PRAMANA
—journal of
physics

Vol. 51, Nos 1 & 2,
July/August 1998
pp. 51–64

Neutrino masses and mixing in the light of experimental data

S M BILENKY, C GIUNTI* and W GRIMUS†
Joint Institute for Nuclear Research, Dubna, Russia, and INFN, Sezione di Torino, Via P. Giuria 1, I–10125 Torino, Italy
* INFN, Sezione di Torino, and Dipartimento di Fisica Teorica, Universitá di Torino, Via P. Giuria 1, I–10125 Torino, Italy
† Institute for Theoretical Physics, University of Vienna, Boltzmanngasse 5, A–1090 Vienna, Austria

Abstract. All the possible schemes of neutrino mixing with four massive neutrinos inspired by the existing experimental indications in favour of neutrino mixing are considered. It is shown that the scheme with a neutrino mass hierarchy is not compatible with the experimental results, likewise all other schemes with the masses of three neutrinos close together and the fourth mass separated by a gap needed to incorporate the LSND neutrino oscillations. Only two schemes with two pairs of neutrinos with close masses separated by this gap of the order of 1 eV are in agreement with the results of all experiments. We carefully examine the arguments leading to this conclusion and also discuss experimental consequences of the two favoured neutrino schemes.

Keywords. Neutrino oscillations; mass spectrum; mixing schemes.

PACS No. 14.60

1. Indications in favour of neutrino oscillations

1.1 *Notation*

Neutrino masses and neutrino mixing are natural phenomena in gauge theories extending the standard model (see, for example, ref. [1]). However, for the time being masses and mixing angles cannot be predicted on theoretical grounds and they are the central subject of the experimental activity in the field of neutrino physics.

In the general discussion, we assume that there are n neutrino fields with definite flavours and that neutrino mixing is described by a $n \times n$ unitary mixing matrix U such that

$$\nu_{\alpha L} = \sum_{j=1}^{n} U_{\alpha j}\, \nu_{jL} \quad (\alpha = e, \mu, \tau, s_1, \ldots, s_{n-3}). \tag{1}$$

Note that the neutrino fields $\nu_{\alpha L}$ other than the three active neutrino flavour fields ν_{eL}, $\nu_{\mu L}$, $\nu_{\tau L}$ must be sterile to comply with the result of the LEP measurement of the number of neutrino flavours. The fields ν_{jL} ($j = 1, \ldots, n$) are the left-handed components of neutrino fields with definite mass m_j. We assume the ordering $m_1 \leq m_2 \leq \cdots \leq m_n$ for the neutrino masses. In eq. (1) and in the following discussion of neutrino oscillations it does not matter if the neutrinos are of Dirac or Majorana type. One should only keep in mind that different types cannot mix.

The most striking feature of neutrino masses and mixing is the quantum-mechanical effect of neutrino oscillations [2]. The probability of the transition $\nu_\alpha \to \nu_\beta$ is given by

$$P_{\nu_\alpha \to \nu_\beta} = \left| \sum_{j=1}^{n} U_{\beta j} U_{\alpha j}^{*} \exp\left(-i \frac{\Delta m_{j1}^2 L}{2p} \right) \right|^2 \tag{2}$$

where $\Delta m_{j1}^2 \equiv m_j^2 - m_1^2$, L is the distance between source and detector and p is the neutrino momentum. Equation (2) is valid for $p^2 \gg m_j^2$ $(j = 1, \ldots, n)$.[1] Evidently, from neutrino oscillation experiments only differences of squares of neutrino masses can be determined. The probability for $\bar{\nu}_\alpha \to \bar{\nu}_\beta$ transitions is obtained from eq. (2) by the substitution $U \to U^*$.

1.2 *Indications in favour of neutrino masses and mixing*

At present, indications that neutrinos are massive and mixed have been found in solar neutrino experiments ([4–7] and [8, 9]), in atmospheric neutrino experiments ([10–12] and [13, 9]) and in the LSND experiment [14]. From the analyses of the data of these experiments in terms of neutrino oscillations it follows that there are three different scales of neutrino mass-squared differences:

- *Solar neutrino deficit*: Interpreted as effect of neutrino oscillations the relevant value of the mass-squared difference is determined as

$$\Delta m_{\text{sun}}^2 \sim 10^{-5}\,\text{eV}^2\ (\text{MSW}) \quad \text{or}\ \Delta m_{\text{sun}}^2 \sim 10^{-10}\,\text{eV}^2\ (\text{vac. osc.})\ [15, 16]. \tag{3}$$

  The two possibilities for Δm_{sun}^2 correspond, respectively, to the MSW [17] and to the vacuum oscillation solutions of the solar neutrino problem.
- *Atmospheric neutrino anomaly*: Interpreted as effect of neutrino oscillations, the zenith angle dependence of the atmospheric neutrino anomaly [10, 13, 9] gives

$$\Delta m_{\text{atm}}^2 \sim 5 \times 10^{-3}\,\text{eV}^2\ [18]. \tag{4}$$

- *LSND experiment*: The evidence for $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations in this experiment leads to

$$\Delta m_{\text{SBL}}^2 \sim 1\,\text{eV}^2 \quad [14] \tag{5}$$

  where Δm_{SBL}^2 is the neutrino mass-squared difference relevant for short-baseline (SBL) experiments.

Thus, at least four light neutrinos with definite masses must exist in nature in order to accommodate the results of all neutrino oscillation experiments. Denoting by δm^2 a generic neutrino mass-squared difference we can summarize the discussion in the following way:

◇ 3 different scales of δm^2 $\Rightarrow$ 4 neutrinos (or more)

Therefore there exists at least one non-interacting sterile neutrino [19–25].

---

[1] There are additional conditions depending on the neutrino production and detection processes which must hold for the validity of eq. (2). See, e.g., ref. [3] and references therein.

However, we must also take into account the fact that in several short-baseline experiments neutrino oscillations were not observed. The results of these experiments allow to exclude large regions in the space of the neutrino oscillation parameters. This will be done in the next section.

