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Layered structure of superfluid ^{4}He at supercritical motion

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Abstract. Landau's criterion plays an important role in the theory of superfluidity. According to this criterion, superfluid motion is possible if $\tilde{\varepsilon}(\mathbf{p}) \equiv \varepsilon(p) + \mathbf{pV} > 0$ along the curve of the spectrum $\varepsilon(p)$ of excitations. For ^{4}He it means that $v < v_c$, $v_c \approx 60$ m/sec. v_s is equal to the tangent of the slope to the roton part of the spectrum. The question of what happens to the liquid when this velocity is exceeded, as far as we know, remains unclear. We shall show that for small excesses above v_c a one-dimensional periodic structure appears in the helium. A wave vector of this structure oriented opposite to the flow and equal to $\rho_c/\hbar$ where ρ_c is the momentum at the tangent point. The quantity $\tilde{\varepsilon}(\mathbf{p})$ is the energy of excitation in the liquid moving with velocity **v**. Inequality of Landau ensures that $\tilde{\varepsilon}$ is positive. If $\tilde{\varepsilon}$ becomes negative, then the boson distribution functio: $n(\tilde{\varepsilon})$ becomes negative, indicating the impossibility of thermodynamic equilibrium of the ideal gas of rotons; therefore the interaction between them must be taken into account. The final form of the energy operator is

$$\hat{H} = \int \{\hat{\psi} + \tilde{\varepsilon}(\mathbf{p})\hat{\psi} + \frac{g}{2}\hat{\psi} + \hat{\psi} + \hat{\psi}\hat{\psi}\} \mathrm{d}^3x, \quad g \sim 2 \cdot 10^{-38} \text{ erg.cm.}$$

Then we can seek the roton ψ-operator in the form $\psi = \eta \exp(i\mathbf{p}_c\mathbf{r}/\hbar)$, determining η from the condition that the energy is minimized. The result is $(\eta)^2 = (v - v_c)\rho_c/g$, for $v > v_c$. The plane wave ψ corresponds to a uniform distribution of rotons. It leads, however, to a spatial modulation of the density of the helium, since the density operator $\hat{n}$ contains a term which is linear in the operator ψ: $\hat{n} = n_0 + (n_0)^{1/2} A/\hat{\psi} \to \hat{\psi}^+)$, where $|A|^2 \sim \rho_c^2/2m\varepsilon(\rho_c)$. Finally we find that the density of helium is modulated according to the law

$$\frac{n - n_0}{n_0} = \left[\frac{|A|^2(v - v_c)\rho_c}{n_0 g}\right]^{1/2} \sin \rho_c x \approx 2,6 \left[\frac{v - v_c}{v_c}\right]^{1/2} \sin \rho_c x.$$

This phenomenon can be observed, in principle, in the experiments on scattering of x-rays in moving helium.

Keywords. Superfluid ^{4}He; supercritical motion; layered structure.

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