

LIMITING POLARIZATION CURVES FOR RADIO WAVE PROPAGATION IN THE IONOSPHERE

R. N. SINGH AND Y. S. N. MURTY
Wireless Laboratory, Dept. of Physics, Banaras Hindu University

THE polarization of a radio wave propagated through the ionosphere in the presence of earth's magnetic field is usually represented by θ and ψ. If the direction of wave propagation is along the positive direction of the X-axis of a right-handed co-ordinate system and the earth's magnetic field is assumed to be in the XZ-plane, then θ is defined by $\tan\theta = -j(h_z/h_y)$ where h_y and h_z are the magnetic vectors of the wave along the Y and Z axes, and ψ is the tilt of the polarization ellipse with respect to the Z-axis. In the present communication we shall discuss the limiting values of θ and ψ, when the wave enters the ionosphere or emerges from it on retracing its path. The observed polarization of a downcoming wave is determined by these limiting values when the electron density N tends to zero.

According to Mary Taylor[1] (1933, 1934), the limiting polarization is given by $\theta = \pm 45°$ and $\psi = 0$. $\theta = \pm 45°$ means that the wave incident on the ionosphere would split up into two circularly polarized components with opposite senses and if they retrace their path they would possess the same polarization on emergence from the ionosphere as on entry into it. The calculations of Martyn[2] (1935) and of Ghosh[3] (1933) for increasing values of the electron density N in the ionosphere (with certain assigned values for the electron collision frequency ν) following the method given by Bailey[4] (1934) have however shown that as N tends to zero, the values of θ and ψ, in the case of vertical propagation tend to certain limiting values which are not $\pm 45°$ and 0° and which depend on the angle between the direction of propagation and the direction of the earth's magnetic field and also on the wavelength and electron collision frequency.

The Bailey's method of conformal representation as applied to the Appleton[5]-Hartree[6] formulæ for the determination of θ and ψ has already been given in detail by Martyn (1935) and Ghosh (1938). We shall give here an outline of this method.

The Appleton-Hartree formulæ for the refractive index μ, absorption coefficient k and polarization R can be written as:

$$M^2 = c^2q^2 = (\mu - i\chi)^2 = 1 + \cfrac{1}{(\alpha+i\beta) - \dfrac{\gamma_T^2}{2(1+\alpha+i\beta)} \pm \sqrt{\left[\dfrac{\gamma_T^4}{4(1+\alpha+i\beta)^2} + \gamma_L^2\right]}} \qquad (1)$$

and

$$R = \frac{h_z}{h_y} = \frac{1}{i\gamma_L}\left[\frac{1}{c^2q^2-1} - \alpha - i\beta\right] \qquad (2)$$

where M = complex refractive index $= cq$

$$\chi = \frac{ck}{p}$$

$$\alpha = -\frac{p^2}{p_0^2} - 1, \quad \beta = \frac{p\nu}{p_0^2}$$

$$\gamma_L = \gamma\cos\theta', \quad \gamma_T = \gamma\sin\theta'$$

in which

$$p_0^2 = \frac{4\pi Ne^2}{m}, \quad \gamma = \frac{pp_H}{p_0^2}, \quad p_H = \frac{He}{mc}$$

and m, e = mass and charge respectively of an electron
N = electron density
H = intensity of earth's magnetic field
θ' = angle between H and direction of propagation
p = angular frequency of the wave
ν = electron collisional frequency.

Now eliminating (c^2q^2-1) from equations (1) and (2) and denoting the value of R with the negative and positive signs as R_0 and R_x corresponding to the ordinary and extraordinary components, we get

$$R_0 + R_x = \frac{\dfrac{i\gamma_T^2}{\gamma_L}}{1+\alpha+i\beta} = \frac{2}{x+iy} \qquad (3)$$

and

$$R_0 R_x = 1 \qquad (4)$$

where

$$x = \frac{\sigma\nu}{p_H}, \quad y = \frac{\sigma p}{p_H}\left(1 - \frac{p_0^2}{p^2}\right)$$

and

$$\sigma = \frac{2\cos\theta'}{\sin^2\theta'}.$$

The major axis of the polarization ellipse is inclined to the Z-axis at an angle ψ and is given by

$$\tan 2\psi = \frac{-2\rho\cos\phi}{(1-\rho^2)} \qquad (5)$$

and the ratio of the axes which we will call $\tan\theta$ can be written as