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Model equations from a chaotic time series

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Abstract. We present a method for obtaining a set of dynamical equations for a system that exhibits a chaotic time series. The time series data is first embedded in an appropriate phase space by using the improved time delay technique of Broomhead and King (1986). Next, assuming that the flow in this space is governed by a set of coupled first order nonlinear ordinary differential equations, a least squares fitting method is employed to derive values for the various unknown coefficients. The ability of the resulting model equations to reproduce global properties like the geometry of the attractor and Lyapunov exponents is demonstrated by treating the numerical solution of a single variable of the Lorenz and Rossler systems in the chaotic regime as the test time series. The equations are found to provide good short term prediction (a few cycle times) but display large errors over large prediction times. The source of this shortcoming and some possible improvements are discussed.

Keywords. Time series; chaos; model equations; strange attractor.

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1. Introduction

In recent years, it has become clear that many chaotic time series observed experimentally in nature, owe their stochasticity to the intrinsic nonlinear dynamics of a system evolving on a low dimensional strange attractor in phase space. Methods of phase space reconstruction (Broomhead and King 1986; Packard *et al* 1980; Takens 1981) allow one to study the geometrical structure of the strange attractor and to determine such important global parameters as the fractal dimension, Lyapunov exponents etc. For many such systems it becomes meaningful to write down model nonlinear equations which may reproduce the observed chaotic behaviour. There is a great deal of interest in this direction in recent years (Farmer and Sidorowich 1987; Casdagli 1989; Crutchfield and McNamara 1987; Abarbanel *et al* 1989). Obtaining model equations (preferably a set of simple differential equations) can serve as an important first step in gaining a better understanding of the underlying physical processes responsible for the observed chaos. They can also form good mathematical simulators and be usefully employed for prediction and control of the chaotic system.

In this paper, we present a method for obtaining a set of differential equations from an experimentally measured chaotic time series of a single physical variable. These equations are found to reproduce reasonably well the geometrical and dynamical features of the attractor reconstructed from the measured time series. The theorems due to Takens (1981) and Whitney (1936) form the theoretical basis for the phase space reconstruction and the determination of embedding dimension respectively.