Pramāṇa - J. Phys., Vol. 28, No. 1, January 1987, pp. L95–L96. © Printed in India.

Some remarks on the finiteness of soliton mass

R K KAUL and R RAJARAMAN

Centre for Theoretical Studies, Indian Institute of Science, Bangalore 560012, India

MS received 29 September 1986

Abstract. Some subtleties regarding regularizations in computing the soliton energy of degenerate systems are discussed.

Keywords. Solitons; 2-D scalar field theories; regularizations

PACS Nos 11·15; 11·30

Some time back we had demonstrated for a class of scalar field theories with classically degenerate vacua that the energy of the quantum soliton is infinite as volume tends to infinity even though its classical energy is finite (Kaul and Rajaraman 1985). This demonstration employed a specific regularization. However, Kumar and Parida (KP) have made the interesting point that there exists a regularization for which the soliton energy is finite (Kumar and Parida 1986). This happens because their specific regularization leaves the vacua degenerate even at the quantum level.

We agree with the calculations of KP and their point is well taken. Strictly speaking within the confines of their model their regularization is as permissible as any other. However, in the larger context, the specific regularization used by them violates a guiding principle in field theory. This is the principle of naturalness. In the Kumar-Parida regularization, the parameter m^2/μ^2 has to be finely tuned and that too afresh at each order in perturbation theory. This is undesirable. For example, the well-known gauge hierarchy problem in grand unified theories arises for the same reason and fine tuning there is not considered desirable.

In fact the fine tuning required by Kumar and Parida is infinitely more stringent than that which arises in the gauge hierarchy problem in grand unified theories. For instance in their one-loop result (equation (16)), the quantity $\ln(\mu^2/m^2) - \frac{4}{3}\ln 4 + 1$ must be tuned to be exactly zero as distinct from that in the case of the gauge hierarchy problem, where the corresponding parameter has to be only small. By contrast the regularization we employed was generic. Our result that the soliton energy diverges as $L \to \infty$ is valid for all regularizations except for the special infinitely fine-tuned one that Kumar and Parida have used.

Furthermore, if one were to apply such models to quasi one-dimensional condensed matter systems, which provide a definite characteristic cut-off Λ, the corresponding parameter m^2/Λ^2 is fixed and not free to be tuned. Therefore, a result from continuum theory will hold only if it is obtained using a generic regularization.