

Embedding of non-simple Lie groups, coupling constant relations and non-uniqueness of models of unification

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Abstract. A general derivation of the coupling constant relations which result on embedding a non-simple group like $SU_L(2) \otimes U(1)$ in a larger simple group (or graded Lie group) is given. It is shown that such relations depend only on the requirement (i) that the multiplet of vector fields form an irreducible representation of the unifying algebra and (ii) the transformation properties of the fermions under $SU_L(2)$. This point is illustrated in two ways, one by constructing two different unification groups containing the same fermions and therefore have same Weinberg angle; the other by putting different $SU_L(2)$ structures on the same fermions and consequently have different Weinberg angles. In particular the value $\sin^2\theta=3/8$ is characteristic of the sequential doublet models or models which invoke a large number of additional leptons like E_6 , while addition of extra charged fermion singlets can reduce the value of $\sin^2\theta$ to $1/4$. We point out that at the present time the models of grand unification are far from unique.

Keywords. Coupling constant relations; embedding in higher symmetries; Weinberg-Salam model; grand unified theories; Shmushkevich relations; Weinberg angle; neutral currents; Schur's lemma; graded Lie groups.

1. Introduction

One of the deficiencies of the standard gauge-model for electroweak interactions (Weinberg 1967; Salam 1968) is that the minimal theory based on the gauge group $SU_L(2) \otimes U(1)$, (Glashow 1961) involves two coupling constants g and g' whose relative magnitude is parametrised by the Weinberg angle

$$\tan \theta_w = g/g' \quad (1)$$

remains an arbitrary parameter. Current experiments (Dydak 1979) give the value

$$\sin^2 \theta_w = 0.23 \pm 0.02 \quad (2)$$

The natural way to remove this arbitrariness is to embed this group in a higher symmetry group (i.e. Lie group or a graded Lie group cf. below) which involves only one coupling constant so that the couplings g and g' corresponding to the $SU_L(2) \otimes U(1)$ subgroup of this higher symmetry, become algebraically related.

Theoretical attempts at this embedding have in the past involved the hadron and lepton sectors separately, while more recently, the so-called grand unified theories (Pati and Salam 1973; Georgi and Glashow 1974) have received increasing attention.