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Rearrangements of bounded variation sequences

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Abstract. Let bv be the set of all bounded variation sequences. In the present paper we deduce from a theorem of Mears a necessary and sufficient condition for the rearrangement $(a_{p(k)})$ to be of bounded variation whenever $(a_k) \in bv$; interestingly it coincides with Pleasants' criterion for convergence-preserving.

Keywords. Rearrangements; sequences.

Let Σa_k be an infinite series of real numbers and p be a permutation of N, the set of all positive integers. The series $\Sigma a_{p(k)}$ is then called a rearrangement of Σa_k. A classical theorem of Riemann states that if Σa_k is a conditionally convergent series and s is any fixed real number (or $\pm\infty$), then there is a permutation p such that $\Sigma a_{p(k)} = s$. Thus it leads us to the problem of characterizing the rearrangements which do not change the sum or the convergence or even the divergence of the series. They were studied in [1]–[9] and by others. Of special interest is a paper by Pleasants [5] giving a characterization of permutations which transform convergent sequences to convergent sequences.

In this paper we consider questions similar to those above, but for rearrangements of bounded variation sequences. We recall some notation before stating the precise problem.

DEFINITION

Let γ denote the set of all convergent series of real numbers. A permutation p on positive integers is then called convergence-preserving (CP for short) if $a_p = (a_{p(k)}) \in \gamma$ for any sequence $a = (a_k) \in \gamma$, [5].

We shall denote the finite consecutive run of integers $i, i+1, \ldots, j-1, j$ by $[i,j]$ and we shall call such a set a block.

Every finite set F of N is a union of disjoint, non-adjacent blocks of consecutive integers. Let $v(F)$ denote the number of such blocks and p^{-1} be the inverse of p. With this terminology, the result of Pleasants on CP permutations can be stated as follows.

Theorem 1. [5; p. 135]. *A permutation p is CP if and only if there is a constant M $(= M(p))$ such that $v(p^{-1}\{1,2,\ldots,k\}) \leqslant M$ for all $k \in N$.*

We now give the following notation similar to the above definition, for sequences of bounded variation.