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Mach's principle and the notion of time

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Abstract. The role of time coordinate in the realization of Mach's principle is highlighted. It is shown that Mach's principle is linked to the definition of a 'particle'. These results suggest a deep connection between quantum gravity and Mach's principle.

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In classical Newtonian mechanics, the "frame of fixed stars" (S) is of crucial importantce. This is the frame of reference in which distant matter in the universe (stars, galaxies, or some such predecided set of objects; here they will be referred to as "stars") is at rest. Newton's laws are usually stated in such a frame. A particle shielded from all external influences will follow an unaccelerated trajectory in this frame. This result can be stated as follows: Let $\boldsymbol{x}_1(t)$, $\boldsymbol{x}_2(t) \cdots \boldsymbol{x}_N(t)$ be the position vectors of N stars and let $\boldsymbol{x}(t)$ be the position of a test particle shielded from external influences. In a reference frame in which $\boldsymbol{x}_i(t) = \boldsymbol{x}_i(0)$ ['distant stars are fixed'], $\boldsymbol{x}(t)$ satisfies the equation

$$\frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}t^2} = 0. \tag{1}$$

Thus, by connecting up the local behaviour of test particles with the state of motion of distant matter, we have brought in (a particular version of) Mach's principle (Mach 1912). In contrast, consider another frame S' in which the distant stars are *not* at rest but moves according to the law

$$\boldsymbol{x}_i'(t) = \frac{1}{2}\boldsymbol{g}t^2. \tag{2}$$

In this frame—in which distant stars are not fixed—we cannot use (1). However, it is easy to make a coordinate transformation which will bring these stars to rest; we can, then, use (1) in such a frame. Transforming back we can find the equation of motion for the free test particle in our original frame S'. By this procedure we will find that $\boldsymbol{x}'$ satisfies the equation:

$$\frac{\mathrm{d}^2\boldsymbol{x}'}{\mathrm{d}t^2} = -\boldsymbol{g}. \tag{3}$$