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Relativistic star clusters with high central redshift

M C DURGAPAL, P S RAWAT and R BANERJI
Department of Physics, Kumaun University, Naini Tal 263 002, India

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Abstract. Collisionless star clusters in dynamical equilibrium are of current interest in general relativity and astrophysics. A step-function distribution is chosen for star clusters. The corresponding equation of state is analogous to a Fermi-gas equation. These clusters are found to be pulsationally unstable for a central redshift of $Z_c \geqslant 0{\cdot}54$. Further, a model of clusters is developed in which the core has an extremely relativistic equation of state. These structures are unstable for $Z_c \geqslant 2{\cdot}55$ when we use Chandrasekhar's technique to study their pulsational stability.

Keywords. Star clusters; general relativity; redshift; Fermigas equation; field equations.

1. Introduction

The normal astrophysical evolution of a galactic nucleus is estimated (Sanders 1970) to lead under certain circumstances to a star cluster so dense that general relativity influences its structure and evolution. Hoyle and Fowler (1967) proposed relativistic cluster as idealized model for quasars. The theory of static, collisionless, relativistic star cluster is constructed along the same line as the corresponding Newtonian theory: A distribution function is constructed from the first integral of the motion of a 'particle', so that this distribution function automatically satisfies the kinetic equation. The Poisson equation in Newtonian case is replaced by Einstein equations relating to the curvature of stress-energy tensor of the distribution of particles. However, collisionless systems of particles cannot give rise to every conceivable equation of state (Zeldovich and Novikov 1971).

For a Newtonian star cluster if we consider the distribution function, $F(E) \sim \exp(-E/\theta)$, we find $P=\theta\rho$ (analogue of an isothermal, ideal gas). For a step-function, $[F(E)=\text{constant},\ E<E_0$ and $F(E)=0,\ E>E_0]$ we obtain $P=\text{constant}\ \rho^{5/3}$ (analogue of a Fermigas). Fackerell (1968) has given a method to obtain distribution function $F(x)$ for a relativistic spherical configuration with isotropic pressure and given internal solutions (that is, known values of ρ, e^{ν}, e^{λ} and P). Alternatively, if the distribution function is given, one can obtain the equation of state. In this paper we have chosen a step-function for the distribution function and obtained the equation of state. This equation of state is analogous to a Fermi-gas and for lower values of P/ρ the behaviour of the equation of state is similar to that of a polytropic gas of index $n=1{\cdot}5$. Further, some equations of state have been obtained which behave like polytropic gas of index n under Newtonian approximation. The distribution function of these new equations of state are positive, that is $F(x)\geqslant 0$ and $\partial F(x)\,/\,\partial x \leqslant 0$.