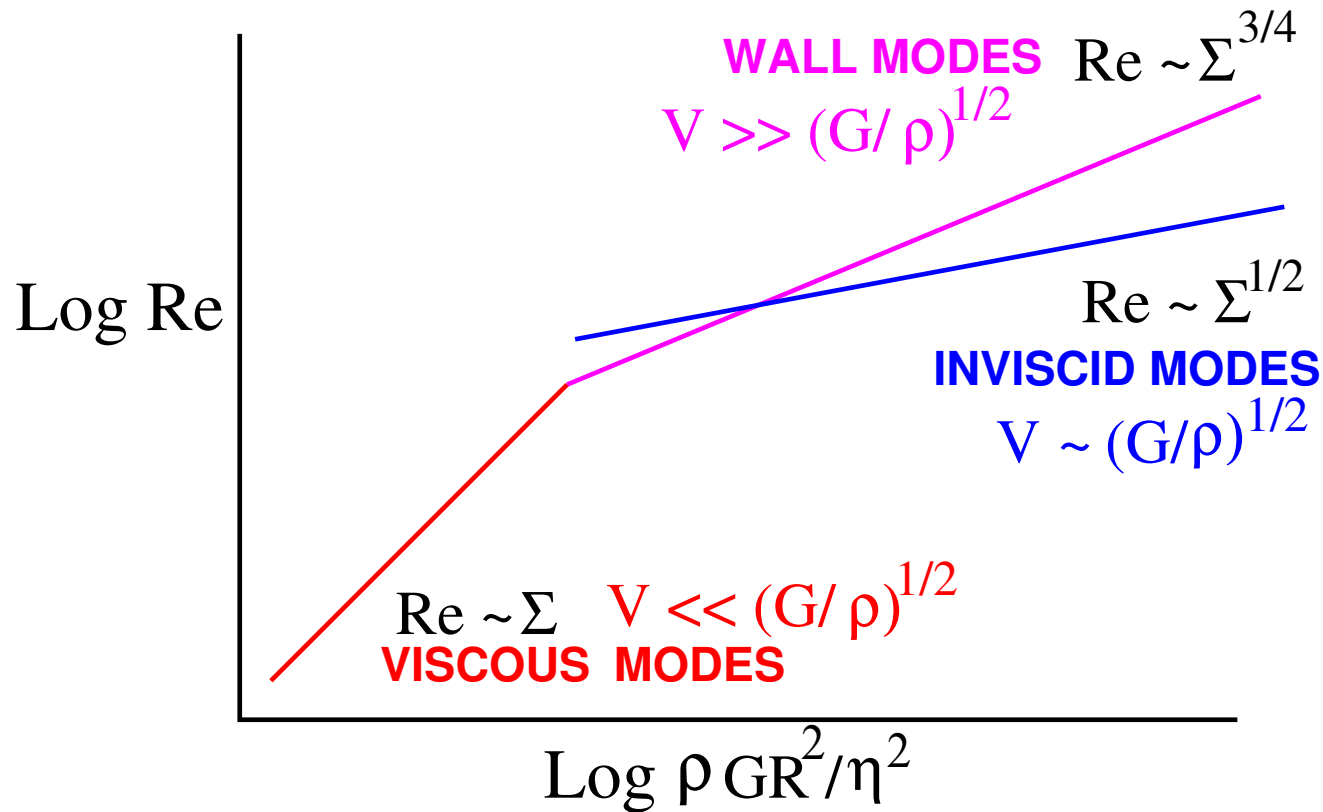
- Present asymptotic analysis: $Re \gg 1$, $Re \propto \Sigma^{3/4}$

Leading order: Fluid shear stresses in the wall layer subdominant; c real, with multiple solutions, $c^* \propto (G/\rho)^{1/2}$.

First correction: Fluid tangential velocity and stresses in the wall layer dominant; Flow unstable when $Re^{-1/3}(\rho V^2/G)^{1/2} \sim O(1)$.

- Instability continues to low Re .
- Importance of tangential wall motion in this instability.
- Independent of the details of the velocity profile: **a generic feature in flow past deformable solid surfaces at $Re \gg 1$.**

