

mechanism of production of light of high quantum efficiency by living matter.

In conclusion it may be stated that the contributions made in the Symposium and the lively discussions accompanying them show that while luminescence is a fundamental phenomenon and much has been done in the way of exploration, there is much more which awaits further investigation, both theoretically and

experimentally, before a general theory of luminescence explaining all facts connected therewith could be formulated. By bringing together prominent workers in the field and publishing the proceedings in full, the Faraday Society has done a great service for the future workers in the subject.

C. S. VENKATESWARAN.

Einstein's Generalisation of Kaluza's Unitary Theory

THE Kaluza-Klein theory introduces a fifth dimension in attempting to derive a unitary theory connecting gravitation and electricity. Einstein has recently attempted to generalise this theory¹ by putting in physical concepts into the purely mathematical structure of Kaluza's theory.

The aim of this theory of Kaluza was to obtain some new physical aspect for gravitation and electricity by introducing a unitary field structure with the aid of a fifth dimension, the essential result being that such a five-dimensional structure could be built up so as to be equivalent to a four-dimensional structure plus a vector field which is the potential vector for the electro-magnetic field. This result, though elegant mathematically, was not productive of new physical ideas, and consequently many attempts were made to retain the essential formal results obtained by Kaluza without sacrificing the four-dimensional character of the physical space. But all such attempts have proved unproductive, and it appears impossible to formulate Kaluza's idea in a simple way without introducing the fifth dimension.

On the basis of these considerations, Einstein and Bergmann have now attempted to introduce the fifth dimension in a very effective manner without its being merely a sort of "catalytic agent" as in the Kaluza theory. To bring out clearly the generalisation proposed by Einstein, let us consider how Kaluza's five-dimensional structure is made equivalent to a four-dimensional one and a vector field. It can be shown that by a suitable characterisation of the 5-space with the metric

$$d\sigma^2 = \gamma_{\mu\nu} dx^\mu dx^\nu \quad (\mu, \nu = 0, 1, 2, 3, 4) \dots (1)$$

the components of the fundamental metric tensor can, by the choice of a special co-ordinate system, be reduced to ten functions g_{mn} and the four functions A_m ($m, n = 1, 2, 3, 4$) which do not depend on x^0 . This reduction gives a four-dimensional description of the space, and the independence of the functions on x^0 shows the purely formal nature of the fifth dimension x^0 which is just put in only to be taken out later. On Einstein's new theory it is shown that,

with a suitable modification of the postulates of the 5-space, it is possible to make an exactly analogous reduction to g_{mn} and A_m with this difference that the components of g_{mn} are in general periodic functions of x^0 . The A_m , however, is independent of x^0 as in the old theory. Remembering that g_{mn} is a four-dimensional metric tensor, this amounts to an intimate physical connection of the space-time with the new dimension. The x^0 which is put in at first is not taken out after the reduction but left behind so as to modify the 4-metric. The periodicity of the components of this 4-tensor in the new co-ordinate enables one to interpret physically the fifth dimension. In a very rough way, one could describe this as a sort of a phase, and the 4-dimensional space-time might be thought of as having been replaced by a 5-dimensional space-time-phase. Since, however, this new co-ordinate is "dimensionless" there arises no contradiction with the empirical four-dimensional character of physical space.

From its very nature, the new theory is essentially complex in its physical aspects, and Einstein and Bergmann have given the derivation of the fourteen field equations starting from a variational principle, and also the identities satisfied by the field equations. The theory involves four universal constants of which one corresponds to the gravitational constant involving a connection between the units of length and mass, another depending on the unit of length, while the remaining two are "genuine" universal constants which cannot be eliminated from the theory.

When looked at from the purely geometrical point of view the new theory introduces some very interesting features. The five-dimensional space defined by the metric (1) is here closed with respect to one dimension, and this closed space will be represented by a space which is open and periodic with respect to this dimension. A point P of the physical space will be represented by an infinite number of points P, P', P'' of the 5-space. This type of non-homeomorphic correspondence between general metric spaces is itself a rich mathematical concept capable of a large number of developments.

¹ Vide *Annals of Mathematics*, July 1938, 39, No. 3, 683.