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Equivalence of stochastic quantization to field theories from supersymmetry

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Abstract. Using the Ward-Takahashi identities from the hidden supersymmetry in Langevin equation we present a very simple proof of the equivalence of stochastic quantization to field theories.

Keywords. Stochastic quantization; supersymmetry; Ward-Takahashi identities.

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The stochastic quantization of Parisi and Wu (1981) scheme has been a subject of intensive study over the past few years. Several authors have proved the equivalence of this method to the conventional quantization method (Cardy 1983; Nakazato *et al* 1983; Grimus and Huffel 1983; Gozzi 1984; Krischner 1984; Gangopadhyay *et al* 1986). It is now well known that there is a superspace of formulation of the Langevin equation which brings out the hidden supersymmetry (SUSY) associated with the Langevin equation (Chaturvedi *et al* 1984a; Gozzi 1983; Egorian and Kalitzin 1983; Feigelman and Tsevlik 1982). This SUSY implies certain Ward-Takahashi identities that were derived by the authors and were used to give a very simple proof of the fluctuation dissipation theorem (Chaturvedi *et al* 1984b). In this paper we shall use the SUSY identities alone to give a very simple and direct proof of the equivalence.

Consider the Langevin equation

$$\frac{\partial\varphi(x,t)}{\partial t}=-\frac{\delta S}{\delta\varphi(x,t)}+\eta(x,t) \tag{1}$$

where $\varphi(x, t)$ is a scalar field and $\eta(x, t)$ a gaussian white noise source. The generating functional for the Green functions $Z(j)$, has been shown to be given by (Chaturvedi *et al* 1984a; Gozzi 1983; Egorian and Kalitzin 1983; Feigelman and Tsevlik 1982).

$$Z(j)=\int\mathscr{D}\varphi\mathscr{D}\pi\mathscr{D}\psi\mathscr{D}\bar{\psi}\exp\left[\int\mathrm{d}x\,\mathrm{d}t\,\mathrm{d}\alpha\,\mathrm{d}\bar{\alpha}\{-\mathscr{L}_{ss}+\mathscr{J}\Phi\}\right]. \tag{2}$$

Here, α, $\bar{\alpha}$ are anticommuting Grassman variables and

$$\mathscr{L}_{ss}=\frac{1}{2}\Phi\frac{\partial}{\partial t}\Phi-\Phi\frac{\partial^2}{\partial\bar{\alpha}\partial\alpha}\Phi-\Phi\alpha\frac{\partial^2}{\partial\alpha\partial t}\Phi+\mathscr{L}(\Phi), \tag{3}$$