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Some aspects of conformal field theories on the plane and higher genus Riemann surfaces

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Abstract. We review some aspects of conformal field theories on the plane as well as on higher genus Riemann surfaces.

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1. Introduction

Conformally invariant two dimensional field theories (CFT) have become the subject of intense investigation in recent years. In this article I shall try to give a general introduction to conformal field theory with special emphasis on a particular class of conformal field theories, known as rational conformal field theories (RCFT). I shall begin by discussing the reasons for the recent upsurge of interest in these theories, and then discuss the various properties of these theories in some detail.

One of the two main applications of two dimensional conformal field theories is that they describe the critical behavior of many known two dimensional statistical mechanical models. In order to understand this connection we must first understand the meaning of conformal invariance. In any dimension, conformal invariance refers to a group of coordinate transformations which leave the angle between any two intersecting lines fixed. Obviously the Poincare group, consisting of translations and rotations have this property, and hence they form part of the conformal group. Another transformation which has this property is the scale transformation. In general the conformal group has other elements also which we shall discuss later, but for the purpose of understanding the connection to the critical behavior of statistical models, the above properties are enough.

In order to understand the behavior of a statistical model at the critical point, let us consider a simple system, the Ising model, which consists of a lattice with classical spin placed at each site, which can point in either the up or the down direction. At very high temperature, the system is completely disordered in the absence of a magnetic field. This may be expressed by saying that the conditional probability that the spin at site i is up given that the spin at the site j is up reduces to the unconditional probability (1/2) as the distance between the sites increases. In fact this approach takes place in an exponential manner, if the distance between the sites is labelled by l, then the above conditional probability is of the form $1/2 + \mathcal{O}\exp(-l/\xi)$, where ξ