The plan of this report is as follows. In § 2 we extensively discuss SBL neutrino oscillations for an arbitrary number of neutrinos. In § 3 we argue that a 4-neutrino mass hierarchy is disfavoured by the experimental data. Thereby, solar and atmospheric neutrino flux data play a crucial role. In § 4 we introduce the two 4-neutrino mass and mixing schemes favoured by all neutrino oscillation experiments. We discuss possibilities to check these schemes in long-baseline (LBL) neutrino oscillation experiments in § 5. Our conclusions are presented in § 6.

2. SBL experiments

2.1 *The oscillation phase*

As a guideline, SBL neutrino oscillation experiments are sensitive to mass-squared differences $\delta m^2 > 0.1\,\mathrm{eV}^2$. A generic oscillation phase is given by

$$\frac{\delta m^2 L}{2p} \simeq 2.53 \times \left(\frac{\delta m^2}{1\,\mathrm{eV}^2}\right)\left(\frac{p}{1\,\mathrm{MeV}}\right)^{-1}\left(\frac{L}{1\,\mathrm{m}}\right). \tag{6}$$

Distinguishing reactor and accelerator experiments and assuming that experiments are roughly sensitive to phases (6) around 0.1 or larger we get the following conditions from $\delta m^2 > 0.1\,\mathrm{eV}^2$:

- Reactors: $p \sim 1\,\mathrm{MeV}$ and therefore $L \gtrsim 10$ m.
- Accelerators: $L \gtrsim 10^3\,\mathrm{m} \times (p/1\,\mathrm{GeV})$.

2.2 *Basic assumption and formalism*

We will make the following basic assumption in the further discussion in this report:

◇ A single δm^2 is relevant in SBL neutrino experiments.

In accordance with eq. (5) we denote this δm^2 by $\Delta m^2_{\mathrm{SBL}}$.

As a consequence of this assumption the neutrino mass spectrum consists of two groups of close masses, separated by a mass difference in the eV range. Denoting the neutrinos of the two groups by $\nu_1, \dots, \nu_r$ and $\nu_{r+1}, \dots, \nu_n$, respectively, the mass spectrum looks like

$$m_1^2 \leq \cdots \leq m_r^2 \ll m_{r+1}^2 \leq \cdots \leq m_n^2 \tag{7}$$

such that

$$\begin{aligned} &\Delta m^2_{kj} \ll \Delta m^2_{\mathrm{SBL}} \quad \text{for} \quad 1 \leq j < k \leq r \quad \text{and} \quad r+1 \leq j < k \leq n, \\ &\Delta m^2_{kj} \simeq \Delta m^2_{\mathrm{SBL}} \quad \text{for} \quad 1 \leq j \leq r \quad \text{and} \quad r+1 \leq k \leq n \end{aligned} \tag{8}$$

for the purpose of the SBL formalism. In eq. (8) we have used the notation $\Delta m^2_{kj} \equiv$

$m_k^2 - m_j^2$. Equation (2) together with eq. (8) gives the SBL transition probability

$$P^{(\mathrm{SBL})}_{\nu_\alpha \to \nu_\beta} = \left| \sum_{j=1}^{r} U_{\beta j} U^*_{\alpha j} + \exp\left(-i \frac{\Delta m^2_{\mathrm{SBL}} L}{2p} \right) \sum_{j=r+1}^{n} U_{\beta j} U^*_{\alpha j} \right|^2 . \tag{9}$$

For the probability of the transition $\nu_\alpha \to \nu_\beta$ ($\alpha \neq \beta$) we obtain from eq. (9)

$$P^{(\mathrm{SBL})}_{\nu_\alpha \to \nu_\beta} = \frac{1}{2} A_{\alpha;\beta} \left(1 - \cos \frac{\Delta m^2_{\mathrm{SBL}} L}{2p} \right) \tag{10}$$

where the oscillation amplitude $A_{\alpha;\beta}$ is given by

$$A_{\alpha;\beta} = 4 \left| \sum_{j \geq r+1} U_{\beta j} U^*_{\alpha j} \right|^2 = 4 \left| \sum_{j \leq r} U_{\beta j} U^*_{\alpha j} \right|^2 . \tag{11}$$

The second equality sign in this equation follows from the unitarity of U. Furthermore, the oscillation amplitude $A_{\alpha;\beta}$ fulfills the condition $A_{\alpha;\beta} = A_{\beta;\alpha} \leq 1$. The second part of this equation is a consequence of the Cauchy–Schwarz inequality and the unitarity of the mixing matrix. The survival probability of ν_α is calculated as

$$P^{(\mathrm{SBL})}_{\nu_\alpha \to \nu_\alpha} = 1 - \sum_{\beta \neq \alpha} P_{\nu_\alpha \to \nu_\beta} = 1 - \frac{1}{2} B_{\alpha;\alpha} \left(1 - \cos \frac{\Delta m^2_{\mathrm{SBL}} L}{2p} \right) \tag{12}$$

with the survivial amplitude

$$B_{\alpha;\alpha} = 4 \left(\sum_{j \geq r+1} |U_{\alpha j}|^2 \right) \left(1 - \sum_{j \geq r+1} |U_{\alpha j}|^2 \right) = 4 \left(\sum_{j \leq r} |U_{\alpha j}|^2 \right) \left(1 - \sum_{j \leq r} |U_{\alpha j}|^2 \right). \tag{13}$$

Conservation of probability gives the important relation

$$B_{\alpha;\alpha} = \sum_{\beta \neq \alpha} A_{\alpha;\beta} \leq 1. \tag{14}$$

The expressions (10) and (12) describe the transitions between all possible neutrino states, whether active or sterile. Let us stress that with the basic assumption in the beginning of this subsection the oscillations in all channels are characterized by the same oscillation length $l_{\mathrm{osc}} = 4\pi p / \Delta m^2_{\mathrm{SBL}}$. Furthermore, the substitution $U \to U^*$ in the amplitudes (11) and (13) does not change them and therefore it ensues from the basic SBL assumption that the probabilities (10) and (12) hold for antineutrinos as well and hence there is no CP violation in SBL neutrino oscillations.

