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A proof of Howard's conjecture in homogeneous parallel shear flows

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Abstract. A rigorous mathematical proof of Howard's conjecture which states that the growth rate of an arbitrary unstable wave must approach zero as the wave length decreases to zero, in the linear instability of nonviscous homogeneous parallel shear flows, is presented here for the first time under the restriction of the boundedness of the second derivative of the basic velocity field with respect to the vertical coordinate in the concerned flow domain.

Keywords. Nonviscous; shear flows.

1. Introduction

The point of inflexion theorem of Rayleigh (1880) and the semicircle theorem of Howard (1961) impose necessary restrictions on the basic velocity field $U(y)$ and the complex wave velocity $C = C_r + iC_i$. These are accessible to an arbitrary unstable $(C_i > 0)$ wave in the linear instability of nonviscous homogeneous parallel shear flows and it is of interest to have a similar restriction on the growth rate kC_i possible for such an unstable wave, k being the wave number and y being the vertical coordinate. In his pioneering contribution (1961; henceforth referred to as HO), Howard established one such estimate in the form

$$k^2 C_i^2 \leqslant \max\left(\frac{\mathrm{d}U}{\mathrm{d}y}\right)^2, \tag{1}$$

and considering its inability to provide the correct qualitative result for plane the Couette flow with $\mathrm{d}U/\mathrm{d}y$ constant, which is known to be neutrally stable with $kC_i \to 0$ as $k \to \infty$, remarked "This estimate is not usually sharp—for example, the Couette flow with $\mathrm{d}U/\mathrm{d}y$ constant, is known to be neutrally stable—but in most cases it will probably give the correct order of magnitude of the maximum growth rate. It is sufficient to show that C_i must approach zero as wavelength decreases to zero, given the boundedness of $\mathrm{d}U/\mathrm{d}y$; but there is likelihood that in fact $kC_i \to 0$ as $k \to \infty$ and...cited in I".

In the present paper we give a rigorous mathematical proof of this conjecture of Howard, namely $kC_i \to 0$ as $k \to \infty$, under the restriction of the boundedness of $\mathrm{d}^2U/\mathrm{d}y^2$ in the concerned flow domain.