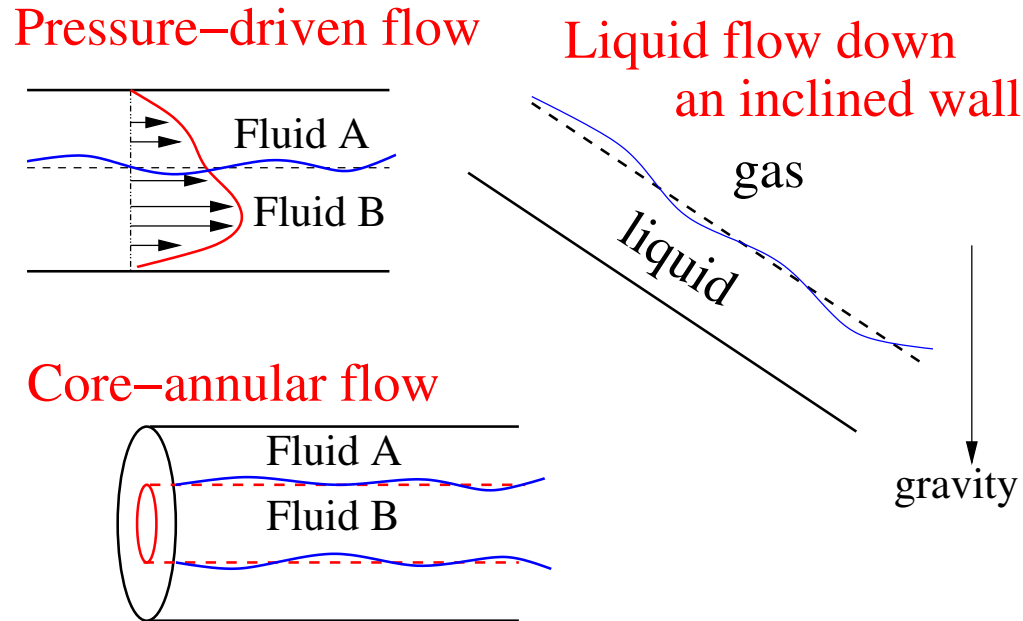
Instabilities in Newtonian flow past a deformable solid



- Instabilities absent in flow through rigid tubes/channels.

Suppression of flow-induced interfacial instabilities in two-layer flows by a soft solid layer

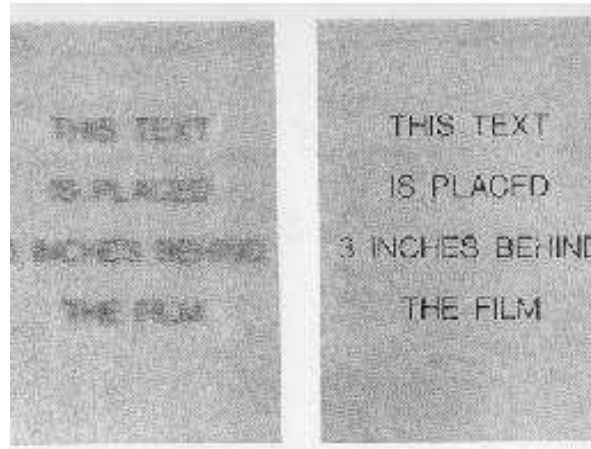
- Coextrusion of polymer melts; lubricated pipelining in oil recovery.



- Interfacial instabilities due to **viscosity** and/or **elasticity stratification**.

Example of an interfacial instability

- Effect on the optical quality of a three-layer co-extruded film.



(Mavridis & Shroff, *Polym. Engg and Science*, 1994)

Can deformable solid layers suppress interfacial instabilities ?

Two-layer flow past a soft solid

Rigid surfaces: Long-wave asymptotic analysis.

$$c = c^{(0)} + kc^{(1)} + \dots$$

$c^{(0)} \rightarrow$ **real**; system neutrally stable to leading order.

$c^{(1)} \rightarrow$ **imaginary**; system unstable under certain configurations.

- Driven by discontinuity of shear rate/normal stress difference at the interface.

Soft solid surface:

At fluid-solid interface: $\tilde{v}_z = -ikc\tilde{u}_z$; If $\tilde{v}_z \sim \tilde{u}_z \sim O(1)$ in the bulk, $\Rightarrow \tilde{v}_z \sim O(k)$ at the interface. Similarly $\tilde{v}_x \sim O(k)$

\Rightarrow $\tilde{v}_z = \tilde{v}_x = 0$ to leading order at the interface.