The oscillation probabilities (10) and (12) look like 2-flavour probabilities. Defining $\sin^2 2\theta_{\alpha\beta} \equiv A_{\alpha;\beta}$, $\sin^2 2\theta_\alpha \equiv B_{\alpha;\alpha}$ and $\sin^2 2\theta_\beta \equiv B_{\beta;\beta}$ for $\alpha \neq \beta$, the resemblance is even more striking. It means that the basic SBL assumption allows to use the 2-flavour oscillation formulas in SBL experiments. However, genuine 2-flavour $\nu_\alpha \leftrightarrow \nu_\beta$ neutrino oscillations are characterized by a single mixing angle given by $\theta_{\alpha\beta} = \theta_\alpha = \theta_\beta$.

2.3 *Disappearance experiments*

For the two flavours $\alpha = e$ and μ results of disappearence experiments are available. We will use the 90% exclusion plots of the Bugey reactor experiment [26] for $\bar{\nu}_e \to \bar{\nu}_e$ disappearance and the 90% exclusion plots of the CDHS [27] and CCFR [28] accelerator experiments for $\nu_\mu \to \nu_\mu$ disappearance. Since no neutrino disappearance has been seen there are upper bounds $B^0_{\alpha;\alpha}$ on the disappearance amplitudes for $\alpha = e, \mu$. These experimental bounds are functions of $\Delta m^2_{\mathrm{SBL}}$. It follows that

$$B_{\alpha,\alpha} = 4\, c_\alpha (1 - c_\alpha) \leq B^0_{\alpha;\alpha} \quad \text{with} \quad c_\alpha \equiv \sum_{j=1}^{r} |U_{\alpha j}|^2 \tag{15}$$

and therefore [29]

$$c_\alpha \leq a^0_\alpha \quad \text{or} \quad c_\alpha \geq 1 - a^0_\alpha \quad \text{with} \quad a^0_\alpha \equiv \tfrac{1}{2}\left(1 - \sqrt{1 - B^0_{\alpha;\alpha}}\right). \tag{16}$$

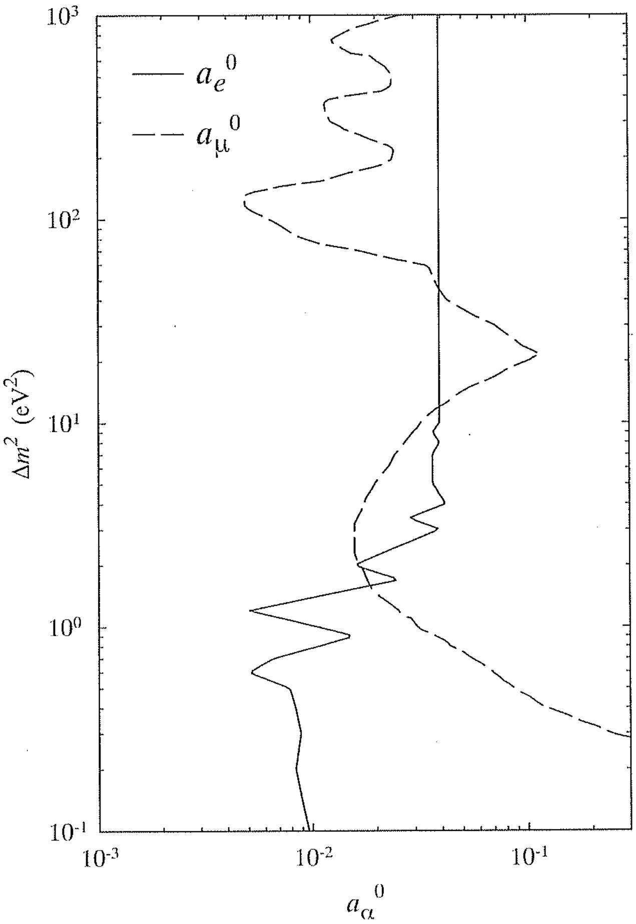

Figure 1. The bounds $a^0_\alpha (\alpha = e, \mu)$.

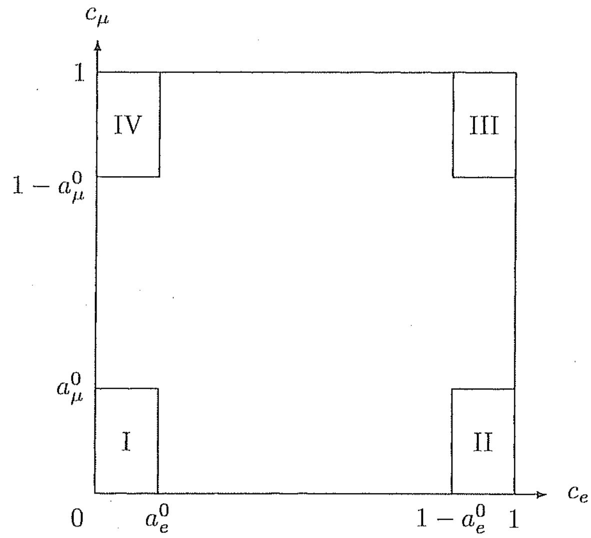

Figure 2. The allowed regions in the $C_e - C_\mu$ plane.

Equation (16) shows that $a_\alpha^0 \leq 1/2$. In figure 1 the bounds a_e^0 and a_μ^0 are plotted as functions of $\Delta m^2_{\rm SBL}$ in the wide range

$$10^{-1} \leq \Delta m^2_{\rm SBL} \leq 10^3 {\rm eV}^2. \tag{17}$$

In this range a_e^0 is small ($a_e^0 \lesssim 4 \times 10^{-2}$) and $a_\mu^0 \lesssim 10^{-1}$ for $\Delta m^2_{\rm SBL} \gtrsim 0.5$ eV^2. This means that in the c_e–c_μ unit square for every $\Delta m^2_{\rm SBL}$ we can distinguish four allowed regions according to $c_\alpha \leq a_\alpha^0$ or $c_\alpha \geq 1 - a_\alpha^0$ (see figure 2).

