

The role of micro-scale inertia in transport processes

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Micro-scale inertia (suspensions and emulsions)

- *Navier-Stokes equations :*

$$Re \underbrace{\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}}_{\text{Inertial acceleration}} = -\underbrace{\nabla p}_{\text{Pressure forces}} + \underbrace{\nabla^2 \mathbf{u}}_{\text{Viscous forces}}$$

$$\text{Reynolds number } Re = \frac{U a \rho}{\mu}$$

a : fibre length
 U : particle (drop) size
 ν : fluid kinematic viscosity

- *Small particles, viscous fluid : $Re \ll 1$*



- *Quasi-steady Stokes equations :*

$$-\nabla p + \nabla^2 \mathbf{u} = 0 \quad \longrightarrow \quad \text{Good approximation for small-scale slow, slowly varying motions}$$

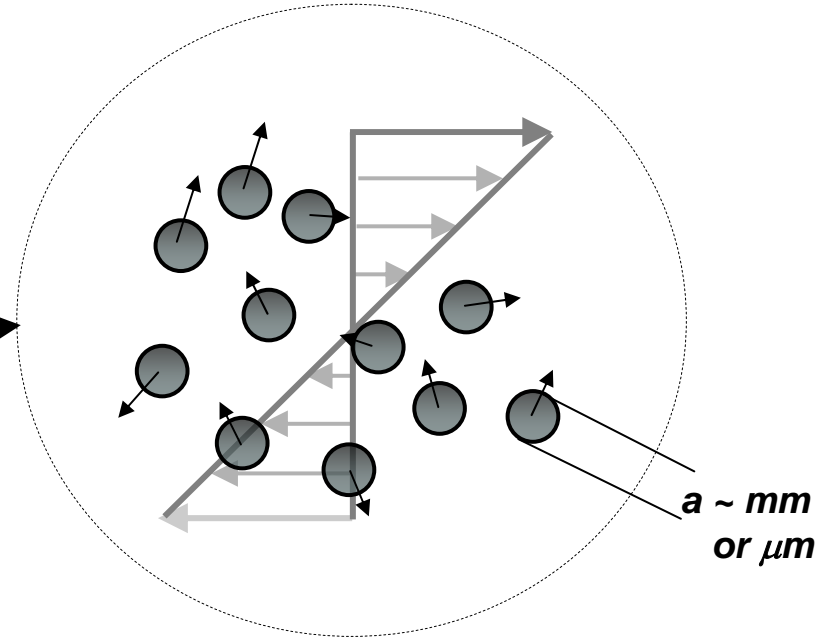
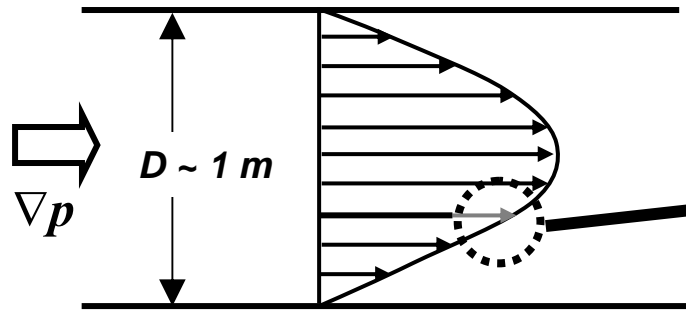
An instantaneous balance between pressure and viscous forces !



Central to the hydrodynamics in suspension rheology

Micro-scale inertia

Macro-scale



Pipe flow of a suspension

$a = 10\text{ micron}$
 $\mu = 1\text{ Pa-sec}$

$Re = O(10^{-9})$

Stokes regime

[Polystyrene/PMMA particles
in honey or glycerine]

Micro-structure

$a = 100\text{ micron}$
 $\mu = 10^{-3}\text{ Pa-sec}$

$Re = O(1)$

Micro-scale inertia important

[Particles or drops in aqueous media,
Cellulose fibres in water – paper and pulp industry]

Applications

Heat (Mass) transfer

- Exothermic reactions on catalyst particles
- Suspension polymerization
- Drug delivery using porous polymer particles
- Transport characteristics of immobilized cells/cell-aggregates in bio-reactors

Momentum transfer

- Pyroclastic flows
- Gravity currents (sediment deposition patterns)
- Rheology

Newtonian : $Re \frac{\partial \mathbf{u}}{\partial t} + Re \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nabla^2 \mathbf{u}$

Averaged equation : $Re \frac{\partial \mathbf{u}}{\partial t} + Re \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nabla \cdot \boldsymbol{\tau}$
(suspension/emulsion) Macro-scale inertia

- Micro-scale inertia determines the rheology of the fluid
- $\boldsymbol{\tau}$ may be regarded as a suitable average over the underlying microstructure
- Velocity fluctuations of the disperse phase may be regarded as contributing to a Reynolds stress

Principle of reversibility

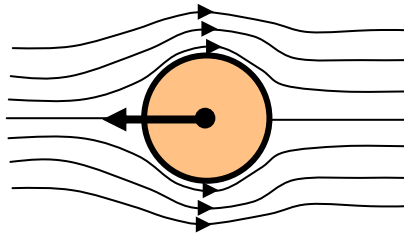
- Mere reversal of boundary conditions should lead to reversal of entire flow field

‘ Reversing boundary motion reverses time ’

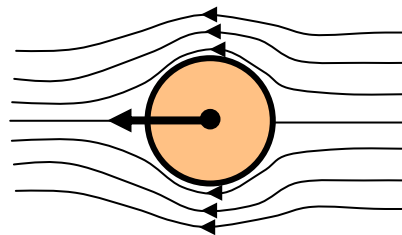
Different from well-known mechanical reversibility in classical mechanics

- **Restrictions imposed by reversibility**

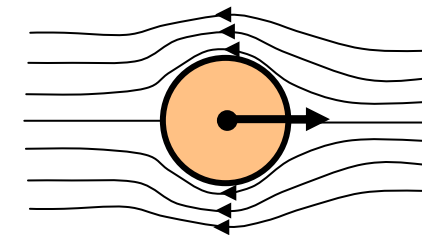
- **Fore-aft symmetric body - Velocity field must be fore-aft symmetric**



1 : *Fore-aft asymmetric velocity field*



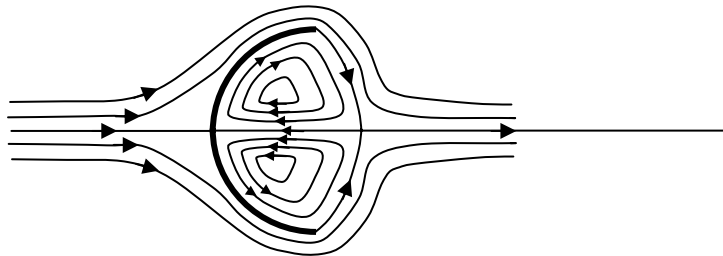
2 : *Reverse time*



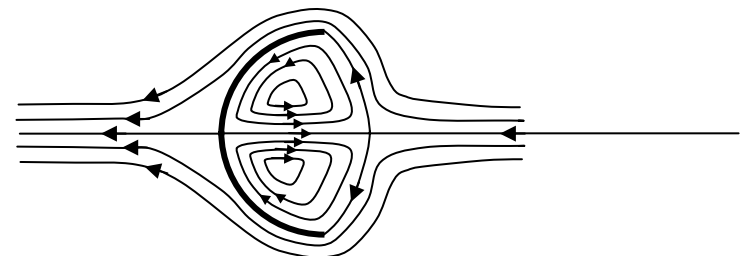
3 : *Reverse boundary motion*

→ *2 and 3 are different streamline patterns !*

- **Both fore and aft-separation possible for fore-aft asymmetric body**



1. *Aft-separation*

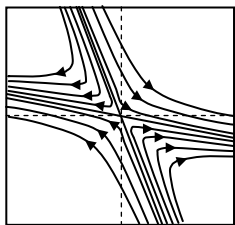
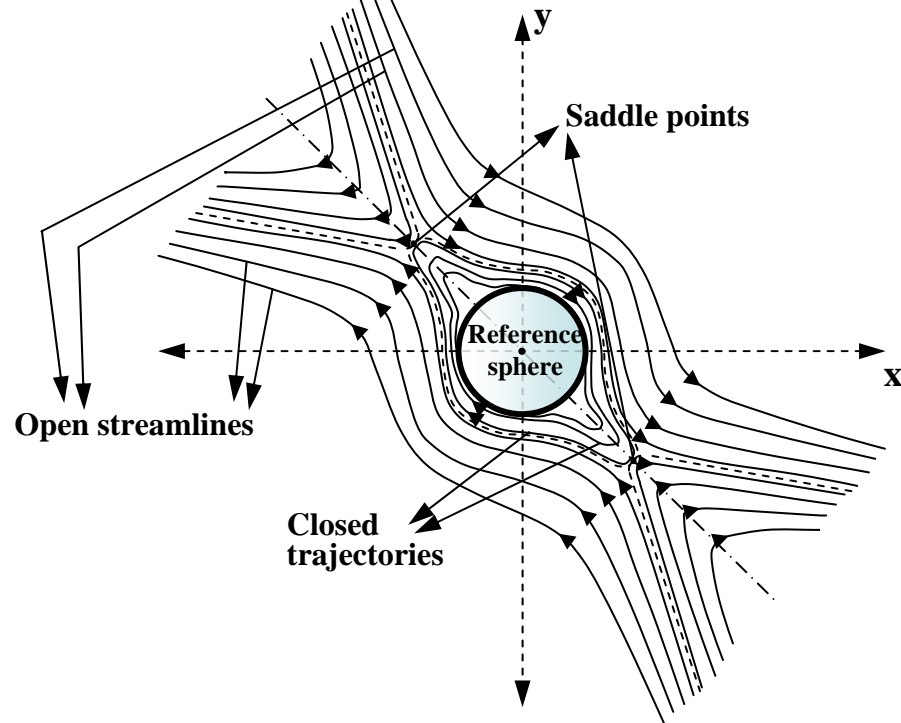
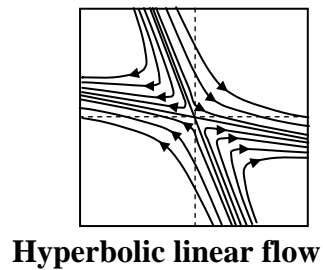


2. *Reverse time: fore-separation !*

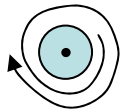
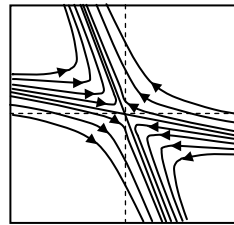
Cross-section of a spherical cap

□ Ubiquitousness of closed streamlines in Stokes flows

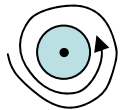
- Planar vortical Stokes flow + rotating particle = closed streamlines around particle



Reverse flow



Assume near-field
Streamlines spiral out



Near-field streamlines
must spiral in

- *Spiralling streamlines are excluded by reversibility*

- **Reversibility leads to closed streamlines and closed particle pathlines**
- **SMALL amount of (micro-scale) inertia has BIG effects**
- **Note that dynamic reversibility does not imply kinematic reversibility**
- **Kinematic irreversibility (chaos) will only become relevant for complex configurations and on sufficiently long time scales in the dilute limit**

Heat (Mass) transfer :

