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Positive values of non-homogeneous indefinite quadratic forms of type (1,4)

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Abstract. Let $\Gamma_{r,n-r}$ denote the infimum of all numbers $\Gamma > 0$ such that for any real indefinite quadratic Q in n variables of type $(r, n-r)$, determinant $D \neq 0$ and real numbers $c_1, \ldots, c_n$ there exist $(x_1, \ldots, x_n) \equiv (c_1, \ldots, c_n) \pmod 1$ satisfying

$$0 < Q(x_1, \ldots, x_n) \leqslant (\Gamma |D|)^{1/n}.$$

All the values of $\Gamma_{r,s}$ are known except $\Gamma_{1,4}$. It is shown that

$$8 \leqslant \Gamma_{1,4} \leqslant 16.$$

Keywords. Quadratic forms; Birch reduction; equivalent forms.

1. Introduction

Let $Q(x_1, \ldots, x_n)$ be a real indefinite quadratic form in n variables of type $(r, n-r)$ and determinant $D \neq 0$. Blaney [9] has shown that there exist Γ, independent of Q and depending only on n and r such that given any real numbers C_n there exist $(x_1, \ldots, x_n) \equiv (c_1, \ldots, c_n) \pmod 1$ such that

$$0 < Q(x_1, \ldots, x_n) \leqslant (\Gamma |D|)^{1/n}. \tag{1}$$

Let $\Gamma_{r,n-r}$ denote the infimum of all such numbers Γ. In this notation the following results are known:

$\Gamma_{1,1} = 4$, Davenport-Heilbronn [11]
$\Gamma_{2,1} = 4$, Blaney [10] and Barnes [7]
$\Gamma_{1,2} = 8$, $\Gamma_{3,1} = \frac{16}{3}$, $\Gamma_{2,2} = 16$, Dumir [12], [13], [14]
$\Gamma_{1,3} = 16$, Dumir and Hans-Gill [15]
$\Gamma_{3,2} = 16$, $\Gamma_{4,1} = 8$, Hans-Gill and Madhu Raka [17], [18]
$\Gamma_{r,n-r}$ for $s = 2r - n = 0, \pm 1, 2, 3$, Bambah, Dumir and Hans-Gill [4], [5], [6]
$\Gamma_{r,r+2}$ and $\Gamma_{r,r+3}$ for $r \geqslant 3$, Aggarwal and Gupta [1], [2]
$\Gamma_{r+4,r}$ for $r \geqslant 1$, Aggarwal and Gupta [3]
$\Gamma_{2,5}$, Dumir and Sehmi [16].

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