2.4 *The* $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ *transition in SBL experiments*

Considering the amplitude $A_{\mu;e}$, with the help of the Cauchy–Schwarz inequality we obtain from eq. (11)

$$A_{\mu;e} \leq 4 \min[c_e c_\mu, (1-c_e)(1-c_\mu)]. \tag{18}$$

Therefore, we immediately see that

$$A_{\mu;e} \leq 4\, a_e^0 a_\mu^0 \quad \text{in regions I and III.} \tag{19}$$

In figure 3 the result of the LSND experiment [14] for the amplitude $A_{\mu;e}$ is shown with 90% CL boundaries (shaded areas). All other experiments measuring this amplitude have obtained upper bounds [30–33]. In addition, the upper bound $B^0_{e;e}$ on the $\bar{\nu}_e \to \bar{\nu}_e$ survivial amplitude of Bugey [26] is indicated by the solid line in figure 3 since the unitarity relation (14) gives $A_{\mu;e} \leq B^0_{e;e}$. Finally, the curve passing through the circles represents the bound (19). Inspecting figure 3 we come to the following conclusion:

◇ Regions I and III are not compatible with the positive result of LSND

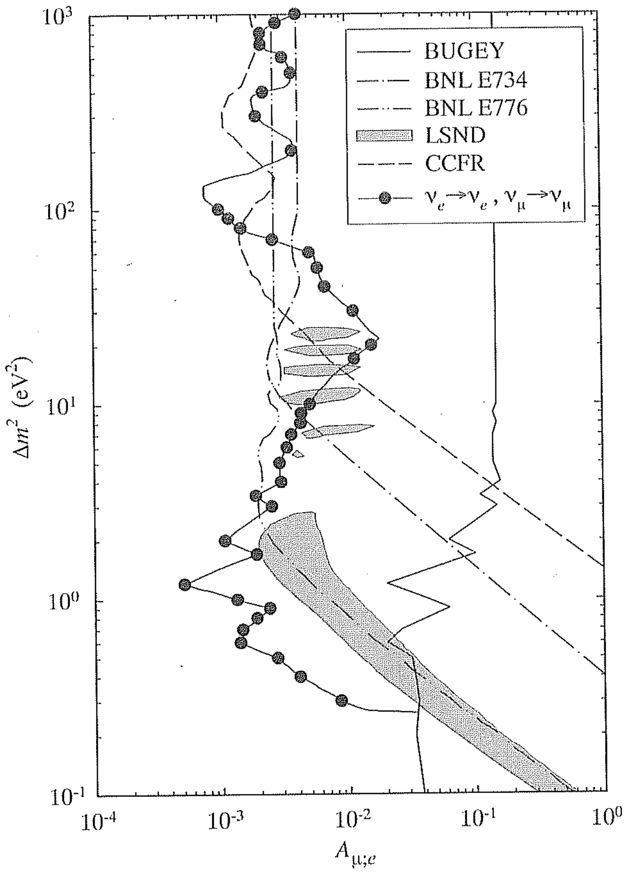

Figure 3. Upper bounds on $A_{\mu;e}$. The curve passing through the circles represents the bound [19]. The shaded areas show the result of the LSND experiment.

indicating $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations and the negative results of all other SBL experiments.

Furthermore, it can be read off from figure 3 that

$$0.27\,\mathrm{eV}^2 \lesssim \Delta m^2_{\mathrm{SBL}} \lesssim 2.2\,\mathrm{eV}^2 \tag{20}$$

is the favoured range for the SBL mass-squared difference. In this range $a^0_\mu \lesssim 0.3$ holds. Let us further mention that for $r = 1$ region III is already ruled out by the unitarity of the mixing matrix. The same is valid for $r = n - 1$ and region I.

3. The 4-neutrino mass hierarchy is disfavoured

In the case of a neutrino mass hierarchy, $m_1 \ll m_2 \ll m_3 \ll m_4$, the mass-squared differences Δm^2_{21} and Δm^2_{32} are relevant for the suppression of the flux of solar neutrinos and for the atmospheric neutrino anomaly, respectively. This case corresponds to $n = 4$

and $r = 3$ (see the formalism in subsection 2.2) with $c_\alpha = \sum_{j=1}^{3} |U_{\alpha j}|^2$. We only have to consider regions II and IV.

We will now take into account information from the solar neutrino anomaly assuming that it is solved by neutrino oscillations. From the fact that the 4th column vector in U pertaining to m_4 is not affected by solar neutrino oscillations we obtain a lower bound on the average survival probability of solar neutrinos given by (see refs [34, 20])

$$P^{\odot}_{\nu_e \to \nu_e} \geq |U_{e4}|^4. \tag{21}$$

In region IV we have $c_e \leq a_e^0$ or $|U_{e4}|^2 \geq 1 - a_e^0$ and therefore $P^{\odot}_{\nu_e \to \nu_e} \gtrsim 0.92$ holds for all solar neutrino energies. Such a large lower bound is not compatible with the solar neutrino data and we conclude:

◇ For a 4-neutrino mass hierarchy region IV is not compatible with the solar neutrino data.

Let us mention that inequality (21) is not completely exact. In the solar neutrino problem the matter background is important and it enters the total Hamiltonian for neutrino propagation. Nevertheless, to very good accuracy the largest eigenvalue of the Hamiltonian is given by $E_4 \simeq m_4^2/2p$ with eigenvector $v_4 \simeq (U_{\alpha 4})$ and corrections to this are of order $a_{CC}/\Delta m^2_{\mathrm{SBL}} \sim 10^{-5}$ where $a_{CC} = 2\sqrt{2} G_F N_e p$, N_e denotes the electron number density in the sun and in the solar core $a_{CC} \sim 10^{-5}\,\mathrm{eV}^2$. Furthermore, the evolution of v_4 in solar matter is adiabatic to an even better accuracy. Thus eq. (21) is accurate for our purpose.