Neutrally buoyant particles or drops in shearing flows

**Collaborator: Don Koch,
Cornell University**

Governing equation for heat transfer - Convection-diffusion equation

$$\underbrace{Pe (u \cdot \nabla T)}_{\substack{\text{Convection} \\ \text{by fluid velocity field}}} = \underbrace{\nabla^2 T}_{\text{Conduction}}$$

$$\text{Peclet number, } Pe = \frac{aU}{D}$$

$$\begin{cases} T = T_0 \longrightarrow \text{on particle surface} \\ T = T_\infty \longrightarrow \text{ambient fluid} \end{cases}$$

- *Physical properties assumed to be constant*
- *u obtained as a solution of the Stokes equations ($Re = 0$) or with weak inertia*
- *High Pe - transfer of heat or mass across a thin boundary layer*

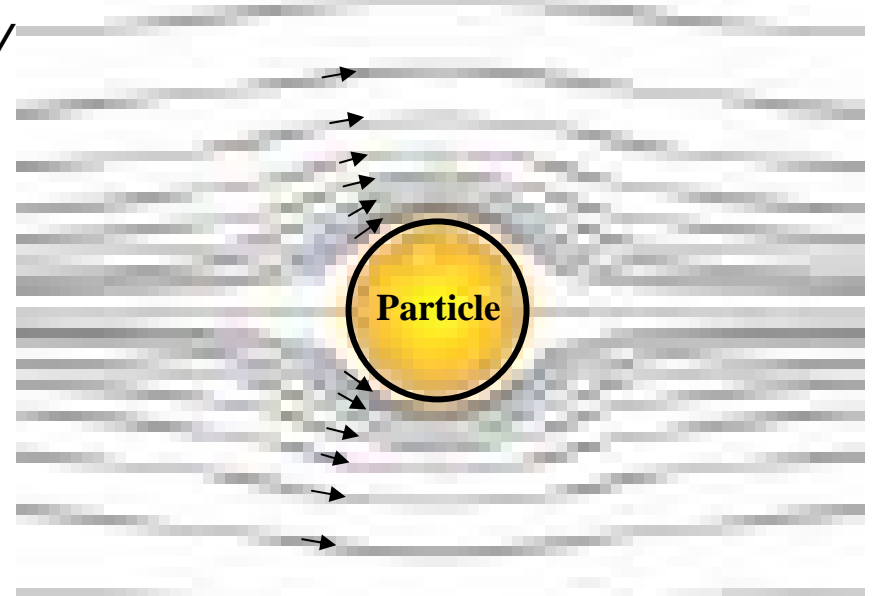
Heat transfer from particle : open streamlines

$$\mathbf{u} = \mathbf{U} + \mathbf{\Gamma} \cdot \mathbf{x} + \text{h.o.t}$$

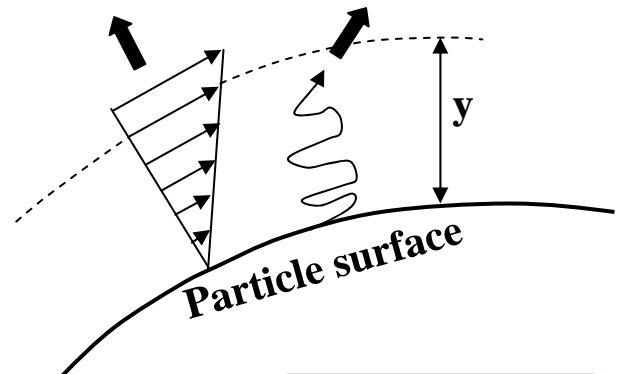
→ Slip velocity

Open streamlines for relative translation of fluid and particle lead to a *thin boundary layer* and the familiar scaling of heat transfer rate with Peclet number

$$Nu = \# Pe^{1/3} \text{ for } Pe \gg 1$$

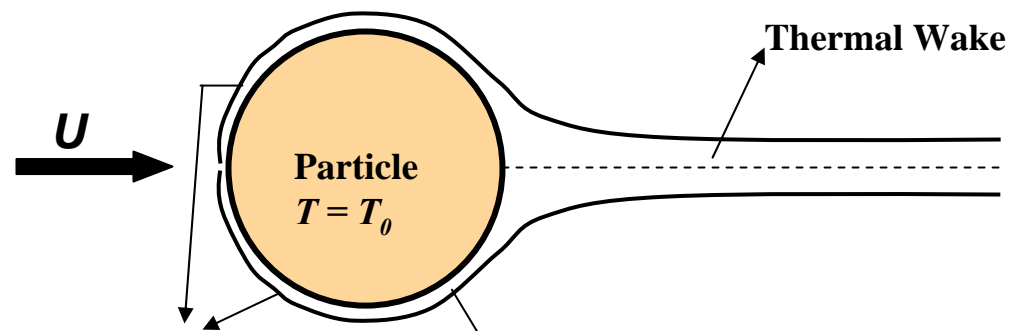


Convection time scale $\sim \frac{a}{(Uy/a)}$ Diffusion time scale $\sim \frac{y^2}{D}$



$$\frac{a}{(Uy/a)} \sim \frac{y^2}{D}$$

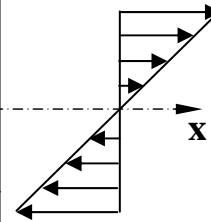
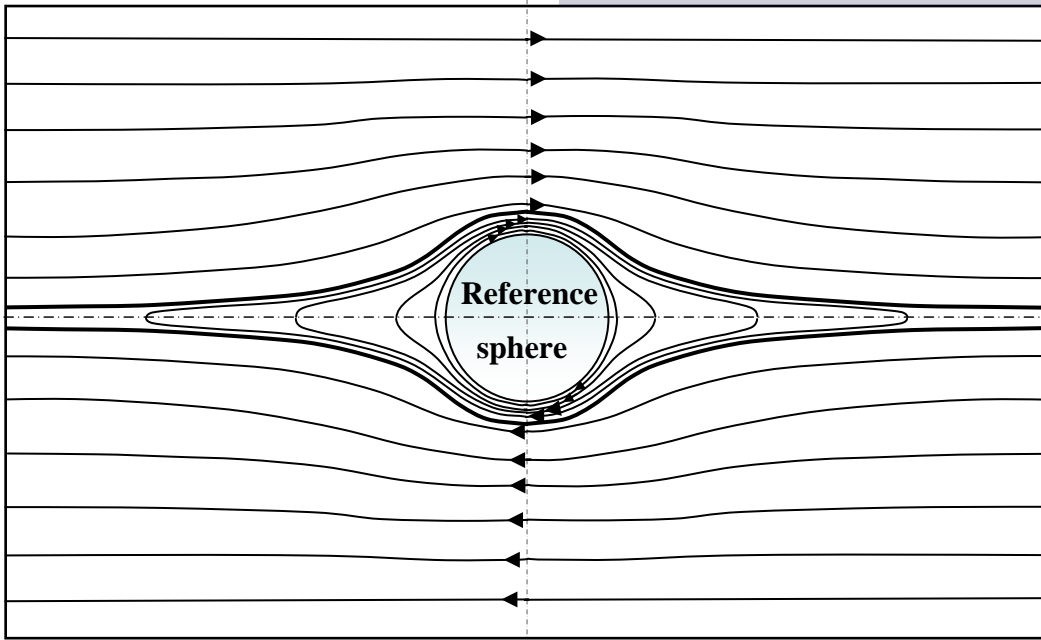
$T = T_\infty$



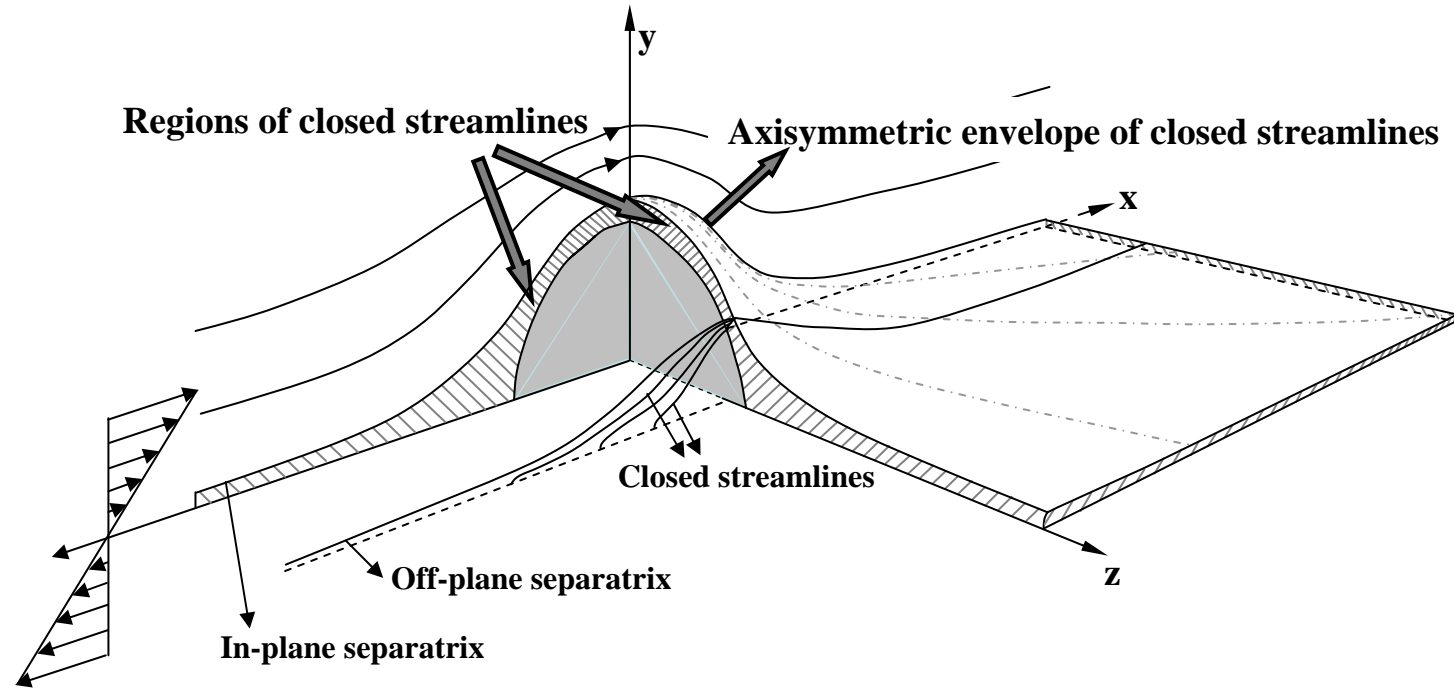
Thermal boundary layer
 $O(Pe^{-1/3})$

Dominant temperature variation occurs in this narrow region

Heat transfer from particle : closed streamlines ($Re=0$)



Simple shear flow ($\alpha = 0$)
 $Re = 0$



Heat transfer from particle : closed streamlines ($Re = 0$)

