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Possible realisation and generalisation of two specific 2×2 forms

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Abstract. For each of a couple of two-dimensional forms for quark mass matrix, it is discussed how that form may be realised in a certain gauge scheme (one of them in the standard model and the other in a scheme based on simple rank two times U(1)) by imposing suitable discrete symmetries and how under a certain small angle approximation that form may be regarded as the simplest member of a family of higher dimensionality forms.

Keywords. quark mass matrix; biunitary diagonalisation; discrete symmetries; higher dimensionality forms.

1. Introduction

In the following, some discrete symmetries are imposed in elementary gauge schemes based on SU(2) (simple rank two) times U(1) as gauge group having four quarks (four ordinary plus four superheavy quarks, their intermixing being forbidden). This allows realisation of two specific forms for quark mass matrix, one in the former case and the other in the latter. The ucds mass matrix being a direct sum of two 2×2 matrices is therefore collectively referred to as a 2×2 form in the following. The 2×2 form realised in the former (latter) case relates Cabibbo angle θ_c to the quark mass ratio m_d/m_s as

$$\theta_c \sim \frac{m_d}{m_s}\left(\theta_c^2 \sim \frac{m_d}{m_s}\right).$$

These forms thus endowed with some physical significance are also observed to have the following property of some mathematical interest, independently of any further gauge model considerations. In analogy with a 2×2 form for mixing between two negatively (and also two positively) charged fermions, one may also envision an $n\times n$ form for mixing between n negatively (as well as n positively) charged fermions. With a specific $n\times n$ form it is found that for arbitrary n under a suitable approximation for $2n$ fermion masses $m_1^{\pm}, m_2^{\pm}, \ldots, m_n^{\pm}$ (which are diagonal entries resulting on biunitary diagonalisation of the $n\times n$ form) there are to leading order of approximation n-1 Cabibbo-like angles related to fermion mass ratios as,

$$\theta_1 \sim \frac{m_1^-}{m_2^-}\,; \qquad \theta_i \sim \frac{m_{i+1}^-}{m_i^-},$$

$$[\theta_1 \sim (m_1^-/m_2^-)^{1/2} - (m_1^+/m_2^+)^{1/2}\,; \qquad \theta_i \sim (m_i^-/m_{i+1}^-)^{1/2} - (m_i^+/m_{i+1}^+)^{1/2}],$$