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Frictional trajectories near a barrier: A dissipationless Newtonian approach

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Abstract. We address the problem of classical frictional motion under a potential V possessing a barrier, apart from other possible confining and nonstationary terms. It is pointed out that the Green's solution of the exact equation of motion can be reduced (under suitable conditions) either to an improved Rayleigh form or a non-Rayleigh form, the latter being outside the scope of the standard large-friction treatment of the Fokker–Planck equation. The resulting dissipationless dynamics involves an appropriately scaled potential which may have promising applications to quantum stochastic phenomena. Genuine dissipative corrections in regions far away from the barrier can be accounted for by the higher-order terms in our asymptotic expansions.

Keywords. Friction; barrier; Rayleigh; Green's solution; dissipationless.

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1. Introduction

The properties of *single-particle*, nonconservative, frictional motion in classical [1], quantum [2], stochastic [3] and statistical [4] mechanics are very important conceptually and quite interesting application-wise. Damping effects on classical trajectories are theoretically introduced via a linear-velocity term [1a] in an extension of Newton's law or a quadratic dissipation function [1b] in a generalization of Lagrange's equations or an explicitly time-dependent factor [1c] in the Bateman–Caldirola–kanai (BCK) Hamiltonian. Numerous applications of frictional trajectories include the calculation of deterministic collision [1d] of nuclear heavy ions and probabilistic description of phenomena-like Brownian movement [3a] of the harmonic oscillator, signal-to-noise ratio for stochastic resonance [3b] in a double well etc. It may be noted that the problems tackled in refs, [1d] and [3b] involve a potential energy which has a pronounced barrier (see figure 1) apart from other possible nonlinear/nonstationary terms.

In the above context, previous workers have employed two different types of theoretical modelling, viz., *the standard frictional equation* and its *overdamped Rayleigh version.* We review their salient features in § 2 and point out that generally these equations have been solved [1d, 3b] by numerical computation/analog simulation and their associated Lagrangians are inconvenient to use in practice. In § 3 we carefully set up the formal Green's solution of the exact equation of motion and show analytically that the