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Symplectic structures on locally compact abelian groups and polarizations

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Dedicated to the memory of Professor K G Ramanathan

Abstract. Let X be a locally compact abelian group and $\omega(\cdot,\cdot)$ a symplectic structure on it. A polarization for (X,ω) is a pair of totally isotropic closed subgroups G, G^* of X such that $X = G\cdot G^*$ and $\omega(\cdot,\cdot)$ defines a dual pairing of G and G^*. In this paper we describe a class of such groups which always admit a polarization and also discuss their structure.

Keywords. Symplectic structures.

1. Introduction

Let $\mathscr{L}$ denote the class of locally compact Hausdorff, abelian and second countable groups. Fox $X\in\mathscr{L}$, consider an alternating bicharacter ω on X, i.e. (i) For each $x, y\to\omega(x,y)$ is character in $y\in X$, and for each $y, x\to\omega(x,y)$ is character in x and (ii) ω is alternating i.e., $\omega(x,x)=1$ for each $x\in X$. Such ω is known to provide a classification of central extensions of X by the circle group T. For the central extension $1\to T\to E_\omega\to X\to 1$ corresponding to the classifying invariant ω, the analogue of the Stone–von Neumann theorem has been proved under two sets of assumptions. The first one is

$$(X,\omega) \text{ is nondegenerate} \tag{1}$$

i.e., $\omega(x_0,y)=1$ for all y implies $x_0=0$ (the identity element of X) and if χ is any continuous character of X, then there exists an $x_0\in X$ such that $\chi(y)=\omega(x_0,y)$ for all y. This assumption makes it possible to apply Mackey's theory of systems of imprimitivity to E_ω to get Stone–von Neumann theorem (see for example [M]). On the other hand, Weil [W1] used a different kind of assumption on (X,ω) to obtain the same result. This assumption may be succinctly described by saying that (X,ω) admits a polarization i.e., there exist closed subgroups G, G^* of X with the following properties:

(i) $G\cap G^*=\langle 1\rangle$, $X=G\cdot G^*$
(ii) $\omega(G,G)=1$, $\omega(G^*,G^*)=1$ i.e., $\omega(x,y)=1$ whenever $x,y\in G$, and also whenever $x,y\in G^*$
(iii) the mapping $x,y\to\omega(x,y)$ of $G\times G^*$ into T is a dual pairing of G and G^*.

Note the assumption (i) means that X is a direct sum (or direct product) of the groups G and G^* and the assumption (iii) identifies G^* as the character group of G.