

## Some remarks on the Jacobian question

SHREERAM S ABHYANKAR

Mathematics Department, Purdue University, West Lafayette IN 47907, USA

Notes by MARIUS VAN DER PUT and WILLIAM HEINZER  
Updated by AVINASH SATHAYE

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**Abstract.** This revised version of Abhyankar's old lecture notes contains the original proof of the Galois case of the  $n$ -variable Jacobian problem. They also contain proofs for some cases of the 2-variable Jacobian, including the two characteristic pairs case. In addition, proofs of some of the well-known formulas enunciated by Abhyankar are actually written down. These include the Taylor Resultant Formula and the Semigroup Conductor formula for plane curves. The notes are also meant to provide inspiration for applying the expansion theoretic techniques to the Jacobian problem.

**Keywords.** Plane curves; Jacobian; automorphism; Newton-Puiseux expansion.

### Introduction

What follows may best be described by altering the famous phrase of Zariski and Samuel – *this paper is the unborn parent of its child*. These notes, based on Abhyankar's lectures, were originally prepared by van der Put in 1972. Since he had to rush back to the Netherlands in the middle of their preparation, they were worked on by William Heinzer to some extent. But there were still unfinished parts and as a result, they stayed buried in Abhyankar's private papers for 21 years. Later, Abhyankar's work on the Jacobian problem was partly reported in the Tata Institute Notes by Balwant Singh [A1]. Avinash Sathaye rearranged the old notes and added some additional topics based on further discourses by Abhyankar. Paul Eakin and David Shannon have also contributed to the current form of these notes by their critical proof-reading and suggestions. Special thanks are also given to the referee whose comments have significantly improved the logical clarity of this exposition.

In the old tradition of Abhyankar's lecture-notes, these notes are also unread by Abhyankar in their final form. The responsibility for the exposition, therefore, rests with the note-takers.

Much has been published about the Jacobian problem in the meantime, but except for the Balwant Singh Notes, the novel technique of using a combination of Newton-Puiseux expansions at infinity and studying the resulting value-semigroups did not get much exposure. Our aim in reviving these old notes is to renew interest in these methods, which, to paraphrase Abhyankar's own words, "never really got stuck, but only got very tiring". Perhaps, this time one of the methods will be carried through!