$c^{(0)}$ **unaffected by solid deformability; same as in rigid surfaces.**

Two-layer flow past a soft solid

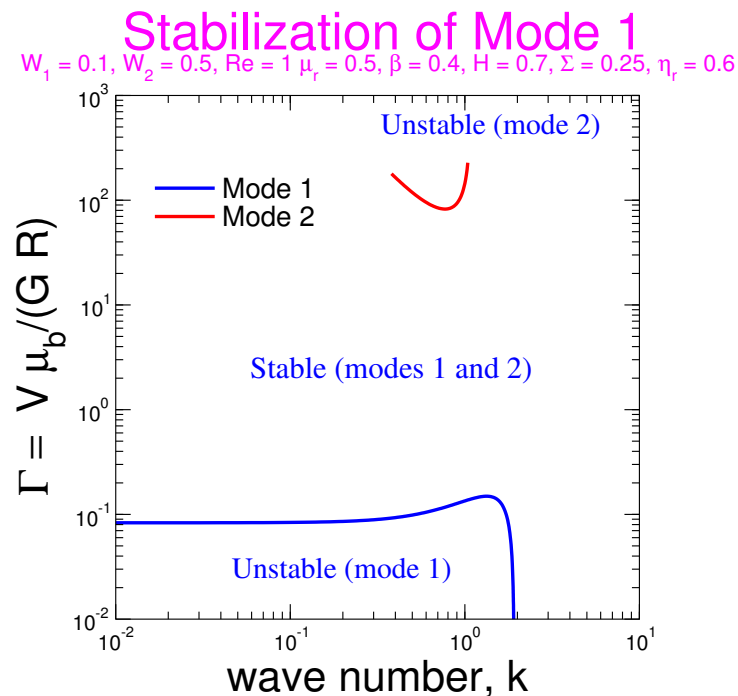
- Fluid stresses at leading order cause deformation in the solid layer.
 \Rightarrow Solid deformation affects the $O(k)$ velocity field in the fluid, through velocity continuity conditions.

$$c = c^{(0)} + ik \left[\text{Re} f_1(\mu_r, \frac{d_1}{d_2}) + f_2(\frac{\lambda_1}{\lambda_2}, \frac{d_1}{d_2}) + \frac{V\eta H}{GR} f_3(H, \mu_r, \frac{d_1}{d_2}) \right]$$

Contribution from soft solid layer: $f_3(H, \mu_r, d_1/d_2)$ can be stabilizing or destabilizing depending on $\mu_r, H, d_1/d_2$.

- Stabilizing mechanism works for different flow-induced interfacial instabilities.
- Does this continue to finite k ?

Results for two-layer viscoelastic Couette flow



- Short-wave fluctuations in the two-fluid interface unaffected by the solid layer; stabilized only by nonzero interfacial tension.
- A wide window in shear modulus where both modes are stable.

High-frequency modes in viscoelastic Couette flow

- Are there qualitatively new instabilities in viscoelastic flows that are absent in Newtonian fluids ?

Viscous diffusion time $(R^2 \rho / \eta) \ll \lambda$ Relaxation time of the fluid.

$$\Rightarrow Re \ll W = \lambda V / R.$$

- Frequency of fluctuations \gg shear rate of base flow.

\Rightarrow Probe only the ‘elastic part’ of the UCM fluid.

- Free shear waves in a viscoelastic fluid: wavespeed $c^* \propto \sqrt{\frac{G}{\rho}} \propto \sqrt{\frac{\eta}{\lambda \rho}}$

$$\Rightarrow c^* / V \propto Re^{-1/2} W^{-1/2} \propto Re^{-1/2} \text{ for } W \sim O(1).$$

Asymptotic analysis in the small parameter $Re^{1/2}$, for $Re \ll 1$.

High-frequency modes in viscoelastic flow

Rigid channel

$$c = Re^{-1/2}c^{(0)} + c^{(1)} + \dots$$

$c^{(0)}$ → multiple real solutions (both upstream and downstream waves)

$c^{(1)}$ → negative imaginary part; fluid viscous effects stabilizing.

Deformable wall

Flow appears even at leading order in $c^{(0)}$.

