

Density of nucleons in heavy nuclei

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Abstract. A shell model description of heavy nuclei is used to show that the density of nucleons in heavy nuclei is of the form $\rho(r) = K(a^2 - r^2)^{3/2}$, K, a being constants. Two broad features of this distribution are mentioned.

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One of the earliest developments in the area of nuclear physics was to study the distribution of nucleons in a nucleus. The model which was put forward in the beginning was that of uniform distribution of nucleons inside a sphere of radius R . It was later established by electron scattering (Hofstadter 1956; Brown *et al* 1979) that the density distribution does not have a sharp cut-off but has a tail. With the development of nuclear shell model it was possible to calculate the form of the shell model density $\rho(r)$ for a number of light nuclei. As the number of nucleons starts increasing one has to carry out summations over a large number of shells. The purpose of the present work is to derive an approximate analytic form of $\rho(r)$ when the number of nucleons becomes large.

To keep the formulation simple we shall treat neutrons and protons alike and consider single nucleon orbits in a harmonic oscillator potential without spin-orbit term. The density of single nucleons $\rho(\vec{r})$ can be written as

$$\rho(\vec{r}) = \frac{4}{A} \sum_i |\phi_i(\vec{r})|^2, \quad (1)$$

here A is the total number of nucleons and ϕ_i are the single nucleon wave functions. The factor 4 arises from spin and isospin. Also $\rho(\vec{r})$ is normalized so that $\int \rho(\vec{r}) d\vec{r} = 1$ and the sum goes over the occupied orbits.

In a three-dimensional isotropic harmonic oscillator (Morse and Feshbach 1953) the radial wave functions $R_{nl}(r)$, $n = 1, 2, \dots$; $l = 0, 1, 2, \dots, n-1$ are given by

$$R_{nl} = N_{nl} \exp(-\alpha r^2) r^l M(-(n-1), l + \frac{3}{2}, 2\alpha r^2), \quad (2)$$

where N_{nl} is the normalization constant, α the harmonic oscillator parameter and $M(a, b, x)$ is the confluent hypergeometric function (Abramowitz and Stegun 1965). The wave functions ϕ_i are products of R_{nl} and spherical harmonics. It turns out that