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A general law for quantum mechanical joint probabilities: generalisation of the Wigner formula and the collapse postulate for successive measurements of discrete as well as continuous observables

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Abstract. The fundamental prescriptions of quantum theory have so far remained incomplete in that there is no satisfactory prescription for the joint probabilities of successive observations of arbitrary sequence of observables. The joint probability formula derived by Wigner is based on the collapse postulate due to Von Neuman and Lüders and is applicable only to observables with purely discrete spectra. Earlier attempts to generalize the collapse postulate to observables with continuous spectra have been unsatisfactory as they lead to only finitely additive (and not σ-additive) joint probabilities in general. In this paper a suitable generalisation of the Wigner joint probability formula is proposed, which is completely satisfactory in the sense that it leads to σ-additive joint probabilities for successive observations of arbitrary sequence of observables, consistent with all the other basic prescriptions of quantum theory. This general law for quantum mechanical joint probabilities is arrived at by a reformulation of earlier results on expectation values in successive measurements. The generalized Wigner joint probability formula is also shown to be a consequence of a general collapse postulate, which allows for changes in state due to measurement from normal states to non-normal states also. As an illustration of our results, the probability distribution of the outcomes of a momentum measurement which immediately succeeds a position measurement is computed, and this seems to shed an entirely new light on the uncertainty principle.

Keywords. Successive measurements; joint probabilities; σ-additivity; collapse postulate; observables with continuous spectrum.

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1. Introduction

One of the outstanding problems of quantum theory, which has eluded a satisfactory solution so far, has been that of extending the conventional description of successive observations which is available only for observables with a purely discrete spectrum to the general case when an arbitrary sequence of observables (with continuous as well as discrete spectra) are measured. This has resulted in a profound 'incompleteness' of the theory at the level of its fundamental prescriptions, so that we have no way of discussing the statistics of successive observations of even the basic observables such as position, momentum, etc., which have a continuous spectrum. In this paper we show that the conventional prescription for the joint probabilities of successive observations, (the Wigner joint probability formula (Wigner 1963) for observables with a purely discrete spectrum) can be extended to a general law for quantum mechanical joint probabilities