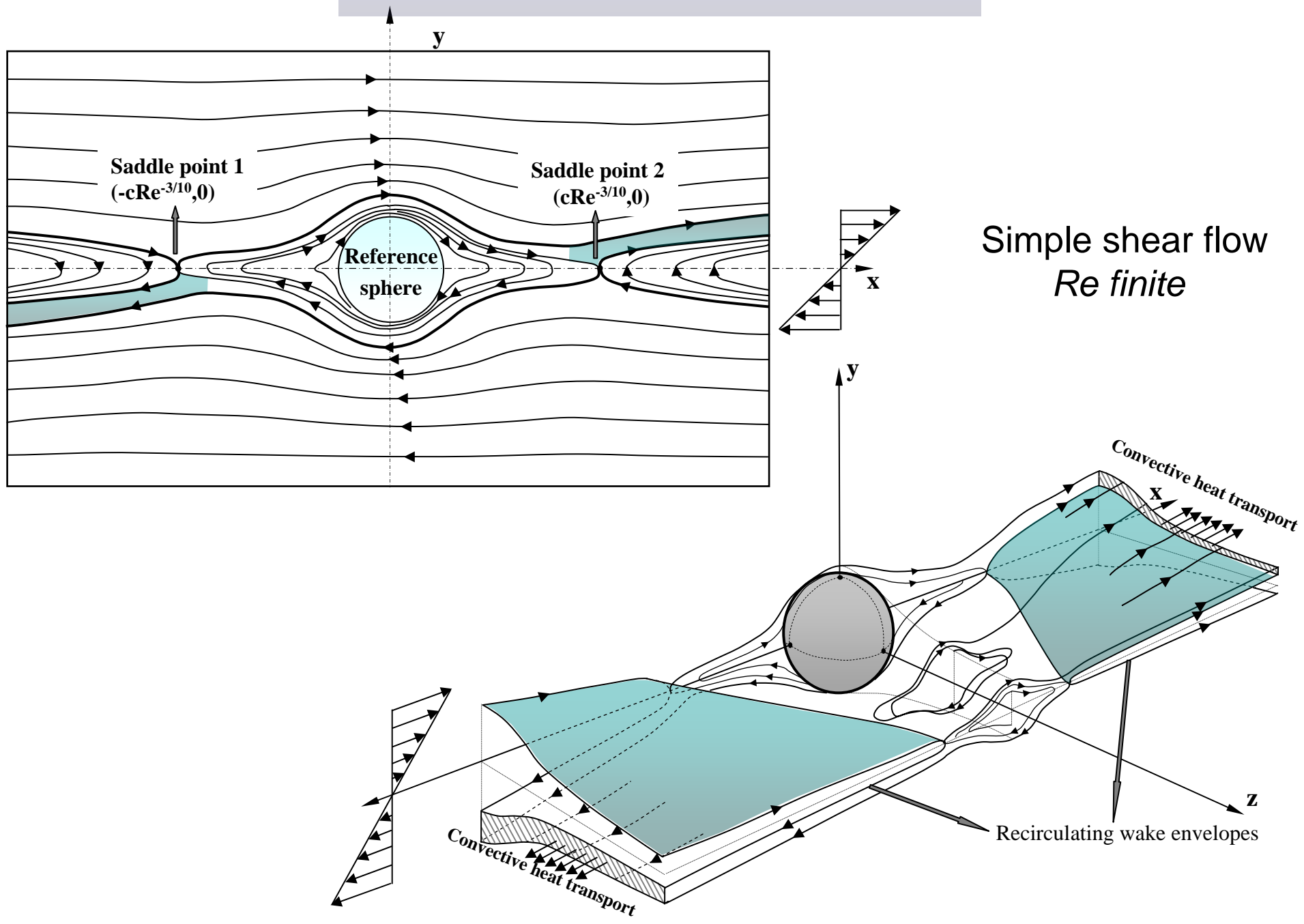
Closed streamlines around the sphere give diffusion limited heat transfer even at large Pe



$$Nu = 4.5 \quad \text{for } Pe \gg 1 \quad \text{and} \quad Re = 0, \alpha = 0$$

- Nu – dimensionless heat transfer in conduction units
- $Nu = 1$ for pure conduction ($Pe = 0$)

Heat transfer from particle : finite Re



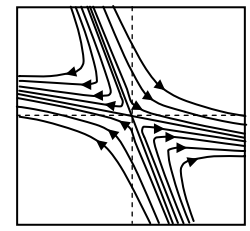
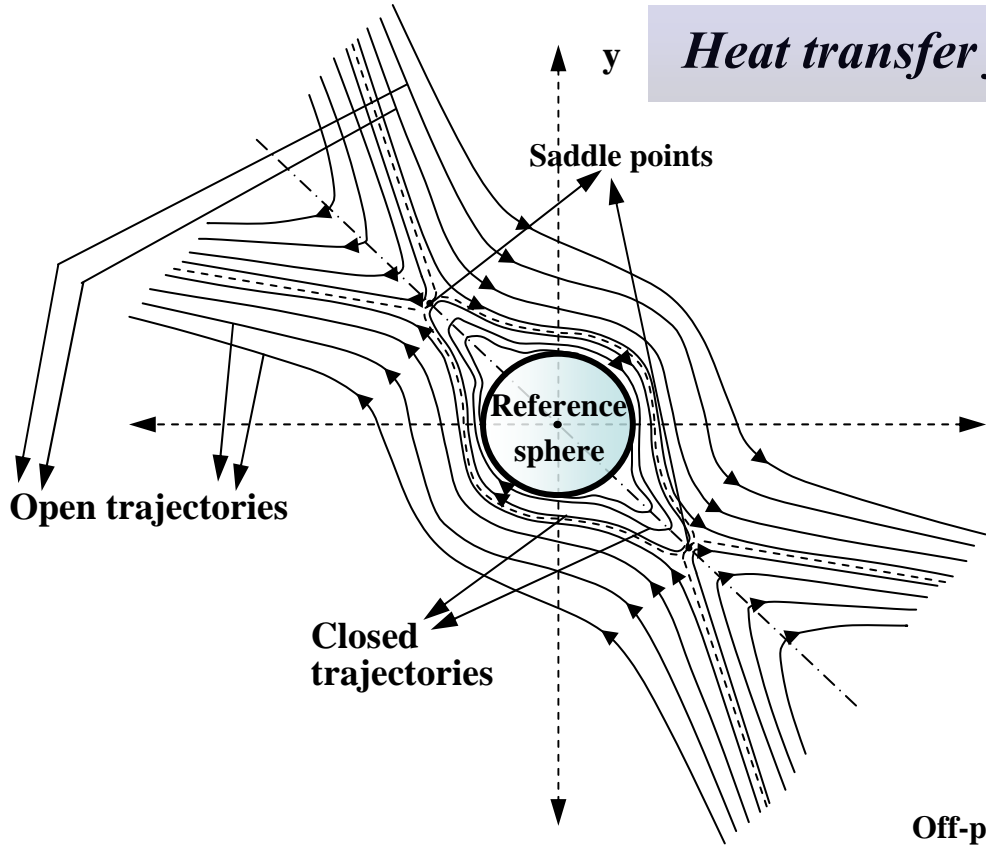
Heat transfer from particle : finite Re

- At small but nonzero Re the flow topology changes.
- Streamlines approach the sphere along the vorticity axis and spiral outward in the flow-gradient plane eventually exiting through the channels indicated in green.
- The open streamlines result in enhanced heat transfer.

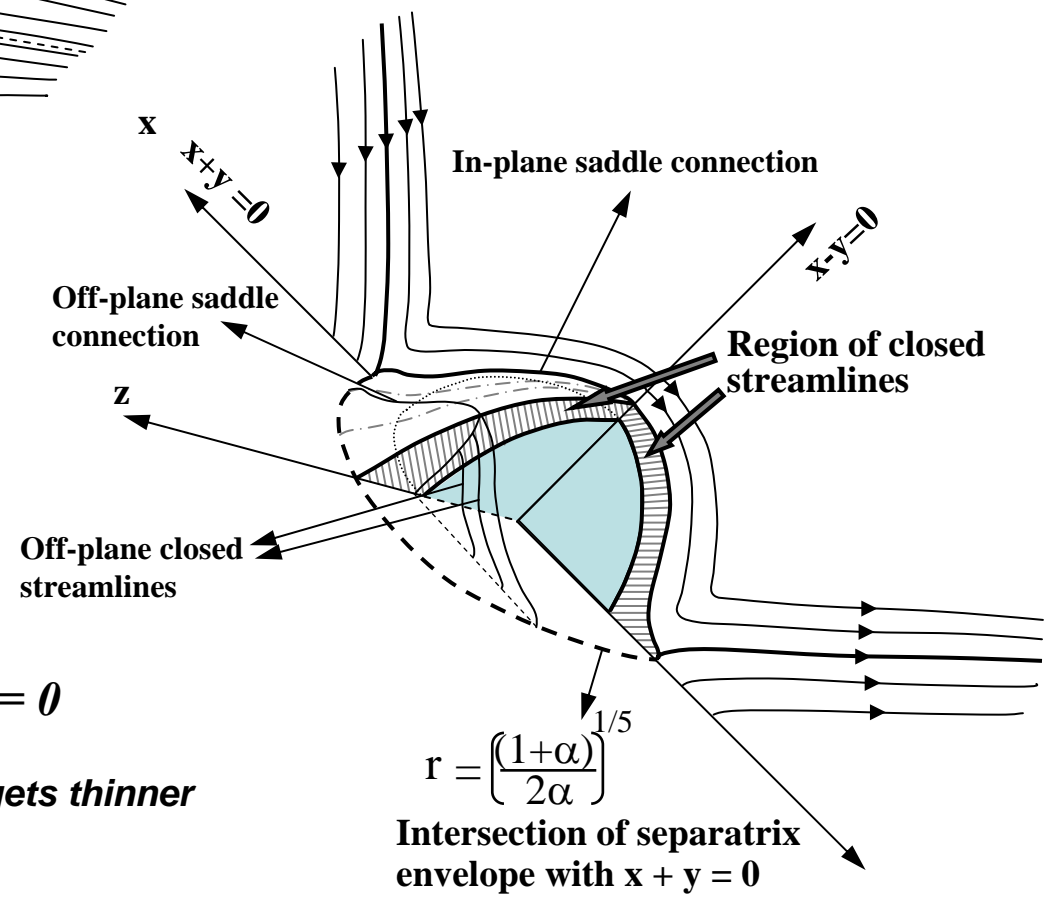


$$Nu = 0.33 (Re Pe)^{1/3} \quad \text{for } Pe \gg 1 \text{ and } Re \ll 1$$

Heat transfer from particle : closed streamlines ($Re=0$)



