

## Kramers-Kronig type of dispersion relation in nonlinear optics

P SEN and P K SEN

Department of Physics, Bhopal University, Bhopal 462 026, India

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**Abstract.** The applicability of Kramers-Kronig (K-K) relation under nearly sharp resonant transition regime in narrow-gap semiconductors has been established and consequently, a generalized dispersion relation for nonlinear optical susceptibility of a dielectric is derived. This relation can be employed in the study of nonlinear optical processes in solids as well as in plasmas over a wide frequency spectrum.

**Keywords.** Kramers-Kronig dispersion relation; nonlinear optical susceptibility; nonlinear refraction; nonlinear absorption.

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### 1. Introduction

Kramers-Kronig (K-K) dispersion relations which connect the dispersive and absorptive aspects of matter-radiation interaction processes are extremely useful in many areas of physics. In particular, the utility of K-K relations in the determination of different optical constants by reflectivity data as well as in the derivation of sum rules is well known and needs very few physically well-founded assumptions (Landau and Lifshitz 1959; Wooten 1972).

The extension of K-K relations to the nonlinear regime is regarded as a difficult and subtle problem because optical nonlinearity of a medium has been the origin of a variety of phenomena in contrast to the simple absorptive and dispersive aspects of the linear material response of the medium. Nonlinear refraction and absorption are two of the vital properties of the medium which play critical roles in the observation of optical bistability, optical phase conjugation via degenerate four-wave mixing and pulse shaping, etc. These phenomena have large technological potential in integrated optics. The investigations of such nonlinear optical effects could be much simplified if an analytical tool similar to the K-K relations is available. Unfortunately, no such relations seems to have yet been established.

The present literature reveals that there is a controversy about the applicability of K-K relations in the nonlinear regime. Flytzanis (1975) commented that these relations are applicable even in the nonlinear optical effects but are not so useful as in linear optics. On the other hand, according to Yariv (1975), these relations are invalid because of the presence of poles in both the upper and lower halves of complex  $\omega$ -plane and violate the causality principle in the nonlinear regime. K-K relations were used by Miller *et al* (1981) probably for the first time in nonlinear optics under the approximation that the quantum mechanical phases of the carriers are randomized by thermalizing collisions. They developed a semi-empirical theory that fits well with