

Zeta functions of prehomogeneous vector spaces with coefficients related to periods of automorphic forms

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Dedicated to the memory of Professor K G Ramanathan

Abstract. The theory of zeta functions associated with prehomogeneous vector spaces (p.v. for short) provides us a unified approach to functional equations of a large class of zeta functions. However the general theory does not include zeta functions related to automorphic forms such as the Hecke L -functions and the standard L -functions of automorphic forms on $GL(n)$, even though they can naturally be considered to be associated with p.v.'s. Our aim is to generalize the theory to zeta functions whose coefficients involve periods of automorphic forms, which include the zeta functions mentioned above.

In this paper, we generalize the theory to p.v.'s with symmetric structure of K_r -type and prove the functional equation of zeta functions attached to automorphic forms with generic infinitesimal character. In another paper, we have studied the case where automorphic forms are given by matrix coefficients of irreducible unitary representations of compact groups.

Keywords. Zeta function; prehomogeneous vector space; functional equation; automorphic form.

Introduction

The theory of zeta functions associated with prehomogeneous vector spaces was developed by Sato and Shintani in [S] and [SS] and generalized later in [S1] to multivariable zeta functions. The theory provides us a unified approach to functional equations of a large class of zeta functions. However the general theory does not include zeta functions related to automorphic forms, even though they can naturally be considered to be associated with prehomogeneous vector spaces. Among those are the Hecke L -functions with Grössencharacters (cf. [Hec1], [Hec2]), the Epstein zeta functions with spherical functions (cf. [E]), the Maass zeta functions attached to quadratic forms (cf. [M1]–[M4]), and the standard L -functions of automorphic forms on $GL(n)$ (cf. [GJ]). Our ultimate purpose is to generalize the theory of zeta functions associated with prehomogeneous vector spaces ([SS], [S1]) to zeta functions whose coefficients involve periods of automorphic forms, which include the zeta functions mentioned above.

In [S6], we have studied the case where automorphic forms are given by matrix coefficients of irreducible unitary representations of compact groups. The simplest example in this case is the Epstein zeta functions with spherical functions (cf. § 2.2, [S4]).

In the present paper, we are concerned mainly with prehomogeneous vector spaces with symmetric structure and prove the functional equations and the analytic