Pramāṇa – J. Phys., Vol. 35, No. 4, October 1990, pp. 349–354. © Printed in India.

Fluctuating hydrodynamics of a fluid with internal rotation

DEBENDRANATH SAHOO
Materials Science Division, Indira Gandhi Centre for Atomic Research, Kalpakkam 603 102, India

MS received 18 May 1990

Abstract. The fluctuating hydrodynamics theory of a fluid possessing internal rotation is set up following the Landau-Lifshitz approach.

Keywords. Fluctuating hydrodynamics; Landau-Lifshitz theory; internal rotation.

PACS Nos 47·90; 05·40

1. Introduction

Landau and Lifshitz (1957) were the first to develop the theory of fluctuating hydrodynamics of a simple liquid (see also: Lifshitz and Pitaevskii 1980a; together with Landau and Lifshitz (1957), jointly referred to hereafter as I). In this classic work they added random 'outside' stress to the stress tensor of the fluid and random 'outside' heat flux to the heat flux vector of the fluid. They wrote down the linearised equations of flutuating hydrodynamics. Treating the fluctuations as Gaussian Markov processes, they then derived the fluctuation-dissipation relations of the random forces in the fluid. The formalism of I has been widely used in the literature for dealing with practical problems such as scattering of light from the fluid. Extension of the formalism to nonlinear hydrodynamics and also to nonequilibrium situations have received much attention (Fox 1978).

In the case of a fluid possessing internal rotation in the hydrodynamic sense, the Landau-Lifshitz formalism has not yet been extended. We have in mind fluids whose molecules may possess structure and thus are capable of rotation as well as translation. The real fluid may consist of two (or more) species of molecules, some possessing rotation and some not being able to rotate. In this case too, the fluid 'particle' in the context of hydrodynamics would possess some 'average' rotation. The deterministic hydrodynamic equations of such fluids have been obtained by Shliomis (1976) (to be referred to hereafter as II) in a penetrating analysis of the problem. It is the purpose of this paper to add fluctuating forces to the hydrodynamic eqution set up in II and to derive the fluctuation-dissipation relations for these random forces.

In this paper, we shall first recapitulate the essential equations of II, following the notation of II. Next we shall set up the formalism of fluctuating hydrodynamics following the procedure of I.