LETTERS TO THE EDITOR

A Note on Self-Reciprocal Functions

I WILL say that a function is $\pm\, R_\nu$, according as it is self-reciprocal or skew-reciprocal for Hankel Transforms of order ν; that is, according as it satisfies the integral equation

$$f(x) = \pm \int_0^\infty \sqrt{xy}\, J_\nu(xy)\, f(y)\, dy, \quad (\nu > -1)$$

with the upper or the lower sign.

In a recent paper[1] I have proved the theorem: If $f(x)$ is R_ν, the function

$$\phi(x) = a^\alpha f(x a^{2\alpha}) \pm \frac{1}{a^\alpha} f\left(\frac{x}{a^{2\alpha}}\right),$$

where $a > 0$, $\alpha > 0$, is $\pm\, R_\nu$.

The object of this note is to give an easy generalisation of this theorem.

2. *Theorem I.*—If $f(x)$ is R_ν, the function

$$\phi(x) = F(a)\, f\{xF^2(a)\} \pm \frac{1}{F(a)} f\left\{\frac{x}{F^2(a)}\right\}, \tag{2·1}$$

where a is a constant, and $F(a) \neq 0$, is R_ν.

We have

$$\int_0^\infty \sqrt{xt}\, J_\nu(xt)\, \phi(t)\, dt$$

$$= \int_0^\infty \sqrt{xt}\, J_\nu(xt) \left[F(a)\, f\{tF^2(a)\} \pm \frac{1}{F(a)} f\left\{\frac{t}{F^2(a)}\right\} \right] dt$$

$$= F(a) \int_0^\infty \sqrt{xt}\, J_\nu(xt)\, f\{t\, F_2(a)\}\, dt$$

$$\pm \frac{1}{F(a)} \int_0^\infty \sqrt{xt}\, J_\nu(at)\, f\left\{\frac{t}{F^2(a)}\right\} dt$$

$$= \frac{1}{F(a)} \int_0^\infty \sqrt{\frac{xu}{F^2(a)}}\, J_\nu\left\{\frac{xu}{F^2(a)}\right\} f(u)\, du$$

$$\pm F(a) \int_0^\infty \sqrt{xu\, F^2(a)}\, J_\nu\{xuF^2(a)\}\, f(u)\, du$$

$$= \frac{1}{F(a)} f\left\{\frac{x}{F^2(a)}\right\} \pm F(a)\, f\{xF^2(a)\}$$

$$= \pm\, \phi(x).$$

Theorem II.—If $f(x)$ is $-R_\nu$, the function (2·1) is $\mp\, R_\nu$.

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1 "A few Self-Reciprocal Functions," *Proc. Physico-Math. Society of Japan*, 1934, 273–74.

Benzylidene-Flavanones considered as Chalkones

PANSE AND WHEELER[1] have shown that Benzylidene-Coumaranones like chalkones condense with acetoacetic ester, desoxybenzoin, cyclohexanone, etc. It is now found that