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Effect of phase fluctuation on a system of rotating superfluid

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Abstract. It has been shown that when the root-mean-square of the gradient of phase fluctuation exceeds the inverse of the coherence length a system of superfluid rotating with angular velocity exceeding the critical angular velocity has an instability.

Keywords. Phase fluctuations; coherent states; stability.

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It was Anderson (1966) who first pointed out the importance of phase in superfluid ^{4}He. He introduced phase as an operator conjugate to the number operator. It was however pointed out by Carruthers and Nioto (1968) that a phase operator conjugate to the number operator does not exist. We take a different approach in introducing the phase. In earlier publications (see Biswas and Rama Rao 1971) we have illustrated our approach of introducing phase in a consistent manner. As stated earlier we start with the boson annihilation operator $\psi(r)$. Let us introduce the coherent states $|\psi\rangle$ from the definition

$$\psi(r)|\alpha\rangle = \alpha(r)|\alpha(r)\rangle. \tag{1}$$

$\alpha(r,t)$ is thus the eigenvalue of the annihilation operator, we write

$$\alpha = \{[J(r,t)]/h\}^{\frac{1}{2}} \exp[i\phi(r,t)]. \tag{2}$$

Let

$$H = -\int \frac{\hbar^2}{2m}\psi^\dagger \nabla \psi^2 \mathbf{d}r + \int V(|r-r'|)\psi^\dagger(r)\psi(r)\psi^\dagger(r')\psi(r')\,\mathrm{d}r\,\mathrm{d}r' \tag{3}$$

be the Hamiltonian. We have shown earlier (Biswas and Rama Rao 1975) that $\phi(r)$ and $J(r)$ satisfy conjugate relations

$$\langle\phi(r)\rangle = \langle \delta H/\delta J(r)\rangle, \tag{4}$$

$$\langle J(r)\rangle = -\langle \delta H/\delta\phi(r)\rangle. \tag{5}$$

The angular brackets in (4) and (5) denote averages with respect to the distribution function $\rho(\alpha^*,\alpha) = \langle\alpha|\rho|\alpha\rangle = \rho(J,\phi,t)$. Equation (4) gives

$$\frac{\partial\langle\phi\rangle}{\delta t} + \frac{\hbar}{2m}\langle(\nabla\phi)^2\rangle = \frac{\hbar}{2m}\left\langle\frac{\nabla^2\sqrt{J}}{\sqrt{J}}\right\rangle - \frac{1}{\hbar}\int V(|r-r'|)\langle J(r')\rangle \mathrm{d}r \tag{6}$$