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Inverse spectral theory for Jacobi matrices and their almost periodicity

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Abstract. In this paper we consider the inverse problem for bounded Jacobi matrices with nonempty absolutely continuous spectrum and as an application show the almost periodicity of some random Jacobi matrices. We do the inversion in two different ways. In the general case we use a direct method of reconstructing the Green functions. In the special case where we show the almost periodicity, we use an alternative method using the trace formula for points in the orbit of the matrices under translations. This method of reconstruction involves analyzing the Abel-Jacobi map and solving of the Jacobi inversion problem associated with an infinite genus Riemann surface constructed from the spectrum.

Keywords. Jacobi matrices; inverse theory; almost periodicity.

1. Introduction

In this paper we address the question of recovering a Jacobi matrix

$$Hu(n) = a_n u(n+1) + b_n u(n) + a_{n-1} u(n-1), \quad u \in l^2(\mathbb{Z}) \tag{1}$$

with $a_n \geqslant 0$ and b_n real, from its spectrum Σ, assuming it to be a compact set, and also the question of the almost periodicity of the sequences a_n and b_n constructed from the spectrum. We consider the case, when the real part of the boundary values of the Green function for the vector δ_0 vanish almost everywhere on the spectrum. In this case we show that there is no uniqueness even when the Dirichlet eigenvalues of the half-line problems are specified.

We use this theory to prove the almost periodicity of some random jacobi matrices with the spectrum having a band structure. The motivation for this work comes from the work on periodic Jacobi matrices and the inverse theory for Schrödinger operators. There is extensive work on inverse spectral theory in the sense of recovering the operators from given spectral quantities, for periodic Schrödinger operators in the literature, the work of McKean–Moerbecke [31], McKean–Trubowitz [32], Trubowitz [38] being the some of these. There is also the work of Dubrovin–Matveen–Novikov [9], Levitan [27], [28], [29] for almost periodic potentials. In a general framework of ergodic potentials, the inverse spectral theory was initiated by Kotani, who showed the existence of ergodic Schrödinger operators associated with a class of spectral functions, and also discussed classical integrable systems in this framework in a series of papers ([18]–[21]). These inversion results of Kotani produced classes of potentials, while the pointwise information, and the nature of the isospectral class obtained were discussed in Kotani–Krishna [23] for ergodic potentials and Craig [8] for a very general class of reflectionless potentials. In the