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A curiosity concerning the role of coherent states in quantum field theory

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Abstract. In the usual Fock quantisation of fields in Minkowski space-time, one has the result that the expectation value of the quantum Hamiltonian in any coherent state equals the energy of the classical field at which the state is peaked. It is shown that this property can be used to *characterise* the usual Fock representation. It is also pointed out that the entire analysis goes through for a substantially more general class of systems including, in particular, Bose fields in arbitrary stationary space-times.

Keywords. Quantum field theory; Fock representations; coherent states.

1. Introduction

Consider, in Minkowski space, a box B of volume V filled with electro-magnetic radiation of frequency ω. For simplicity, let us work in the Coulomb gauge (in the rest frame of B) and assume that the vector potential A_a vanishes on the walls of B. Then, the classical energy $\mathscr{E}_c$ of this A_a is given by

$$\mathscr{E}_c = (\tfrac{1}{2}) \int_B (E_a E^a + H_a H^a) dV = \omega^2 k,$$

where E_a and H_a are the electric and magnetic parts of the given Maxwell field in the rest frame of B, and,

$$k = \int_B A^a A_a \, dV.$$

On the other hand, one may choose to describe the content of the box in the framework of quantum field theory. Then, this content is best described by a coherent state of the quantised Maxwell field (see e.g. Glauber 1963; Sudarshan 1963). Denote this state by Ψ. The energy $\mathscr{E}_q$ of this state is given by:

$$\mathscr{E}_q = \langle \Psi, \Psi \rangle^{-1} \langle \Psi, \mathscr{H} \Psi \rangle = \hbar\omega \langle \Psi, \Psi \rangle^{-1} \langle \Psi, \mathscr{N} \Psi \rangle = \hbar\omega \langle A, A \rangle = \omega^2 k,$$

where $\langle , \rangle$, $\mathscr{H}$ and $\mathscr{N}$ are respectively, the inner-product the Hamiltonian and the total-number operator on the Fock space of quantum states of the electromagnetic