Planar linear flow
 $Re = 0$

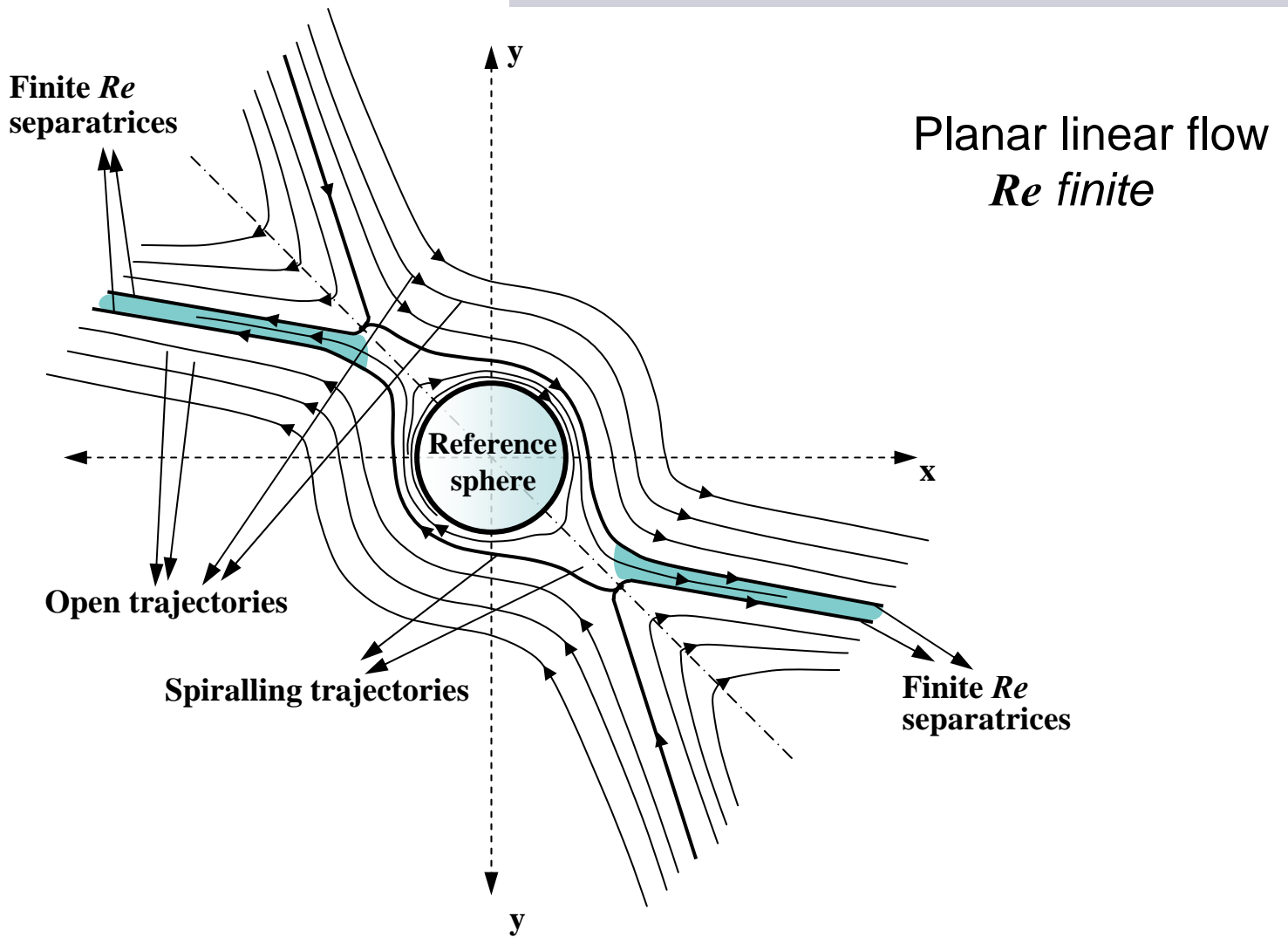


$Nu \propto (1-\alpha)^{-1}$ for $Pe \gg 1$ and $Re = 0$

*The closed streamline region gets thinner
Higher temperature gradients*

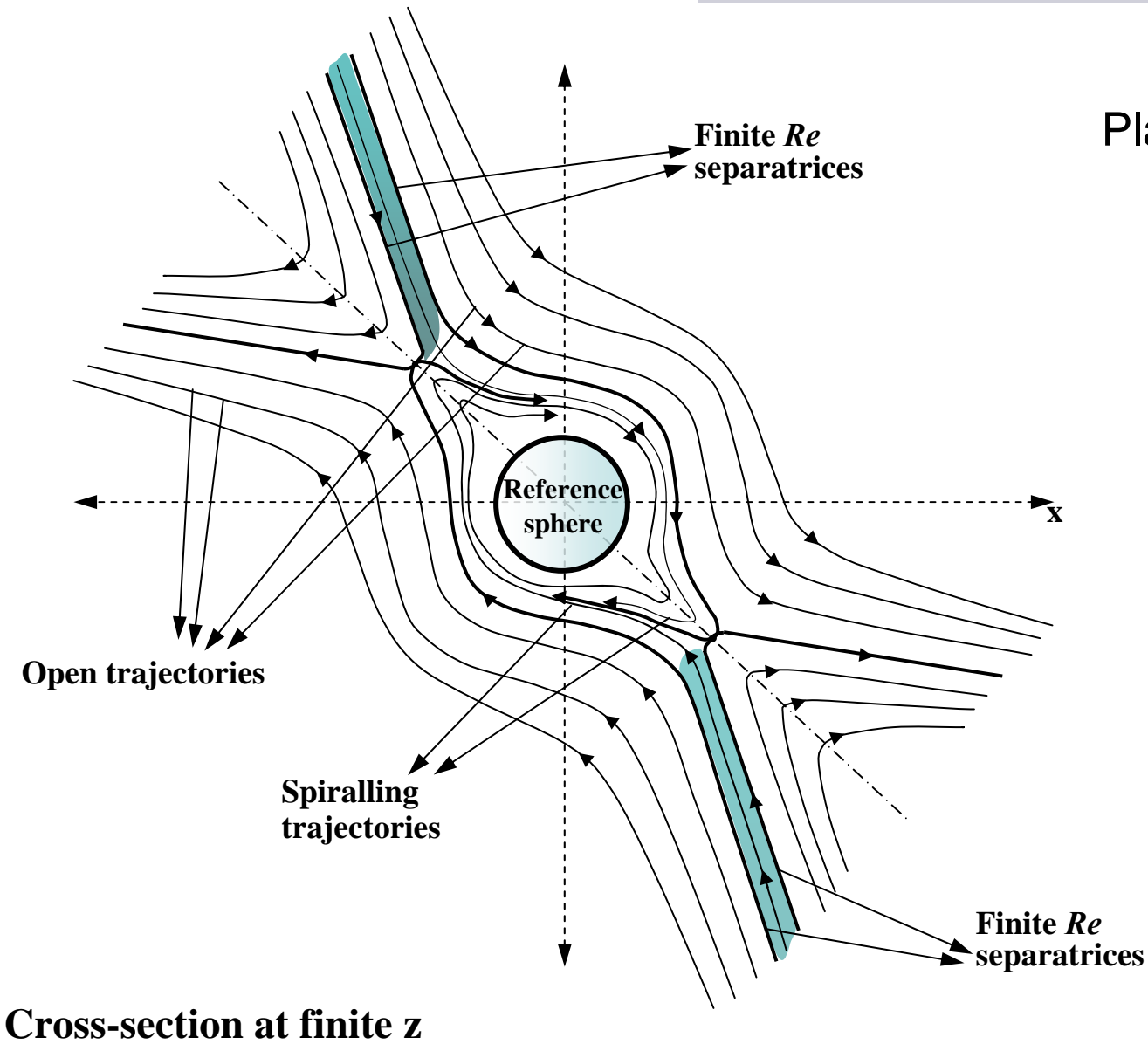
Intersection of separatrix envelope with $x + y = 0$

Heat transfer from particle : finite Re



Plane of symmetry ($z = 0$)

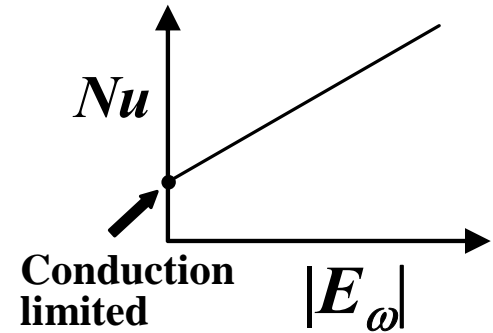
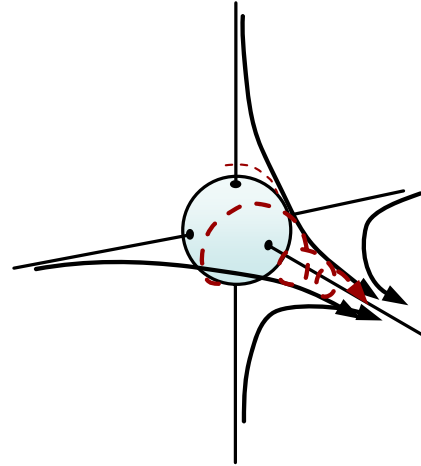
Heat transfer from particle : finite Re



Cross-section at finite z

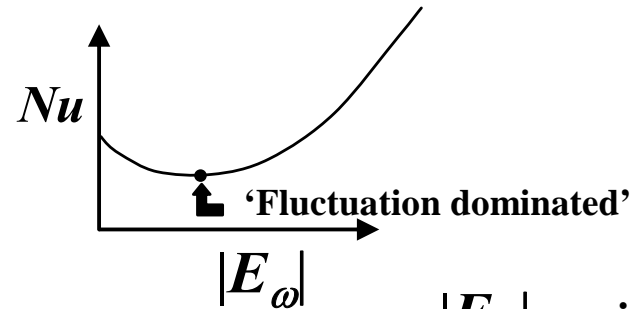
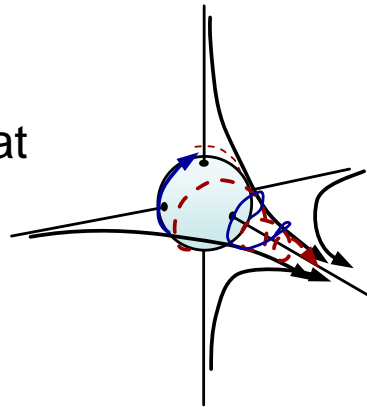
Heat transfer to small particles in a turbulent flow

If the particle Re is zero (particle much smaller than the Kolmogorov scale), Batchelor showed that the heat transfer depends on the extension rate parallel to the vorticity axis



$Re = 0$

At finite Re , microscale Inertia induces a flow that opposes the imposed axial extension reducing the heat transfer



Finite Re

$|E_\omega|$: axial extension

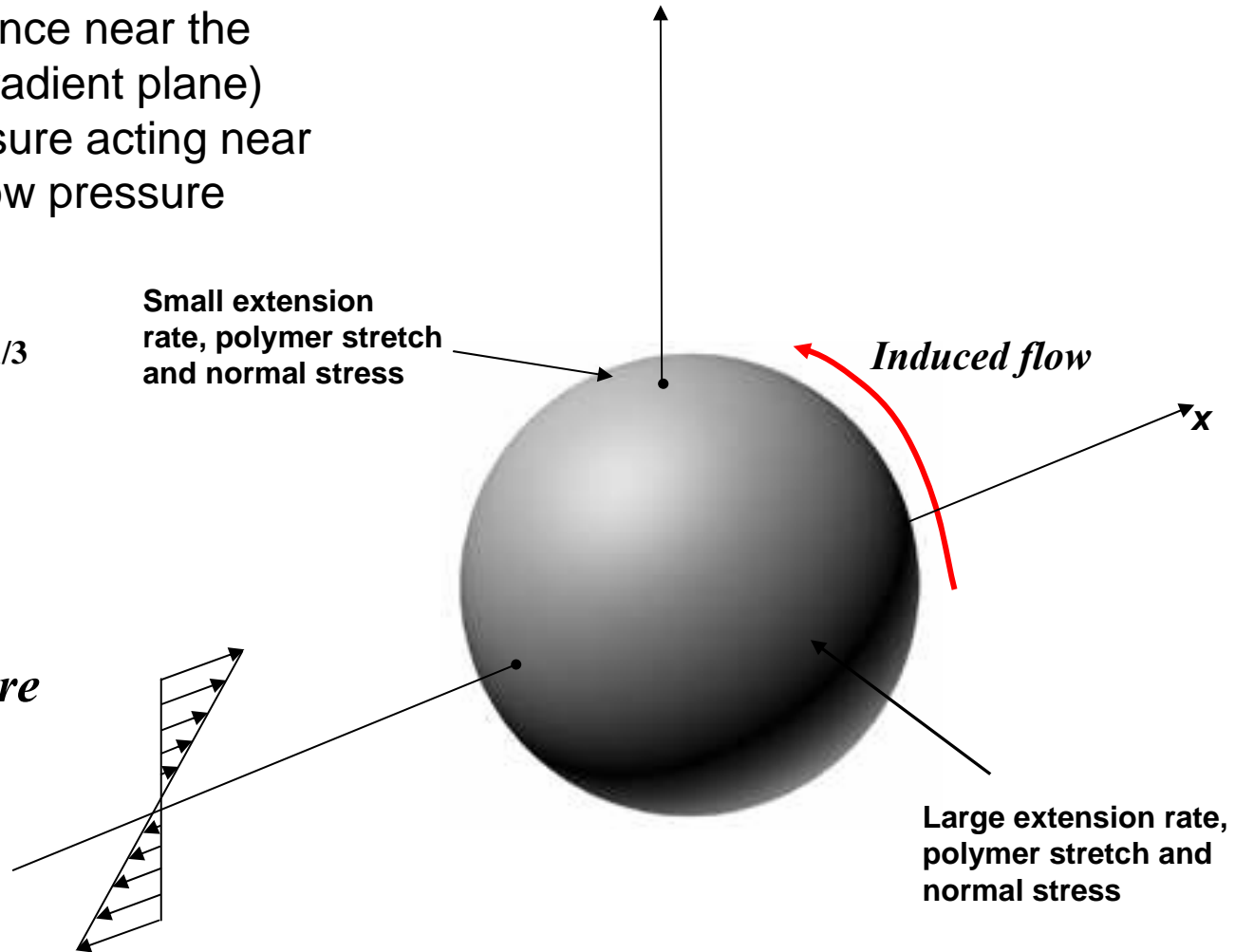
Heat transfer from particle in a viscoelastic fluid

