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On the short-time behaviour of quantum unstable systems

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Abstract. We consider the survival amplitude $A(t)$ for a normalized decaying state whose energy spectral density vanishes asymptotically as an inverse power. By using simple calculus a Taylor expansion of $A(t)$ is derived around $t = 0$, the form of the remainder term identified, and a physical significance given to the other coefficients. It is shown that the Taylor remainder may contain logarithms of t besides powers.

Keywords. Unstable particles; survival amplitude; Taylor expansion.

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1. Introduction

The problem of describing the time evolution and decay of quasi-stationary systems has drawn the attention of physicists ever since the advent of quantum mechanics. The enormous amount of work done on this delicate subject has been reviewed by Goldberger and Watson (1964), Newton (1966), Fonda *et al* (1978), Parravicini *et al* (1980), Exner (1985) and in a large number of research papers (Khalfin 1958; Fleming 1973; Degasperis *et al* 1974; Chiu *et al* 1977; Sudarshan *et al* 1978; Haake 1978; Peres 1980, 1984; Affleck 1981; Horwitz and Katznelson 1983; Menon and Lagu 1983; Grotz and Klapdor 1984; Alvarez-Estrada *et al* 1985). In the case of a *closed* system (i.e., one which develops in isolation) it has been demonstrated that the survival probablility ($\hbar = 1$ units)

$$P(t) = |A(t)|^2, \tag{1}$$

must deviate from the classical radioactive law at times much smaller (Fleming 1973; Chiu *et al* 1977; Sudarshan *et al* 1978; Peres 1980) or much bigger (Khalfin 1958; Fonda *et al* 1978) than the lifetime $\tau = 1/\Gamma$ where $A(t)$ is the amplitude of finding the system to be undecayed at the instant t. If the system is *open* (i.e., if the decaying particle interacts with the surroundings) the exponential law gets effectively restored with a changed lifetime τ', say (Degasperis *et al* 1974; Fonda *et al* 1978). Recently there has been a spurt of activity in examining the implications of these theories for resonances (Menon and Lagu 1983), conceptual as well as practical aspects of processes such as proton decay (Peres 1984; Horwitz and Katznelson 1983), double β decay (Grotz and Klapdor 1984), $n\bar{n}$ oscillations in nuclei (Alvarez-Estrada *et al* 1985) and quantum-statistical fluctuations (Haake 1978; Affleck 1981).

Several pioneering workers such as Fleming (1973), Chiu *et al* (1977) and Peres (1980) have particularly emphasized the need to study the form of the survival amplitude $A(t)$