It remains to discuss region II. To this end we consider the atmospheric neutrino anomaly which is expressed through the deviation of the double ratio

$$R = \frac{(\mu/e)_{\mathrm{data}}}{(\mu/e)_{\mathrm{MC}}} = \frac{P^{\mathrm{atm}}_{\nu_\mu \to \nu_\mu} + r^{-1} P^{\mathrm{atm}}_{\nu_e \to \nu_\mu}}{P^{\mathrm{atm}}_{\nu_e \to \nu_e} + r P^{\mathrm{atm}}_{\nu_\mu \to \nu_e}} \tag{22}$$

from 1. In eq. (22) $(\mu/e)_{\mathrm{MC}} \equiv r$ is the ratio of muon and electron events without neutrino oscillations. It is obtained by a Monte Carlo calculation which gives $r \simeq 1.57$ for sub-GeV events. For atmospheric neutrinos matter effects are non-negligible. Analogously to eq. (21) we have the lower bound

$$P^{\mathrm{atm}}_{\nu_\mu \to \nu_\mu} \geq |U_{\mu 4}|^4. \tag{23}$$

Let us assume for the moment that $P^{\mathrm{atm}}_{\nu_e \to \nu_\mu} = P^{\mathrm{atm}}_{\nu_\mu \to \nu_e}$. This is the case if CP is conserved or if the oscillating parts in the probabilities occurring in eq. (23) drop out because of averaging processes involving neutrino energy and distance between source and detector. Then it is easily shown by eqs (22) and (23) that [21]

$$R \geq P^{\mathrm{atm}}_{\nu_\mu \to \nu_\mu} \geq (1 - c_\mu)^2 \tag{24}$$

for all energy ranges and zenith angle bins. In this case in region II we obtain

$$R \geq (1 - a_\mu^0)^2. \tag{25}$$

The assumption $P^{\mathrm{atm}}_{\nu_e \to \nu_\mu} = P^{\mathrm{atm}}_{\nu_\mu \to \nu_e}$ is not fully satisfactory because it is not clear if or how well it is fulfilled. Let us therefore dispense with it now. The evolution of oscillation probabilities with a matter background has the general form [34, 20]

$$P_{\nu_\alpha\to\nu_\beta}(x_1,x_0) = |U(x_1)_{\beta k}B_{kj}U(x_0)^*_{\alpha j}|^2 \tag{26}$$

where B is a unitary matrix and $U(x)$ diagonalizes the Hamiltonian for neutrino propagation in matter at the location x. Note that eq. (26) is the generalization of eq. (2) referring to vacuum oscillations where B has only diagonal elements given by $\exp(-i\Delta m^2_{j1}(x_1-x_0)/2p)$ and $U(x_0)=U(x_1)=U$. Because $\Delta m^2_{\mathrm{SBL}} \gg a_{CC}, \Delta m^2_{\mathrm{atm}}, \Delta m^2_{\mathrm{sun}}$ the matrix B decomposes approximately into a 3×3 and a 1×1 block and therefore (see the discussion after eq. (21))

$$P_{\nu_\alpha\to\nu_\beta}(x_1,x_0) \simeq \sum_{j,k=1}^{3} |U(x_1)_{\beta k}B_{kj}U(x_0)^*_{\alpha j}|^2 + |U_{\beta 4}|^2|U_{\alpha 4}|^2. \tag{27}$$

This consideration leads to

$$R \geq \frac{(1-c_\mu)^2 + r^{-1}(1-c_e)(1-c_\mu)}{c_e^2 + (1-c_e)^2 + r[c_e c_\mu + (1-c_e)(1-c_\mu)]} \geq \frac{(1-c_\mu)^2}{1+rc_\mu}. \tag{28}$$

For $c_e \geq 1-a_e^0$, the central expression of eq. (28) has the minimum with respect to c_e at $c_e = 1$. This explains the second part of the inequality. Equation (28) represents a general bound valid for all energy ranges and zenith angles, whether assumption $P^{\mathrm{atm}}_{\nu_e\to\nu_\mu} = P^{\mathrm{atm}}_{\nu_\mu\to\nu_e}$ is fulfilled or not. Its right-hand side is a decreasing function in c_μ and therefore in region II we arrive at

$$R \geq \frac{(1-a_\mu^0)^2}{1+ra_\mu^0}. \tag{29}$$

Let us take advantage of the $\cos\zeta = -0.8$ bin (ζ is the zenith angle) of the sub-GeV super-Kamiokande events where $R \lesssim 0.48$ (90% CL) [35]. Here R is particularly small. In figure 4 the horizontal lines indicate R with its 90% CL interval taken from ref. [35], the dashed line represents the bound (25) and the solid line the general bound (29). Taking into account that the SBL experiments and, in particular, LSND restrict $\Delta m^2_{\mathrm{SBL}}$ to the range (20) ($\Delta m^2_{\mathrm{SBL}} \gtrsim 0.27$ eV^2) we see that the bound (25) rules out region II. However, the general bound (29) it is not tight enough around $\Delta m^2_{\mathrm{SBL}} \sim 0.3$ eV^2 to fully exclude region II with a neutrino mass hierarchy because a_μ^0 gets too large there.

There is a possiblity to improve the bound around 0.3 eV^2 in the following way. For a mass hierarchy we have $A_{\mu;e} = 4(1-c_e)(1-c_\mu)$ or $c_\mu = 1 - A_{\mu;e}/4(1-c_e) \leq 1 - A^{\min}_{\mu;e}/4a_e^0$ where $A^{\min}_{e;\mu}$ is the minimum measured by LSND. Thus we get

$$R \geq \frac{(1-\bar{a}_\mu^0)^2}{1+r\bar{a}_\mu^0} \quad \text{with} \quad \bar{a}_\mu^0 \equiv \min\left(a_\mu^0, \frac{A^{\min}_{\mu;e}}{4a_e^0}\right). \tag{30}$$

The dash-dotted curve in figure 4 which branches off from the solid curve corresponds to the part of the lower bound (30) originating from $A^{\min}_{\mu;e}$. Therefore, comparing the lower bounds on R obtained by using 90% CL data, namely the solid and the dash-dotted lines, with the uppermost horizontal line which corresponds to the 90% CL experimental upper bound on R we see that only a tiny allowed triangle is left in figure 4. Thus we arrive at

