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Use of harmonic oscillator potential in the analysis of muonic transition energies

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Abstract. Energy levels: $1s_{1/2}$, $2p_{1/2}$, $2p_{3/2}$, $3d_{3/2}$, and $3d_{5/2}$ of the muon in the spherical nuclei: ^{120}Sn, ^{197}Au and ^{208}Pb have been calculated under the assumption of harmonic oscillator potential. The levels are corrected for vacuum polarisation. The agreement with experimental values is better than 0·5%. An accurate method of solving the Dirac equation to obtain the energy eigenvalues is outlined. The importance of choosing the 'classical turning point' as the radius for matching the interior and exterior solutions is discussed.

Keywords. Muonic atom; harmonic oscillator potential; classical turning point; vacuum polarisation.

1. Introduction

Since the pioneering work of Fitch and Rainwater [1953] great progress has been made in the study of muonic atoms which is now an established technique for probing into the electromagnetic structure of the nuclei. This has been made possible owing to the high accuracy achieved by the introduction of solid state detectors and the improved estimates of quantum electrodynamical corrections to the calculated muonic energy levels. In practice the Dirac equation is numerically solved for the potential corresponding to the assumed charge distribution and the parameters are adjusted until the calculated transition energies agree with the experimental values. The quantum electrodynamical corrections, important among which are the vacuum polarisation and Lamb shift corrections, are allowed in the calculations by standard procedures. The x-ray transition energies in muonic atoms are strongly affected by the nuclear size. In a typical heavy muonic atom, an appreciable part of the muon $1s_{1/2}$ wave function is immersed within the nucleus itself. Consequently, the transition energies for the low lying orbits ($2p$—$1s$, $3d$—$2p$) are strongly sensitive to the nuclear charge distribution. Most of the authors have analysed the experimental data by means of a two- or three- parameter Fermi charge distribution. The analysis of muonic transition energies with an analytical function for the charge distribution leads to the estimation of parameters such as t, the skin thickness and C, the half-radius as well as the root mean square radius. In all these cases, however, the choice of one specific analytical form for the charge density rather than another is questionable since there are no *a priori* criteria for the choice. The choice of Fermi distribution for medium and heavy nuclei is mainly dictated by the results of the high energy electron scattering experiments, and the parameters are well deter-