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Existence theory for linearly elastic shells

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Dedicated to the memory of Professor K G Ramanathan

Abstract. We review existence and uniqueness results, recently obtained for three of the most important linear two-dimensional shell models: Koiter's model, the bending model and the membrane model. They rely on a crucial lemma of J L Lions, used in an essential way for establishing in each case a generalized Korn's inequality, which is then combined with a generalized rigid displacement lemma of a geometrical nature.

Keywords. Linearly elastic shells; bending shell model; membrane shell model; Koiter's model.

1. Geometrical and mechanical preliminaries

In what follows, Greek indices and exponents vary in the set $\{1,2\}$, Latin indices and exponents vary in the set $\{1,2,3\}$, and the repeated index or exponent convention for summation is used. The Euclidean inner product, the vector product and the Euclidean norm, of vectors $\mathbf{u}$, $\mathbf{v}\in\mathbf{R}^3$ are denoted as $\mathbf{u}\cdot\mathbf{v}, \boldsymbol{\mu}\times\mathbf{v}$, and $|\mathbf{u}|$.

Let ω be an open, bounded, connected subset of $\mathbf{R}^2$ with a Lipschitz-continuous boundary γ, the set ω being locally on one side of γ. Let $y=(y^1,y^2)$ denote a generic point of the set $\bar{\omega}$ and let $\partial_\alpha=\partial/\partial y^\alpha$. We consider a surface S in $\mathbf{R}^3$, of the form $S=\boldsymbol{\varphi}(\bar{\omega})$, where $\boldsymbol{\varphi}:\bar{\omega}\to\mathbf{R}^3$ is a given, injective, smooth enough mapping. We assume that the two vectors $\mathbf{a}_\alpha=\partial_\alpha\boldsymbol{\varphi}$ are linearly independent at all points of $\bar{\omega}$.

The vectors $\mathbf{a}_\alpha$ form the covariant basis of the tangent plane, and the vectors $\mathbf{a}^\alpha$, defined by the relations $\mathbf{a}^\alpha\cdot\mathbf{a}_\beta=\delta^\alpha_\beta$, form its contravariant basis. The three vectors $\mathbf{a}^i$, where $\mathbf{a}^3=\mathbf{a}_3=(\mathbf{a}_1\times\mathbf{a}_2)/|\mathbf{a}_1\times\mathbf{a}_2|$ form the contravariant basis at each point of S. The Christoffel symbols are defined by

$$\Gamma^\rho_{\alpha\beta}=\mathbf{a}^\rho\cdot\partial_\alpha\mathbf{a}_\beta,$$

and the first, second and third fundamental forms of S are defined by

$$a_{\alpha\beta}=\mathbf{a}_\alpha\cdot\mathbf{a}_\beta \text{ or } a^{\alpha\beta}=\mathbf{a}^\alpha\cdot\mathbf{a}^\beta,$$

$$b_{\alpha\beta}=-\mathbf{a}_\alpha\cdot\partial_\beta\mathbf{a}_3,$$

$$c_{\alpha\beta}=b^\rho_\alpha b_{\rho\beta}, \text{ where } b^\rho_\alpha=a^{\rho\sigma}b_{\sigma\alpha}.$$

Note that $\Gamma^\rho_{\alpha\beta}=\Gamma^\rho_{\beta\alpha}$, $a_{\alpha\beta}=a_{\beta\alpha}$, $b_{\alpha\beta}=b_{\beta\alpha}$, $c_{\alpha\beta}=c_{\beta\alpha}$. Finally, we let

$$a=\det(a_{\alpha\beta}).$$