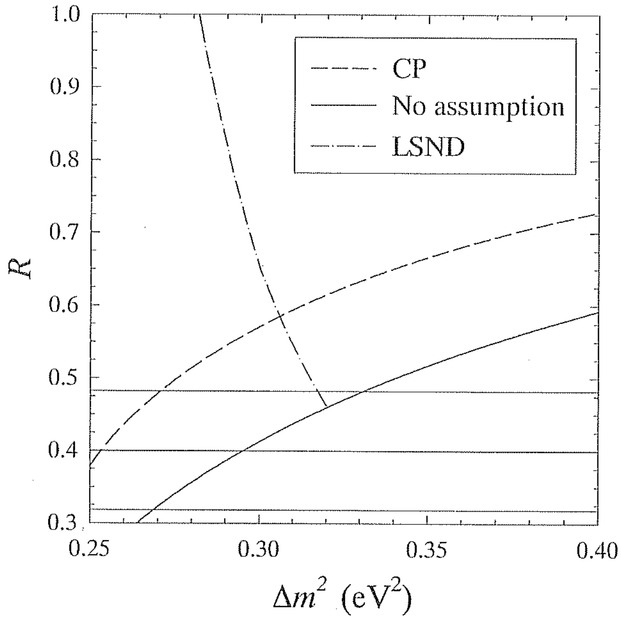

Figure 4. Lower bounds on R. See eqs. (25), (29) and (30).

the conclusion:

◇ With a 4-neutrino mass hierarchy region II is strongly disfavoured by the atmospheric neutrino data and the results of all SBL neutrino oscillation experiments.

Let us summarize our findings for a 4-neutrino mass hierarchy:

- Region I: Excluded by the unitarity of U.
- Region II: Strongly disfavoured by atmospheric neutrino data.
- Region III: Ruled out by LSND.
- Region IV: Ruled out by solar neutrino data.

It is easy to show that with the arguments presented here all neutrino mass schemes where three masses are clustered and the fourth one is separated by the 'LSND' gap are disfavoured by the present data [21, 22].

4. The favoured non-hierarchial 4-neutrino mass spectra

Now we are left with only two possible neutrino mass spectra in which the four neutrino masses appear in two pairs separated by ~ 1 eV:

$$\text{(A)} \quad \underbrace{\overbrace{m_1 < m_2}^{\text{atm}} \ll \overbrace{m_3 < m_4}^{\text{solar}}}_{\text{LSND}} \quad \text{and} \quad \text{(B)} \quad \underbrace{\overbrace{m_1 < m_2}^{\text{solar}} \ll \overbrace{m_3 < m_4}^{\text{atm}}}_{\text{LSND}}. \tag{31}$$

We have to check that these mass spectra are compatible with the results of all neutrino oscillation experiments.

In schemes A and B the quantities c_α (15) are defined with $r = 2$. Clearly, regions I and III (see figure 2) are ruled out by LSND (see subsection 2.4). Let us first consider scheme A. For the survival probability of solar ν_e's have [34, 20]

$$P^{\odot}_{\nu_e\to\nu_e} = \sum_{i=1,2} |U_{ei}|^4 + (1-c_e)^2 P^{(3;4)}_{\nu_e\to\nu_e} \tag{32}$$

where $P^{(3;4)}_{\nu_e\to\nu_e}$ is the ν_e survival probability involving ν_3, ν_4 only. If $c_e \geq 1 - a_e^0$, it follows from eq. (32) that the survival probability $P^{\odot}_{\nu_e\to\nu_e}$ of solar ν_e's practically does not depend on the neutrino energy and $P^{\odot}_{\nu_e\to\nu_e} \gtrsim 0.5$. This is disfavoured by the solar neutrino data [36]. Consequently, regions II and III are ruled out by the solar neutrino data. This argument does not apply to region IV and one can easily convince oneself that also the atmospheric neutrino anomaly is compatible with this region. Furthermore, looking at eq. (18) we see that this upper bound on $A_{\mu;e}$ is linear in the small quantity a_e^0 in region IV. Since $a_e^0 \gtrsim 5 \times 10^{-3}$ for all values of $\Delta m^2_{\mathrm{SBL}}$, in the case of scheme A the bound (18) is compatible with the result of the LSND experiment. For scheme B the analogous arguments lead to region II. Therefore we come to the conclusion that [21, 22]

$$\begin{aligned} &\text{Scheme A}: \quad c_e \leq a_e^0 \quad \text{and} \quad c_\mu \geq 1 - a_\mu^0, \\ &\text{Scheme B}: \quad c_e \geq 1 - a_e^0 \quad \text{and} \quad c_\mu \leq a_\mu^0. \end{aligned} \tag{33}$$

Schemes A and B have different consequences for the measurement of the neutrino mass through the investigation of the end-point part of the ^{3}H β-spectrum. From eq. (33) it follows that in the case of scheme A the neutrino mass that enters in the usual expression for the β spectrum of ^{3}H decay is approximately equal to the 'LSND mass', i.e., $m_\nu(^3\mathrm{H}) \simeq \mathrm{m}_4$. If scheme B is realized in nature and m_1, m_2 are very small, the mass measured in ^{3}H experiments is at least two orders of magnitude smaller than m_4 [21, 22].

5. Checks of the favoured neutrino schemes in LBL experiments

LBL neutrino oscillation experiments are sensitive to the so-called 'atmospheric δm^2 range' of 10^{-2}–10^{-3} eV^2. For reactor experiments with $p \sim 1$ MeV this amounts to $L \sim 1$ km [37, 38] whereas in accelerator experiments with $p \sim 1$–10 GeV the length of the baseline is of order $L \sim 1000$ km [39–41] (see eq. (6)). Let us consider scheme A for definiteness. Then in vacuum the probabilities of $\nu_\alpha \to \nu_\beta$ transitions in LBL experiments are given by

$$P^{(\mathrm{LBL,A})}_{\nu_\alpha\to\nu_\beta} = \left| U_{\beta 1} U^*_{\alpha 1} + U_{\beta 2} U^*_{\alpha 2} \exp\left(-i \frac{\Delta m^2_{21} L}{2p} \right) \right|^2 + \left| \sum_{k=3,4} U_{\beta k} U^*_{\alpha k} \right|^2. \tag{34}$$

This formula has been obtained from eq. (2) taking into account the fact that in LBL experiments $\Delta m^2_{43} L/2p \ll 1$ and dropping the terms proportional to the cosines of phases much larger than 2π ($\Delta m^2_{kj} L/2p \gg 2\pi$ for $k = 3, 4$ and $j = 1, 2$). Such terms do not contribute to the oscillation probabilities averaged over the neutrino energy spectrum.

