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Use of Padé approximants in the evaluation of α and δ for one-dimensional maps

G AMBIKA and K M VALSAMMA

Department of Physics, Cochin University of Science and Technology, Cochin 682 022, India

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Abstract. Recently an analytic algorithm for evaluating the Feigenbaum indices of one-dimensional maps was developed using a perturbative expansion. We find that the use of Padé approximants in the resulting asymptotic series, significantly improves the technique.

Keywords. Feigenbaum scenario; universal constants; asymptotic series; Padé approximants.

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1. Introduction

It is well known that most nonlinear dissipative dynamical systems follow the period doubling route to irregular or chaotic behaviour. Such systems can be modelled by one-dimensional maps of the form,

$$x_{n+1} = f_a(x_n) = 1 - a|x_n|^z \tag{1}$$

in the interval $(-1, 1)$. Their transition to chaos is characterized by two universal indices α and δ (Feigenbaum 1980); The scale factor α is defined as

$$\alpha = \lim_{n\to\infty} \frac{\Delta x_n}{\Delta x_{n+1}}, \tag{2}$$

where Δx_n is half the separation between the fixed points in the 2^n-cycle and the rate of accumulation of bifurcations

$$\delta = \lim_{n\to\infty} \frac{a_{n+1} - a_n}{a_{n+2} - a_{n+1}}. \tag{3}$$

At $a = a_\alpha$, $f^{2^n}(0)$ approaches zero geometrically as $1/\alpha^n$ where f^{2^n} represents 2^n iterations of f. It has been established by Feigenbaum that f^{2^n} in the neighbourhood of $x = 0$ which is the extremum of the map, asymptotically approaches a universal function $g(x)$ (Feigenbaum 1983). The function $g(x)$ is the fixed point function of the doubling and rescaling operator defined by the relation (Feigenbaum 1978),

$$g(x) = -\alpha g(g(x|\alpha)). \tag{4}$$