The sphere reduces the extension rate and normal stress difference near the sphere's equator (flow-gradient plane) leading to a higher pressure acting near the sphere's poles and low pressure near the equator

$$Nu = 6.01 (Pe De(1+\varepsilon))^{1/3}$$

Small extension rate, polymer stretch and normal stress

Induced flow



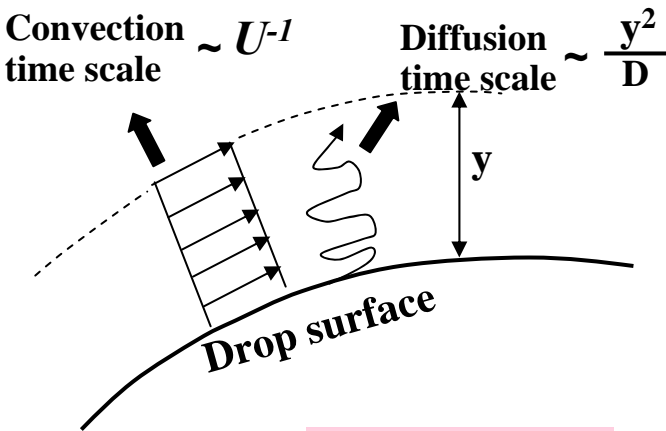
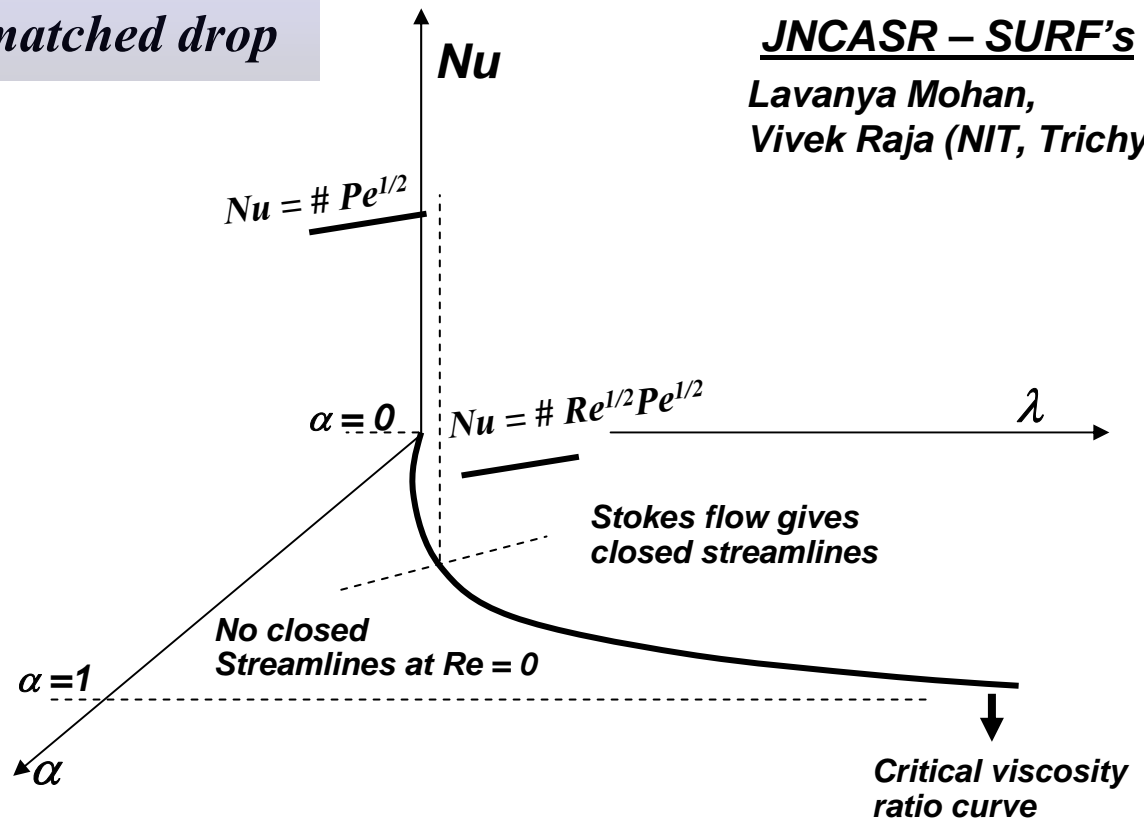
Large extension rate, polymer stretch and normal stress

De : dimensionless measure of a relaxation time

Heat transfer from a density matched drop

JNCASR – SURF's
 Lavanya Mohan,
 Vivek Raja (NIT, Trichy)

- Flow inside drop likely to be chaotic
- Enhanced drop mixing
- Controlling resistance lies outside the drop



