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L^p-Estimates for Schrödinger operators

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Abstract. L^p-estimates of the resolvent for a large class of Schrödinger operators are proved. Combining this with the almost analytic continuations, we obtain L^p-estimates for functions of Schrödinger operators.

Keywords. Schrödinger operators; L^p-estimates; functional calculus.

1. Introduction

The purpose of these lectures is to present some recent results on L^p-estimates for Schrödinger operators. In particular, we will discuss estimates for the resolvents and, more generally, functions of Schrödinger operators in L^p with $1 \leqslant p \leqslant \infty$. Most of the result in these lectures are outcome of the joint work of Arne Jensen and the author [JN1, JN2], and presented in a slightly generalized form.

We consider Schrödinger operators: $H = -\Delta + V(x)$, which is primarily defined as a self-adjoint operator on $L^2(\mathbf{R}^d)$ with $d \geqslant 1$. We always suppose that the potential $V(x)$ satisfies the following assumption:

Assumption. $V(x)$ is a real-valued function on $\mathbf{R}^d$ such that $V(x) = V_+(x) - V_-(x)$, $V_\pm(x) \geqslant 0$, $V_+ \in L^1_{\mathrm{loc.}}(\mathbf{R}^d)$ and $V_-(x) \in K_d$, where K_d is the Kato-class of functions defined by: $V \in K_d$ if

(i) If $d = 1$,

$$\sup_{x \in \mathbf{R}} \int_{|x-y| \leqslant 1} |V(y)|\,dy < \infty;$$

(ii) If $d = 2$,

$$\lim_{\alpha \downarrow 0} \left[\sup_{x \in \mathbf{R}^2} \int_{|x-y| \leqslant \alpha} \log(|x-y|^{-1})|\, V(y)|\,dy \right] = 0;$$

(iii) If $d \geqslant 3$,

$$\lim_{\alpha \downarrow 0} \left[\sup_{x \in \mathbf{R}^d} \int_{|x-y| \leqslant \alpha} \frac{|V(y)|}{|x-y|^{d-2}}\,dy \right] = 0.$$

Then it is well-known that the Friedrichs extension with the form domain $Q(H) = H^1(\mathbf{R}^d) \cap D(|V|^{1/2})$ exists, and it is bounded from below, i.e., $H \geqslant C$ with some