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SU(3) representation for the polarisation of light

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Abstract. A new mathematical representation for discussing the state of polarisation of an arbitrary beam of partially polarised light is described which makes use of the generators of the group SU(3). This representation is sufficiently general to describe not only physical photons which are transverse but also virtual photons. The correspondence between our representation and the conventional Stokes parameter representation is established and this leads to an equivalent geometrical description of partially polarised light in terms of diametrically opposite points on a Poincarè sphere with radius equal to the degree of polarisation. The connection with the spherical tensor representation is also discussed and this leads to a simple geometrical interpretation of the bounds on the parameters characterizing an arbitrary beam of partially polarised light.

Keywords. Photons; polarisation; density matrix, Stokes parameters; SU(3) representation; bounds.

1. Introduction

New mathematical representations for the state of polarisation of light or photons are of considerable interest in several areas of physics like crystal optics, nuclear theory or elementary particles. The well-known review article by Ramachandran and Ramaseshan (1961) discusses exhaustively several methods, starting with the Poincarè sphere and its connection with the Stokes parameters. The review articles by Fano (1957) and McMaster (1961) based on quantum mechanical ideas show how polarisation of light can be represented using the concept of the density matrix. Although the spin of the photon is one, it is found sufficient here, to use 2×2 matrices in view of the fact that light is a transverse wave and consequently the longitudinal state of polarisation is physically absent. However, in dealing with interactions between charged particles, it is well-known from quantum electrodynamics (Feynman 1962) that longitudinal state is also involved along with the two transverse states for the photons; in fact, the well-known Coulomb law between two charged particles is the result of an exchange of a ‘longitudinal photon’. Therefore, in several physical problems as in the case of electroproduction of pions (Dombey 1971), for example, it is advantageous to use 3×3 density matrices to represent the state of photon polarisation. A representation using the Kemmar algebra has been proposed by Roman (1959a, b) to describe the (3×3) density matrix for a stationary quasi-monochromatic field which is not a plane wave.

The distinct advantage which the density matrix formalism shares with the description in terms of Stokes parameters is that it can describe partially polarised as