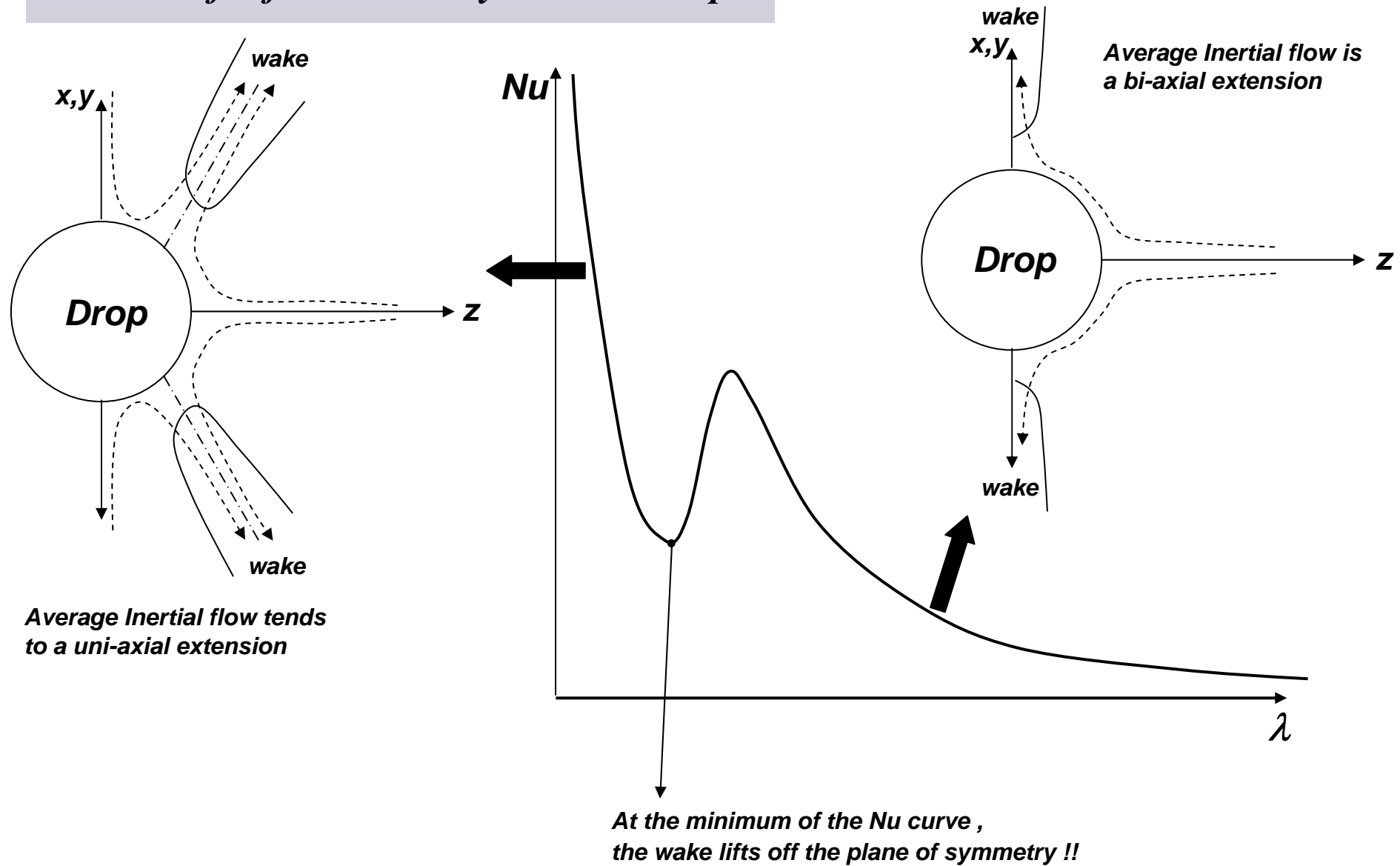
$$U^{-1} \sim \frac{y^2}{D}$$

$$Nu = \# Pe^{1/2} \text{ for } Pe \gg 1 \quad \text{Open streamlines}$$

$$Nu = \# (Re Pe)^{1/2} \text{ for } Pe \gg 1$$

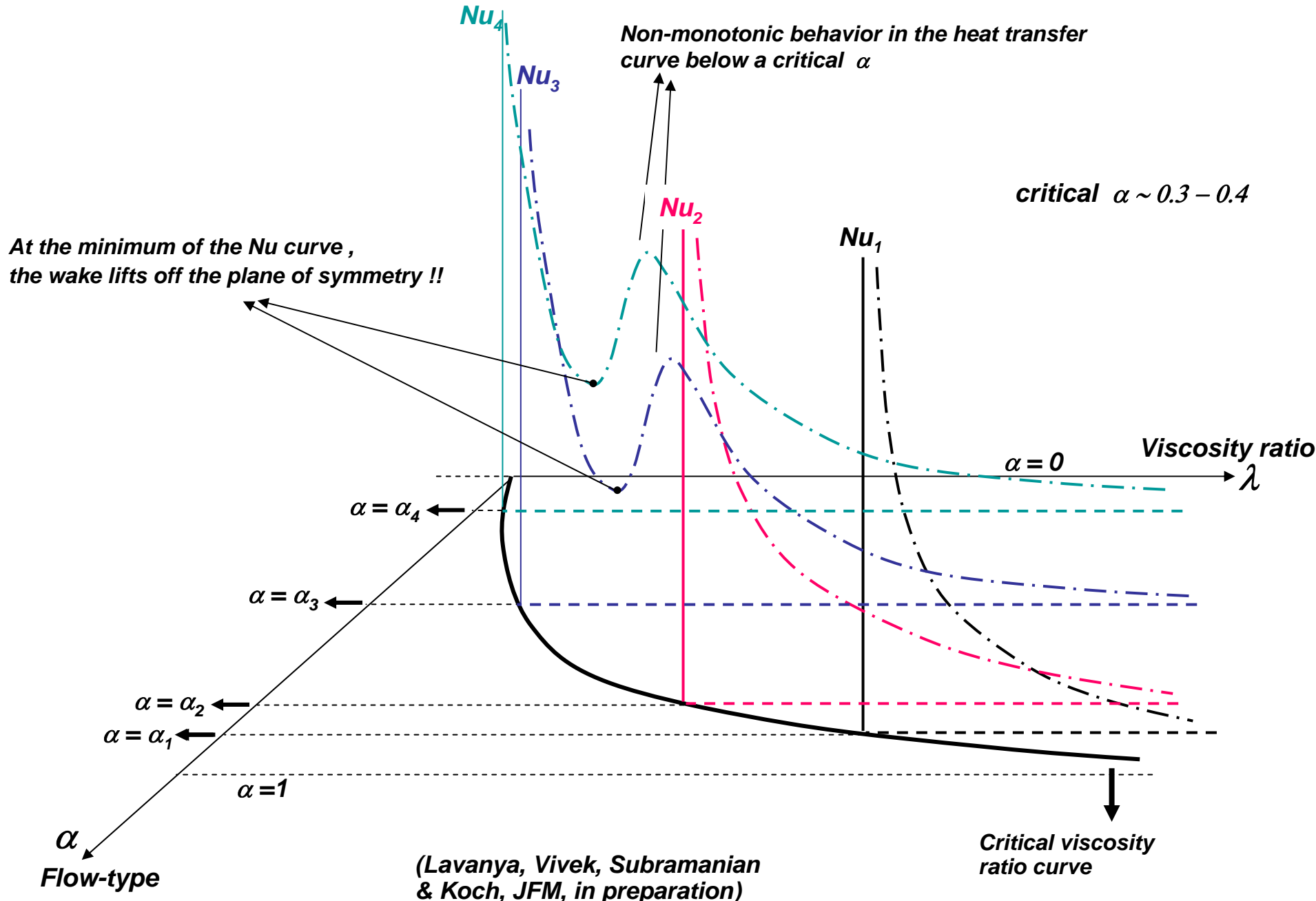
\longrightarrow Function of flow type and viscosity ratio

Heat transfer from a density matched drop



Nu curve for α less than critical

Heat transfer from a density matched drop



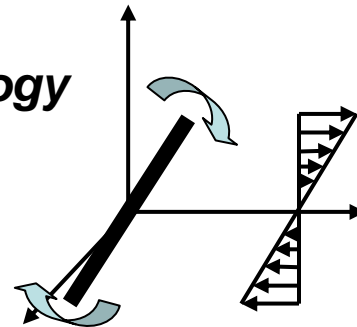
Momentum transfer :

Orientation dynamics of anisotropic particles

Orientation distribution in Sedimentation



Orientation distribution in shearing flows - rheology

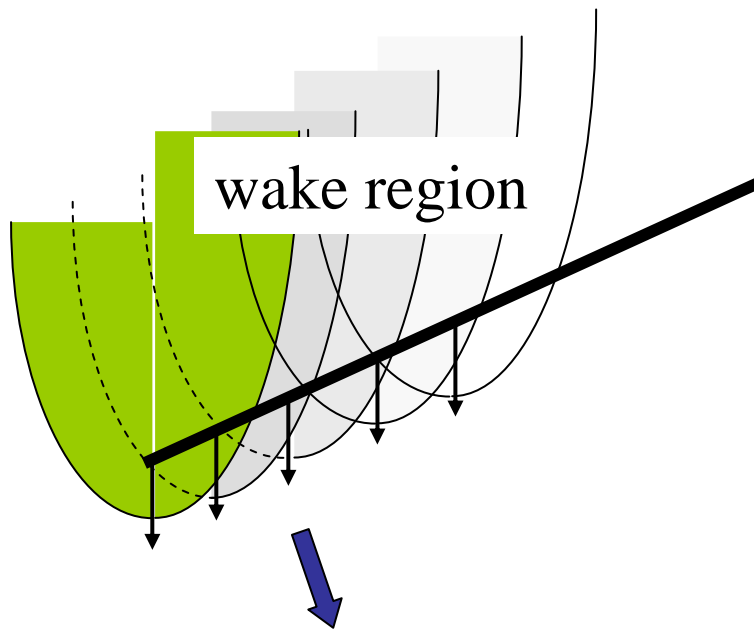


Irreversibility due to

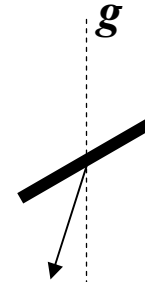
- *Inertia*
- *non-Newtonian rheology*
- *Interactions*
- *Brownian motion*

Sedimenting particle

Inertial effects in sedimentation →



Lagging part of the particle is shielded by the wake



$Re = 0$

**Constant orientation
(Reversibility)**



- *Degenerate scenario*
- *Mean sedimentation velocity not unique*

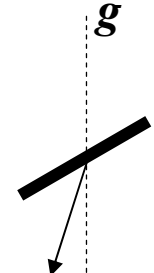


Finite Re

**Particle settles
broadside-on**

Sedimenting particle

Viscoelastic effects in sedimentation
(Tensioned streamlines) →



$De = 0$

Constant orientation
(Reversibility)

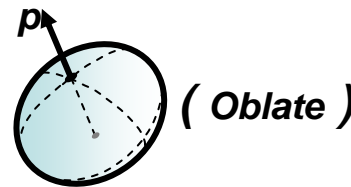


Finite De

Particle settles
longside-on

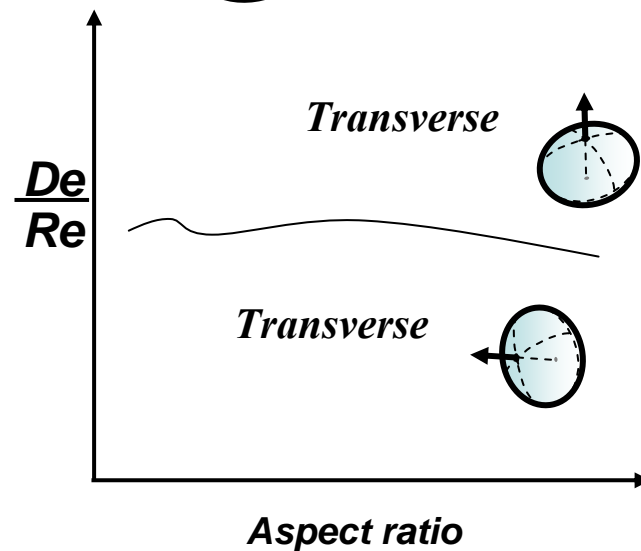
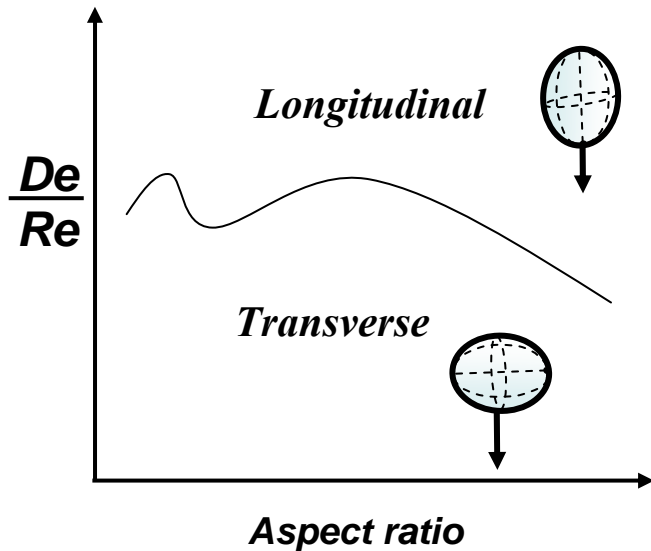


(Prolate)



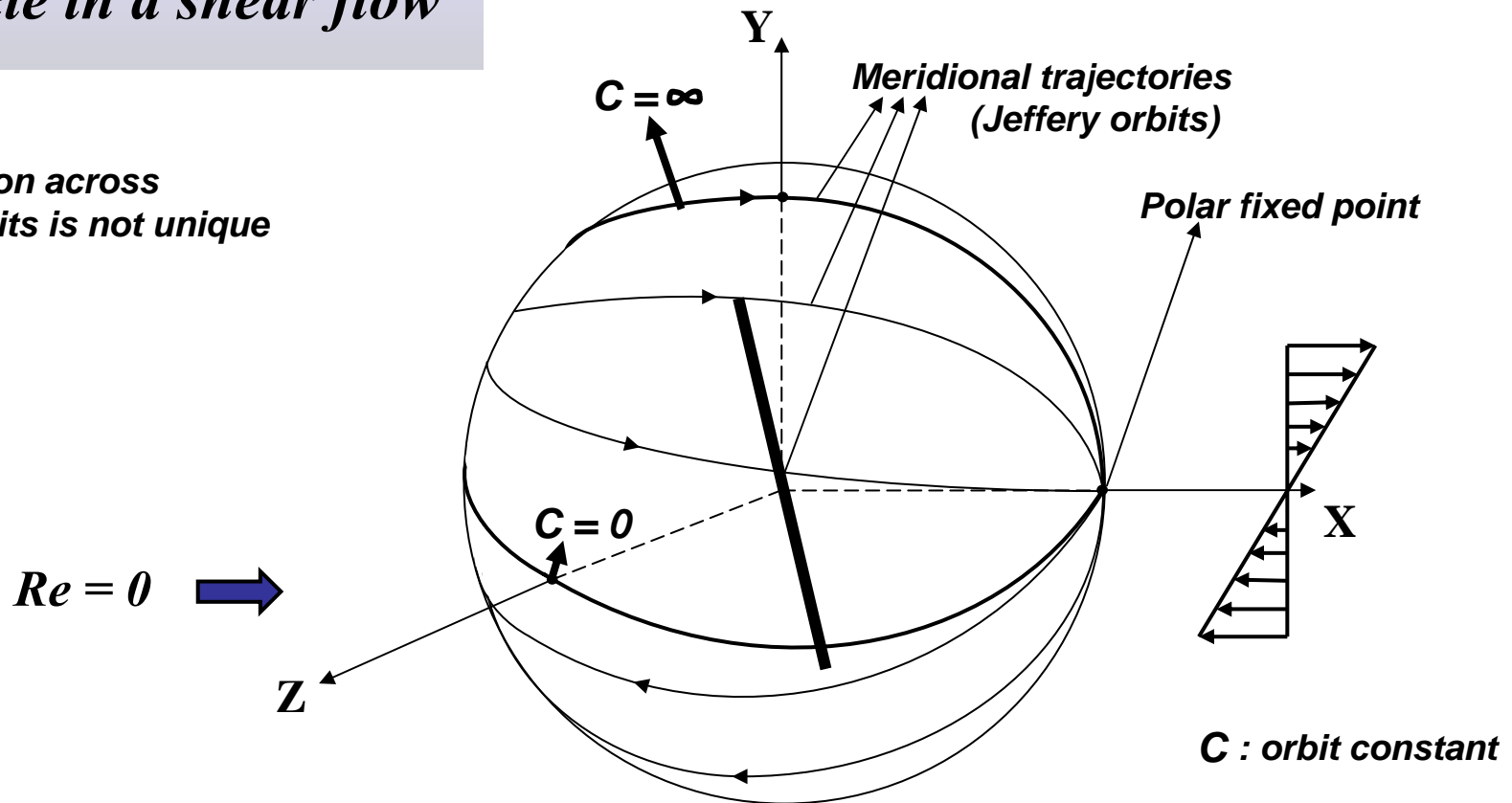
(Oblate)