To obtain limits on the LBL oscillation probability (34) from the results of the SBL oscillation experiments, we employ the Cauchy–Schwarz inequality on the term with the summation over $k = 1, 2$ and use c_α (15) with $r = 2$ to find the inequalities

$$(1 - c_\alpha)^2 \le P^{(\mathrm{LBL,A})}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \quad \text{and} \quad c_\alpha^2 \le P^{(\mathrm{LBL,B})}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \tag{35}$$

and confining ourselves to scheme A

$$P^{(\mathrm{LBLA})}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \le c_\alpha c_\beta + \frac{1}{4} A_{\alpha;\beta} \quad (\alpha \neq \beta). \tag{36}$$

Both equations also hold for antineutrinos. Considering reactor experiments and taking into account eq. (33) we obtain the bound

$$1 - P^{(\mathrm{LBL})}_{\bar{\nu}_e \to \bar{\nu}_e} \le a_e^0 (2 - a_e^0) \tag{37}$$

which holds for both schemes. Inserting the numerical values of the function a_e^0 (see figure 1) it turns out that the upper bound (37) is below the sensitivity of the CHOOZ experiment in the preferred range (20) of $\Delta m^2_{\mathrm{SBL}}$. For the accelerator experiments matter effects have to be taken into account. We have shown [23] that the matter-corrected version of eq. (36) leads to stringent bounds on $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ and $\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_\tau$ LBL transition probabilities of the order of 10^{-2} to 10^{-1} depending on the value of $\Delta m^2_{\mathrm{SBL}}$ and on the energy of the neutrino beam (for a study of LBL CP violation in schemes A and B see ref. [24]).

6. Conclusions

In this report we have discussed the possible form of the neutrino mass spectrum that can be inferred from the results of all neutrino oscillation experiments, including the solar and atmospheric neutrino experiments. The crucial input are the three indications in favour of neutrino oscillations given by the solar neutrino data, the atmospheric neutrino anomaly and the result of the LSND experiment. These indications, which all pertain to different scales of neutrino mass-squared differences, require that apart from the three well-known neutrino flavours at least one additional sterile neutrino (without couplings to the W and Z bosons) must exist. In our investigation we have assumed that there is one sterile neutrino and that the 4-neutrino mixing matrix (1) is unitary. We have considered all possible schemes with four massive neutrinos which provide three scales of δm^2. We have argued that a neutrino mass hierarchy is not compatible with the above-mentioned indications in favour of neutrino oscillations together with the negative results of all other SBL neutrino oscillation experiments other than LSND. The same holds for all mass spectra with three squares of neutrino masses clustered together, such that the gap between the cluster and the remaining mass-squared determines $\Delta m^2_{\mathrm{SBL}}$ relevant in SBL experiments.

Thus only two possible spectra of neutrino masses, denoted by A and B (see eq. (31)), with two pairs of close masses separated by a mass difference of the order of 1 eV are compatible with the results of all neutrino oscillation experiments. The positive result of the LSND experiment confines the SBL mass-squared difference to the interval $0.27\,\mathrm{eV}^2 \lesssim \Delta m^2_{\mathrm{SBL}} \lesssim 2.2\,\mathrm{eV}^2$ (see figure 3). If, of the two neutrino schemes defined by eqs (31) and (33), scheme A is realized in nature, the neutrino mass that is measured in

^{3}H β-decay experiments coincides with the 'LSND mass'. If the massive neutrinos are Majorana particles, in the case of scheme A, the experiments on the search for $(\beta\beta)_{0\nu}$ decay have good chances to obtain a positive result. Furthermore, schemes A and B have severe consequences for long-baseline neutrino oscillations: the $\overset{(-)}{\nu_e}$ survivial probability is close to one and the $\overset{(-)}{\nu_\mu} \to \overset{(-)}{\nu_e}$ and $\overset{(-)}{\nu_e} \to \overset{(-)}{\nu_\tau}$ transitions are strongly constrained.

Finally, we can ask ourselves what happens if not all experimental input data leading to schemes A and B are confirmed in future experiments. Among the many questions in this context, the two most burning ones concern LSND and the zenith angle variation in the atmospheric neutrino flux. Clearly, if LSND is not confirmed, three neutrinos are sufficient. If one nevertheless requires a 4th neutrino with a mass in the eV range for cosmological reasons then the neutrino spectrum is likely to be hierarchial because region III (see figure 2) cannot be excluded in this case. If, on the other hand, the zenith angle variation in the atmospheric neutrino flux is not confirmed, a 3-neutrino mixing scheme with $\Delta m^2_{\mathrm{SBL}} \equiv \Delta m^2_{\mathrm{atm}} \sim 0.3$ eV^2 and other definite predictions are possible [42]. We have to wait for future experimental results to see if the present interesting and puzzling situation concerning the neutrino mass and mixing pattern persists.

Acknowledgement

The author (WG) would like to thank the organizers of the workshop for their great hospitality and the stimulating and pleasant atmosphere.

References

[1] R N Mohapatra and P B Pal, *Massive Neutrinos in Physics and Astrophysics*, World Scientific Lecture Notes in Physics (World Scientific, Singapore, 1991) vol. 41

[2] B Pontecorvo, *Sov. Phys. JETP* **26**, 984 (1968)
S M Bilenky and B Pontecorvo, *Phys. Rep.* **41**, 225 (1978)
S M Bilenky and S T Petcov, *Rev. Mod. Phys.* **59**, 671 (1987)

[3] C Giunti *et al*, *Phys. Rev.* **D48**, 4310 (1993)
K Kiers, S Nussinov and N Weiss, *Phys. Rev.* **D53**, 537 (1996)
W Grimus and P Stockinger, *Phys. Rev.* **D54**, 3414 (1996)
C Giunti and C W Kim, hep-ph/9711363
H Burkhardt *et al*, hep-ph/9803365
C W Kim and A Pevsner, *Neutrinos in Physics and Astrophysics*, Contemporary Concepts in Physics, (Harwood Academic Press, Chur, Switzerland, 1993) vol. 8

[4] B T Cleveland *et al*, *Nucl. Phys.* **A38**, 47 (1995)

[5] K S Hirata *et al*, *Phys. Rev.* **D44**, 2241 (1991)

[6] GALLEX Coll, W Hampel *et al*, *Phys. Lett.* **B388**, 384 (1996)

