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On the structure of stable random walks

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Abstract. We show that the Cauchy random walk on the line, and the Gaussian random walk on the plane are similar as infinite measure preserving transformations.

Keywords. Stable random walks; Gaussian random walk.

1. Introduction

A measure preserving transformation T is considered acting on a *standard* measure space (X_T, B_T, m_T) (a complete, separable metric space equipped with its Borel sets and a σ-finite, non-atomic measure). It is known that standardness is unaffected by replacing X_T with a T-invariant subset $X'_T \in B_T$ of full measure, and we shall consider T acting on (X_T, B_T, m_T) to be the same as T acting on $(X'_T, B_T \cap X'_T, m_T)$.

Let S and T be measure preserving transformations. A *factor map* from S to T is a map $\pi: X_S \to X_T$ such that

$$\pi S = T\pi, \quad \pi^{-1} B_T \subset B_S, \quad \text{and} \quad m_S o \pi^{-1} = c m_T$$

where $0 < c < \infty$.

In this situation (denoted by $\pi: S \to T$), one says that T is a *factor* of S and that S is an extension of T (both denoted $S \to T$).

It is necessary to consider factor maps with $c \neq 1$ as our measure spaces are not normalized. The constant c can be thought of as a relative normalization of the transformations concerned.

Two measure preserving transformations are said to be *similar* if they have a common extension, that is: if there is another measure preserving transformation of which they are both factors; and they are said to be *strongly disjoint* if they have no common extension. We denote the statement that S and T are similar by $S \sim T$.

Any two transformations preserving finite measures are similar, their Cartesian product being a common extension. Invariants for similarity are given in [A1, A2], where it is shown that similarity is an equivalence relation. Examples of conservative, ergodic, measure preserving transformations which are strongly disjoint from their inverses are given in [A2].

In this paper, we consider random walks on $\mathbb{R}$ and $\mathbb{R}^2$. For f a probability on G, a locally compact second countable abelian group, the *random walk* on G with *jump distribution* f can be defined as follows: Let S_f be the shift on $G^{\mathbb{Z}}$ considered with the S_f-invariant product measure $m_f = \Pi f$. The *random walk* on G with *jump distribution*