Proc. Indian Acad. Sci. (Math. Sci.), Vol. 104, No. 4, November 1994, pp. 599–651.

Scattering theory for Stark Hamiltonians

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Abstract. We give an introduction to the spectral and scattering theory for Schrödinger operators. An abstract short range scattering theory is developed. It is applied to perturbations of the Laplacian. Particular attention is paid to the study of Stark Hamiltonians. The main result is an explanation of the discrepancy between the classical and the quantum scattering theory for one-dimensional Stark Hamiltonians.

Keywords. Spectral theory; scattering theory; Schrödinger operators.

1. Introduction

In this paper we give a short survey of potential scattering theory in nonrelativistic quantum mechanics. Particular attention is paid to Stark Hamiltonians. The reason for this is a remarkable discrepancy between classical and quantum scattering in the one-dimensional case. In the last section this discrepancy is resolved.

Let H be a selfadjoint operator, called the Hamiltonian, on a Hilbert space $\mathscr{H}$. The time-dependent Schrödinger equation

$$i\frac{\mathrm{d}}{\mathrm{d}t}\varphi(t) = H\varphi(t), \tag{1.1}$$

$$\varphi(0) = \varphi_0, \tag{1.2}$$

is solved by

$$\varphi(t) = U(t)\varphi_0 = e^{-itH}\varphi_0. \tag{1.3}$$

In potential scattering theory we have a free Hamiltonian H_0 and a potential V such that the full Hamiltonian is $H = H_0 + V$. The free evolution is given by $U_0(t) = \exp(-itH_0)$. In scattering theory we compare the two evolutions $U_0(t)$ and $U(t)$ for large positive and negative times t, using the wave operators

$$W_\pm = \underset{t\to\pm\infty}{s\text{-lim}}\, U(-t)U_0(t)P_{\mathrm{ac}}(H_0) = \underset{t\to\pm\infty}{s\text{-lim}}\, e^{itH}e^{-itH_0}P_{\mathrm{ac}}(H_0). \tag{1.4}$$

Here $P_{\mathrm{ac}}(H_0)$ denotes the projection onto the subspace of absolute continuity of H_0. An important property of the wave operators is the intertwining relation

$$e^{itH}W_\pm = W_\pm e^{itH_0}. \tag{1.5}$$

Assume that the wave operators exist. They are said to be complete, if