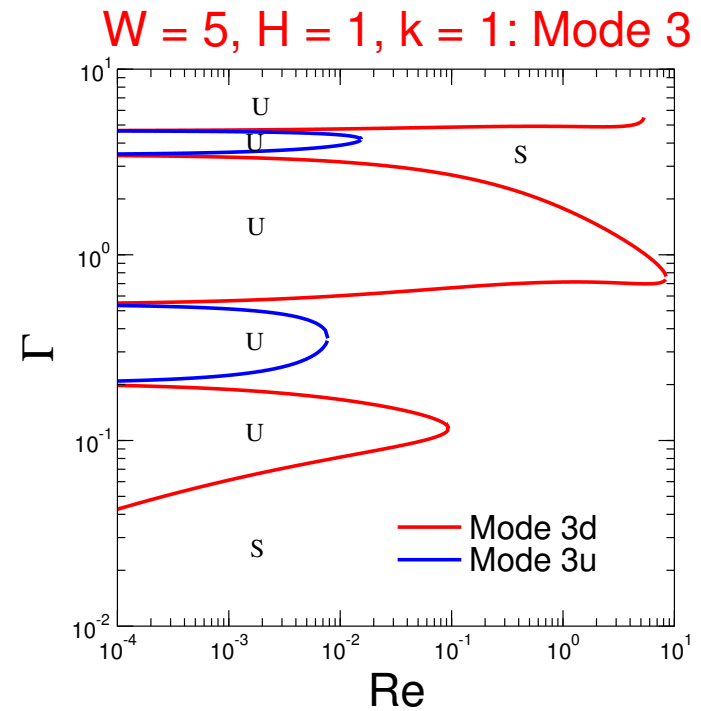
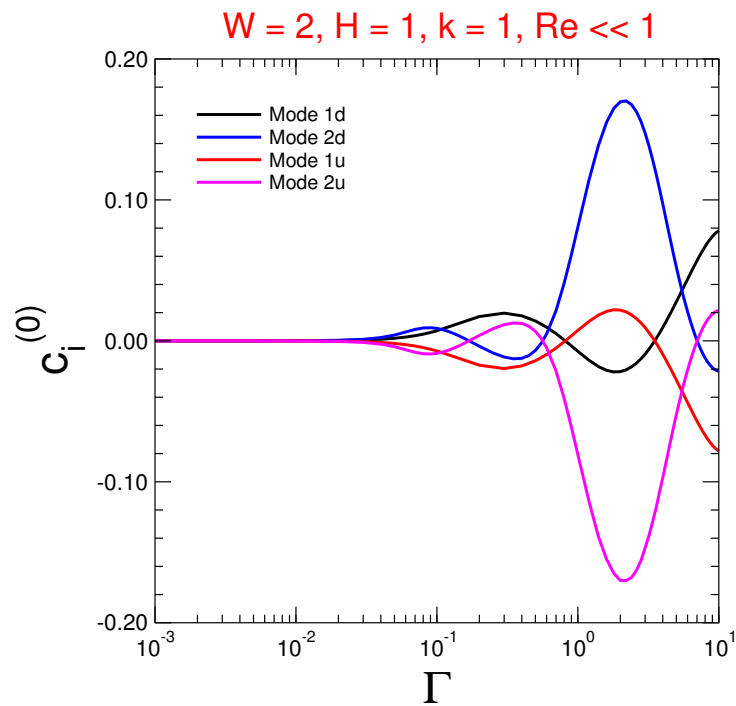
Flow destabilizing for downstream waves, with $c_i^{(0)} \propto \Gamma^2 = (V\eta/(GR))^2$ for small Γ .

First correction $c^{(1)}$: Stabilizing due to viscous effects

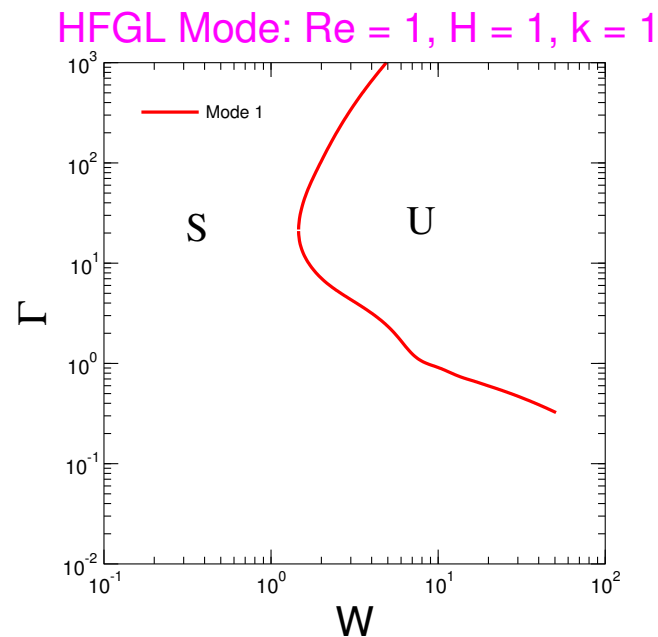
⇒ For small Re and $W \sim O(1)$, $\Gamma \propto Re^{1/4}$ or $Re \propto \Sigma^{4/3}$

Results for Growth rate and Neutral stability

- Oscillatory variation of $c_i^{(0)}$ with Γ for $\Gamma \sim O(1)$.



- Instability exists only upto $Re \sim 1$.

Results: Γ - W Neutral curve

- Instability absent in Newtonian fluids.
- Solvent viscosity has a stabilizing effect on this instability \Rightarrow Instability expected in concentrated polymer solutions and melts at low Re .

Summary and outlook

- Qualitatively new instabilities at high Re in Newtonian fluid flow past deformable solid surfaces.
- Soft, deformable solid layer coatings can suppress two-fluid interfacial instabilities in viscoelastic and Newtonian fluids.
- New class of unstable modes in viscoelastic flow past a deformable wall (absent in Newtonian fluids).

Open questions

- Flow in the nonlinear regime ?
- Both computations and experiments can throw more light.

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