

The trousers problem revisited

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MS received 14 December 1987

Abstract. Anderson and DeWitt considered the quantization of a massless scalar field in a spacetime whose spacelike hypersurfaces change topology and concluded that the topology change gives rise to infinite particle and energy production. We show here that their calculations are insufficient and that their propagation rule is unphysical. However, our results using a more general propagation rule support their conclusion.

Keywords. Trousers topology; propagation rules; massless scalar field; energy density.

PACS Nos 11·10; 03·70

1. Introduction

The trousers topology was introduced by Anderson and DeWitt (1986; see also DeWitt 1985) to model the effects of topology change in quantum gravity. Instead of the gravitational field, they consider a massless scalar field on a background spacetime whose spatial cross-sections change topology. By comparing the "natural" Fock vacua before and after the topology change, they conclude that infinite particle and energy production occurs, and that the topology change therefore does not take place.

We re-examine their procedure and show that it is incomplete. Fundamental to the solution of the problem is a choice of propagation rule as well as of "in" and "out" mode functions. We show that their propagation rule is unphysical and furthermore that their choice of mode functions is incomplete.

More specifically, we introduce as a physical constraint on the Bogolubov transformations between in and out modes that they be time-independent and show that this determines the propagation rule completely up to a single free parameter. The shadow rule of Anderson and DeWitt (1986) does *not* satisfy this constraint. Anderson and DeWitt only give a general argument for the presence of an energy density proportional to the square of a delta function; we calculate the energy density explicitly and attempt (unsuccessfully) to use the free parameter to set the coefficient of this term to zero.

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