Spheroidal harmonics
formalism allows an
analytical prediction



Particle in a shear flow

- *Distribution across jeffery orbits is not unique*



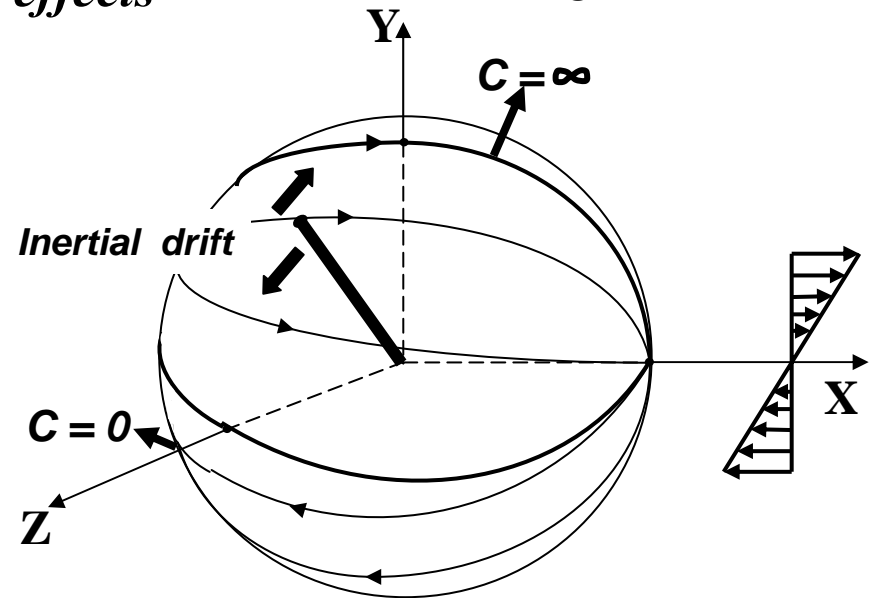
- *Orientation distribution of the suspended particles is a key determinant of macroscopic properties*
- *The distribution of orientation of a single particle in simple shear flow under inertialess conditions is indeterminate!*
- *'Microscale inertia' eliminates this indeterminacy.*

Particle in a shear flow : Inertial effects

C : orbit constant

Finite Re →

- Inertial drift leads to a drift across Jeffery orbits
- A determinate orientation distribution
- Direction of drift depends on aspect ratio



- For a fibre, first effects of inertia lead to a drift towards the shearing (XY) plane
($C = \infty$: tumbling)
- For a nearly spherical particle, drift is towards the vorticity axis
($C = 0$: log-rolling)

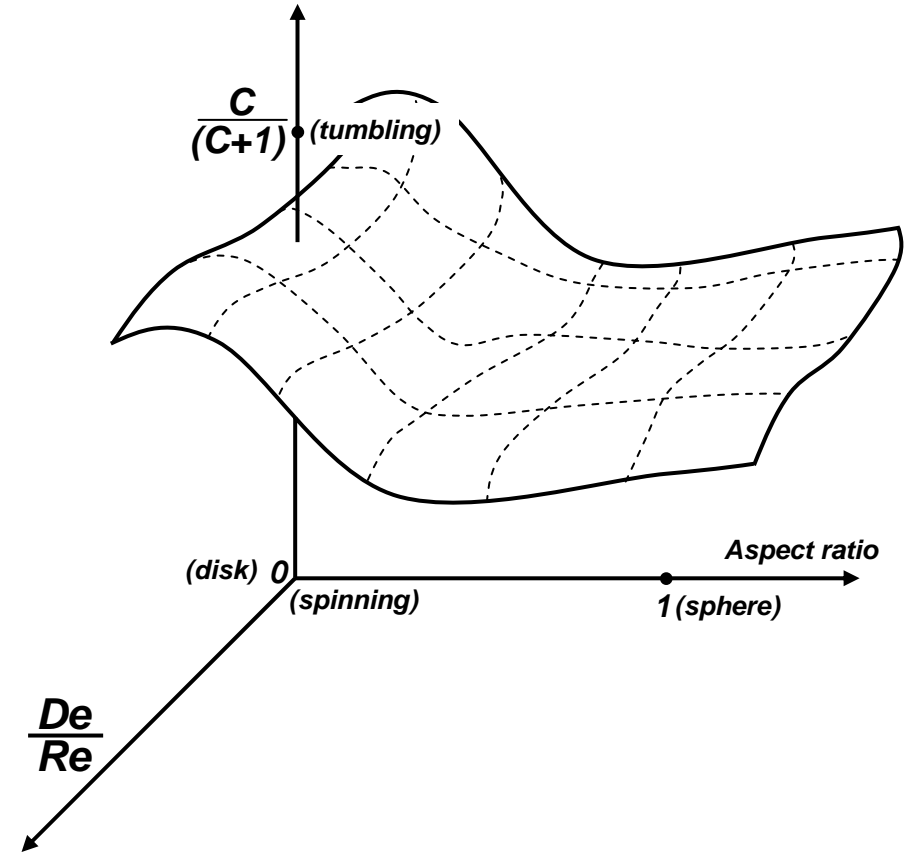
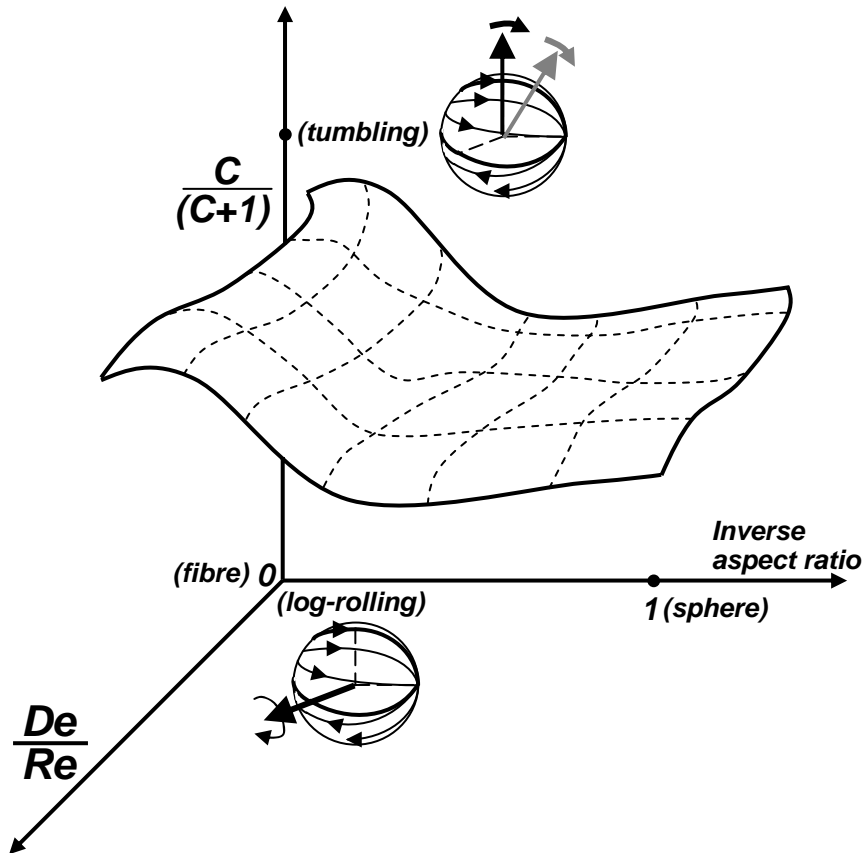
• Implied bifurcation in orientation dynamics for a critical aspect ratio !

(Subramanian & Koch, JFM, 2005)

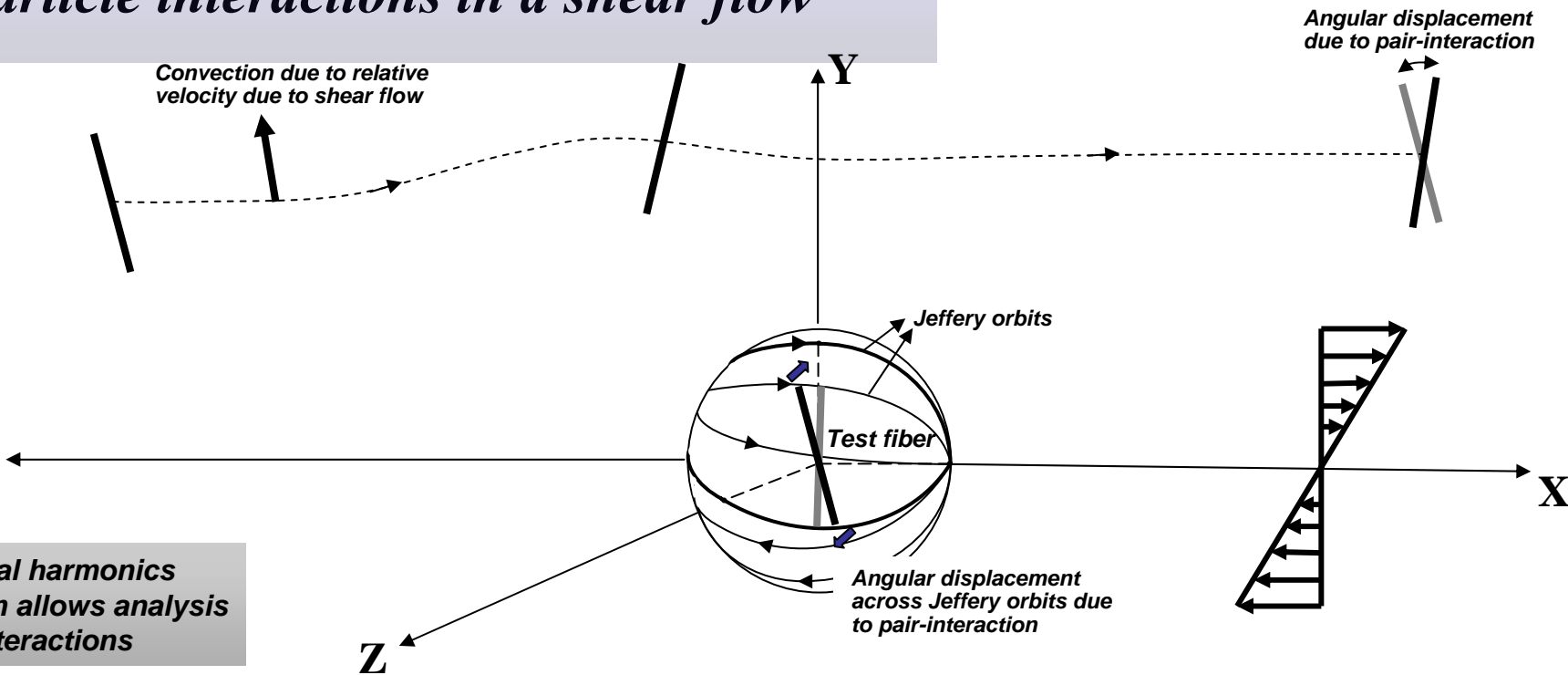
(Subramanian & Koch, JFM, 2006)