[7] SAGE Coll, J N Abdurashitov *et al*, *Phys. Rev. Lett.* **77**, 4708 (1996)

[8] K Inoue, Talk presented at the 5th International Workshop on *Topics in Astroparticle and Underground Physics* (Gran Sasso, Italy, September 1997) (http://www-sk.icrr.u-tokyo.ac.jp/doc/sk/pub/pub_sk.html)
R Svoboda, Talk presented at the Conference on *Solar Neutrinos: News About SNUs*, 2–6 December 1997, Santa Barbara, California (http://www.itp.ucsb.edu/online/snu/)

[9] M Nakahata, Talk presented at the APCTP Workshop: *Pacific Particle Physics Phenomenology*, Seoul, Korea, October 31 – November 2, 1997 (http://www-sk.icrr.u-tokyo.ac.jp/doc/sk/pub/pub_sk.html)

[10] Y Fukuda *et al*, *Phys. Lett.* **B335**, 237 (1994)

[11] R Becker-Szendy *et al*, *Nucl. Phys.* **B38**, 331 (1995)
[12] W W M Allison *et al*, *Phys. Lett.* **B391**, 491 (1997)
[13] Super-Kamiokande Coll., Y Fukuda *et al*, ICRR-Report-411-98-7 (hep-ex/9803006)
[14] C Athanassopoulos *et al*, *Phys. Rev. Lett.* **77**, 3082 (1996)
[15] N Hata and P Langacker, *Phys. Rev.* **D56**, 6107 (1997)
[16] G L Fogli, E Lisi and D Montanino, hep-ph/9709473
[17] S P Mikheyev and A Yu Smirnov, *Yad. Fiz.* **42**, 1441 (1985) [*Sov. J. Nucl. Phys.* **42**, 913 (1985)]; *Il Nuovo Cimento* **C9**, 17 (1986)
L Wolfenstein, *Phys. Rev.* **D17**, 2369 (1978); **20**, 2634 (1979)
[18] M C Gonzalez-Garcia *et al*, hep-ph/9801368
[19] J T Peltoniemi and J W F Valle, *Nucl. Phys.* **B406**, 409 (1993)
D O Caldwell and R N Mohapatra, *Phys. Rev.* **D48**, 3259 (1993)
Z Berezhiani and R N Mohapatra, *Phys. Rev.* **D52**, 6607 (1995)
J R Primack *et al*, *Phys. Rev. Lett.* **74**, 2160 (1995)
E Ma and P Roy, *Phys. Rev.* **D52**, R4780 (1995)
R Foot and R R Volkas, *Phys. Rev.* **D52**, 6595 (1995)
E J Chun *et al*, *Phys. Lett.* **B357**, 608 (1995)
J J Gomez-Cadenas and M C Gonzalez-Garcia, *Z. Phys.* **C71**, 443 (1996)
S Goswami, *Phys. Rev.* **D55**, 2931 (1997)
A Yu Smirnov and M Tanimoto, *Phys. Rev.* **D55**, 1665 (1997)
E Ma, *Mod. Phys. Lett.* **A11**, 1893 (1996)
E J Chun, C W Kim and U W Lee, hep-ph/9802209
[20] S M Bilenky, C Giunti, C W Kim and S T Petcov, *Phys. Rev.* **D54**, 4432 (1996)
[21] S M Bilenky, C Giunti and W Grimus, *Proc. of Neutrino 96*, Helsinki, June 1996, edited by K Enqvist *et al*, (World Scientific, Singapore, 1997) p. 174; *Eur. Phys. J.* **C1**, 247 (1998)
[22] N Okada and O Yasuda, *Int. J. Mod. Phys.* **A12**, 3669 (1997)
[23] S M Bilenky, C Giunti and W Grimus, *Phys. Rev.* **D57**, 1920 (1998)
[24] S M Bilenky, C Giunti and W Grimus, hep-ph/9712537, to be published in *Phys. Rev.*
[25] V Barger, T Weiler and K Whisnant, hep-ph/9712495
S C Gibbons, R N Mohapatra, S Nandi and A Raychaudhuri, hep-ph/9803299
[26] B Achkar *et al*, *Nucl. Phys.* **B434**, 503 (1995)
[27] F Dydak *et al*, *Phys. Lett.* **B134**, 281 (1984)
[28] I E Stockdale *et al*, *Phys. Rev. Lett.* **52**, 1384 (1984)
[29] S M Bilenky, A Bottino, C Giunti and C W Kim, *Phys. Lett.* **B356**, 273 (1995); *Phys. Rev.* **D54**, 1881 (1996)
[30] L A Ahrens *et al*, *Phys. Rev.* **D36**, 702 (1987)
[31] L Borodovsky *et al*, *Phys. Rev. Lett.* **68**, 274 (1992)
[32] J Kleinfeller, *Nucl. Phys.* **B48**, 207 (1996)
[33] A Romosan *et al*, *Phys. Rev. Lett.* **78**, 2912 (1997)
[34] X Shi and D N Schramm, *Phys. Lett.* **B283**, 305 (1992)
[35] E Kearns, Talk presented at the Conference on *Solar Neutrinos: News About SNUs* 2–6 December 1997, Santa Barbara, California (http://www.itp.ucsb.edu/online/snu/)
[36] P I Krastev and S T Petcov, *Phys. Rev.* **D53**, 1665 (1996)
[37] C Y Cardall and G M Fuller, *Phys. Rev.* **D53**, 4421 (1996)
C Y Cardall, G M Fuller and D B Cline, *Phys. Lett.* **B413**, 246 (1997)
E Ma and P Roy, hep-ph/9706309
[38] M Appolonio *et al*, hep-ex/9711002
[39] F Boehm *et al*, *The Palo Verde experiment*, 1996 (http://www.cco.caltech.edu/~songhoon/Palo-Verde/Palo-Verde.html)
[40] Y Suzuki, *Proc. of Neutrino 96*, Helsinki, June 1996, edited by K Enqvist *et al*, p. 237 (World Scientific, Singapore, 1997)
[41] MINOS Coll., NUMI-L-63, February 1995
[42] ICARUS Coll., LNGS-94/99-I, May 1994