opinion this is marred by an over-filling of numerical work.

Here are two specimens (not typical, though) of *Descriptive Mathematics* :—

(*a*) "On March 21 the number of seconds taken by the sun to cross the horizon in latitude l is $32/15 \sec l$; find this for $l = 19°$ and $l = 60°$" (p. 58).

Is this illustration going to create a new interest in the student for the secant function, or is it going to be the nucleus of his future astronomical studies, or does it just show off the pedantry of the author ?

(*b*) "Note sets of words like "due, duty, dutifully" which are useful in teaching time in music. The periods of the syllables of these words are as 4 : 2 : 1. Consider the possibility that rhythm in speech and in prose may be partly and automatically determined by the essential periods and intensities of syllables. (In poetic rhythm there is of course deliberate selection of combinations of syllables)". (p. 60.)

This example follows problems like sketching the curves corresponding to $\cos^2 x$, $\cos^3 x$, etc. I have no music in me, but frankly, this piece of mathematics is beyond me.

C. N. S.

Research Notes.

On Ternions in Geometry.

HANS BECK (*Math. Zeit.*, **40**, 4, pp. 509-520) has investigated the occurrence of the linear transformation group of the system of non-commutative ternions, under various forms in several places in geometry. Let A, B, C, D be four ternions, then the linear transformation is $X' = (Cx + D)^{-1}(Ax + B)$. Now it is known that apart from the non-commutative system of the ternions, there exist two other commutative systems of ternions. In the latter cases, the linear transformation-group reduces itself to one of nine parameters. This is not of so much importance as the group in the non-commutative case, of eleven proper parameters. Beck has shown that this group occurs in the following places in geometry : (1) A special collineation group of a linear-complex ; (2) A cremona group in affine space ; (3) The group of Laguerre transformations of directed planes ; and (4) The group of rotations (in the same sense) in the four dimensional Euclidean space, etc.

A ternion of the system is represented as $A = A_0E_0 + A_1E_1 + A_2E_2$ and the multiplication table is

$$\left|\begin{matrix} E_0 & E_1 & E_2 \\ E_1 & E_0 & E_2 \\ E_2 & -E_2 & O_0 \end{matrix}\right|$$

E_0 can be taken to be the scalar unit. The norm $N(A) = A_0^2 - A_1^2$ (Hence reducible). If $\xi_0, \xi_1, \xi_2, \xi_3$ are the co-ordinates of a point in a projective R_3 and the Plucker's co-ordinates of a line are $P_{ik} = \xi_i \eta_k - \eta_i \xi_k$, then the ternions A, can be made to correspond to the lines of R_3 with co-ordinates $P_{01} : P_{02} : P_{03} : P_{23} : P_{31} : P_{12} = 0 : 1 : A_0 - A_1 : A_2 : -(A_0^2 - A_1^2) : (A_0 + A_1)$. (The transformation is not one-one.) By means of this transformation he has shown that the group is identical with the collineation which transforms $\xi_0 = 0$, $\xi_1 = 0$ into itself. Here is a nice geometrical representation of ternions.

He has also shown that the group is holomorphic with the group of the minimal complex—the straight lines having proper intersection with the conic-absolute in an Euclidean R_3.

The first representation of ternions is such that an ∞^2 st. lines of the projective space R_3 did not correspond to ternions at all. Then by considering a geometrical entity as corresponding to a ratio of two ternions, he obtains a representation in which the geometrical entities are points in an affine space ; the exceptional points for which ratio of ternions do not correspond belong to a plane which is naturally considered as the special-plane of the affine space (*Uneigentliche-Ebene*). The work is a very striking illustration of the unity in geometry stressed by Klein in his epoch-making Erlangen-Programme.

K. V. I.

Cauchy-Riemann Conditions.

MENCHOFF (*Fund. Math.* **25**, pp. 59-97) has extended Looman's classic result (*Gott. Nach.*, 1923) about the sufficient conditions for the analyticity of $f(z) = P + iQ$ in a given simply connected region. Looman had shown that if the Cauchy-Riemann partial differential equations were valid for almost all points in the region then $f(z)$ was analytic. This amounted to assuming that the derivates in two perpendicular directions (directions same

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