Particle in a shear flow : Inertial + Viscoelastic effects

- Competing inertial and viscoelastic effects may lead to intermediate orbits
- Orientation distribution is a function of (De/Re) and the particle aspect ratio



Particle interactions in a shear flow



- Orientation de-correlation for very slender particles is a local rotary diffusion process

Inertialess suspension of orientable particles : [Position-orientation space]

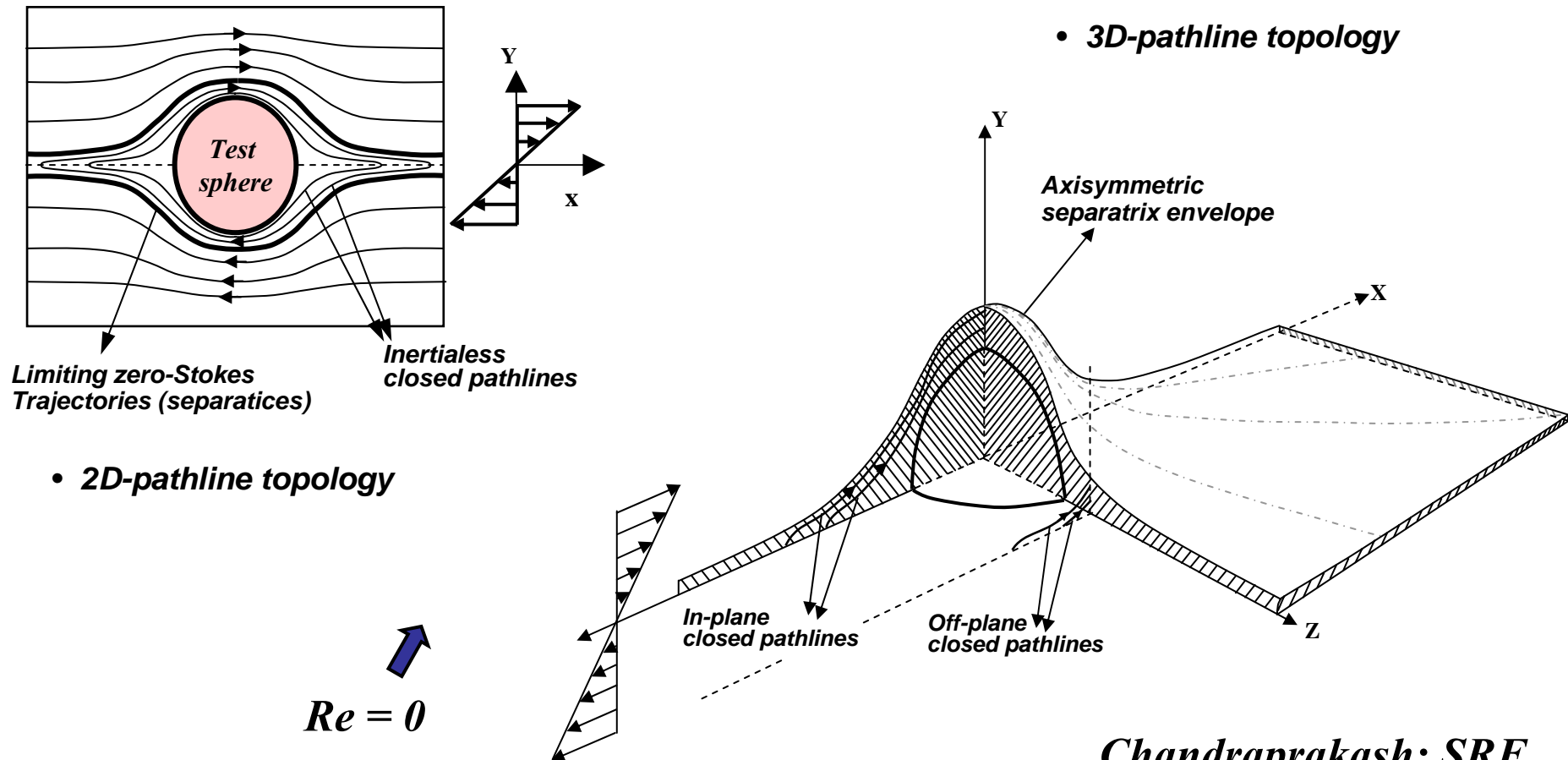
- Non-local interactions in orientation space for an arbitrary aspect ratio
- Complex distributed (long-ranged) pair-interactions lead to orientation de-correlation
- Orientation de-correlation leads to a determinate single-particle orientation distribution

Molecular gas of (elastic) hard-spheres or Granular gases : [Position-velocity space]

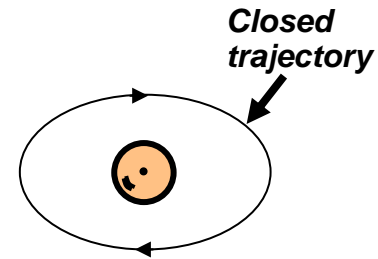
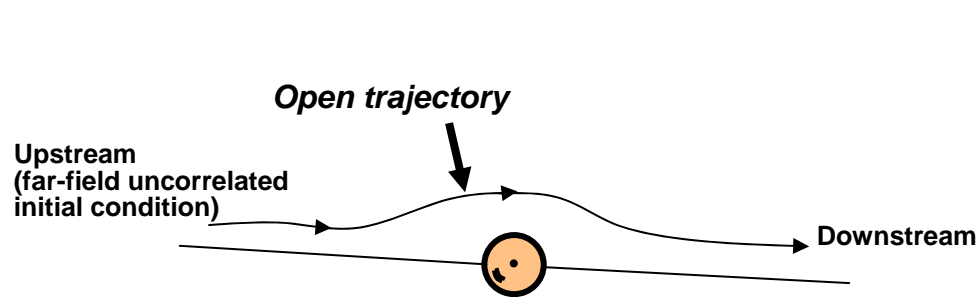
- Non-local interactions in velocity space - Boltzmann equation
- Impulsive pair-interactions lead to velocity de-correlation
- Velocity de-correlation leads to a Maxwellian distribution

Pair-hydrodynamic interactions of spherical particles in shear flow

- Reversibility again leads to closed pair-particle pathlines in the absence of inertia
- Closed pathlines lead to an indeterminacy in the pair-distribution function : $g(r)$
- Similar to the original heat transfer problem, slow Brownian diffusion across 'path-surfaces' determines pair-probability across closed pathlines in dilute suspensions
- Brownian diffusion of particles/drops across pathlines also controls the rate of coagulation

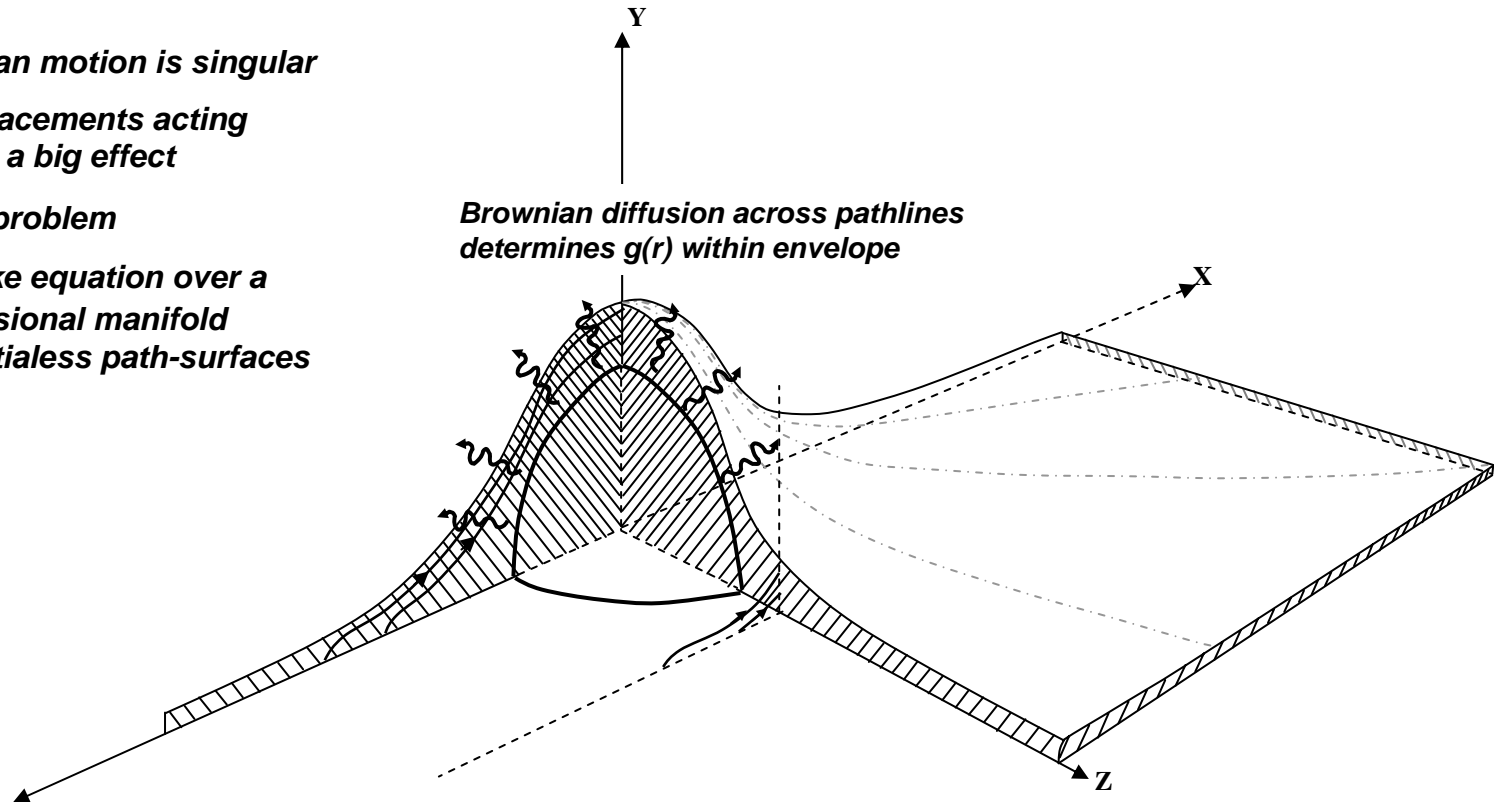


Pair-hydrodynamic interactions of spherical particles in shear flow



Where and what does one Specify as the initial condition ? Indeterminacy

- *Limit of weak Brownian motion is singular*
- *Small Brownian displacements acting over a long time have a big effect*
- *Interesting diffusion problem*
- *To solve a Laplace- like equation over a curvilinear two-dimensional manifold corresponding to inertialess path-surfaces*



Conclusions

A SMALL amount of inertia, or in general, a small source of irreversibility at the micro-scale can make a BIG difference

Questions