where $\square$ stands for
$$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial t^2} \text{ and } \phi_1 \text{ for } \frac{\partial \phi}{\partial x}.$$
It can be verified that
$$\square\left(\frac{\partial \sigma_x}{\partial x}+\frac{\partial \sigma_y}{\partial y}+\frac{\partial \sigma_z}{\partial z}+\frac{\partial \rho}{\partial t}\right)=0 \qquad (6)$$

We have similarly obtained the ten gravitational field equations showing how the gravitational potentials a_{ij} are modified on account of the electrostatic and electromagnetic potentials. The details and further deductions will be published elsewhere.

Benares Hindu University, V. V. NARLIKAR.
October, 19, 1948. RAMJI TIWARI.

* Einstein, A., "A Generalized Theory of Gravitation", *Rev. Mod. Phys.*, 1948, **20**, 35. A reference may also be made to the earlier papers in *Ann. Math.*, II (1945), **46**, 578 and (1946), **47**, 731.

ON HOTELLING'S WEIGHING PROBLEM

IN the June issue of the *Annals of Mathematical Statistics*, Kempthorne approached the construction of the orthogonal matrix X required in Hotelling's weighing design through fractional replicates, the original discussion of which was given by Finney. While referring to weighing three objects, Kempthorne mentions about the three-fourth replicate of a 2^n factorial experiment without giving any details. It has been shown that the efficiency of such designs bears a constant ratio to that of the designs given by the completely orthogonalised matrices.

In a three-fourth fractional replicate, the treatment contrasts will divide themselves into groups of 4 contrasts each and in each such group only three contrasts will be independent. The contrasts in any group will be orthogonal to all the contrasts in the other groups but non-orthogonal to one another within the group itself. One contrast in the group of 4, preferably that due to the highest order interaction may be left out. The matrix X'X will take in this case the following form:

$$\begin{bmatrix} x & a & a & 0 & 0 & 0 & \cdot\cdot \\ a & x & a & 0 & 0 & 0 & \cdot\cdot \\ a & a & x & 0 & 0 & 0 & \cdot\cdot \\ 0 & 0 & 0 & x & a & a & \cdot\cdot \\ 0 & 0 & 0 & a & x & a & \cdot\cdot \\ 0 & 0 & 0 & a & a & x & \cdot\cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\cdot \end{bmatrix}$$

where the order of the matrix $N=(3/4)2^n$ is of the form $3t$ $(t=2^{n-2})$ and $x=3\cdot 2^{n-2}$) $a=-\frac{1}{2}2^n+2^{n-2}=-2^{n-2}$. The value of the above determinant$=(x-a)^{2t}(x+2a)^t$ and that of the determinant suppressing the first row and the first column $=(x-a)^{2t-1}(x+a)(x+2a)^{t-1}$. Hence the variance factor $a^{ii}=(x+a)/[(x-a)(x-2a)]=\frac{1}{2^{n-1}}$, substituting for x and a.

The variance of each estimate will therefore be $\sigma^2/2^{n-1}$, whereas the variance of the estimates given by an orthogonal matrix of the order $N=(3/4)\,2^n=3\cdot 2^{n-2}$ will be equal to $\sigma^2/3\cdot 2^{n-2}$. The ratio of the two variances $=2/3$. Therefore, the efficiency of a design given by a three-fourth replicate would always bear the constant ratio of 66·66 per cent of the efficiency of a design given by a completely orthogonal matrix. In general, when the fraction is of the type $(2\beta-1)/2\beta$ of 2^n, ($\beta=1, 2, \cdots n$), the variance of an estimate is $\sigma^2/2^{n-1}$, the same as above. When a completely orthogonalised matrix is available, the variance of an estimate will be $\sigma^2/[2^{n-\beta}(2\beta-1)]$. The ratio of the two variances $=2^{\beta-1}/(2\beta-1)$, which shows how the efficiency of the weighing design decreases with the increasing value of the fraction. When $\beta=1$, *i.e.*, in a half replicate, the efficiency is cent per cent. The value of the fraction is never less than ½.

Pusa, Bihar, K. S BANERJEE.
November 13, 1948.

1. Harold Hotelling, "Some improvements in weighing and other experimental techniques," *Annals of Math. Stat.*, 1944, **15**, 297-306. 2. K. Kishen, "On the design of experiments for weighing," *Ibid.*, 1945, **16**, 294-301. 3. A. M. Mood, "On Hotelling's weighing problem," *Ibid.*, 1946, **17**, 432-446. 4. R. L. Plackett and J. P. Burman, "The design of optimum multifactorial experiments," *Biometrika*, 1946, **33**, 305-325. 5. O. Kempthorne, "The factorial approach to the weighing problem" *Annals of Math. Stat.*, 1948, **19**, 238-245. 6. D. J. Finney, "The fractional replication of factorial arrangements," *Annals of Eugenics*, 1945, **12**, 291-301. 7. K. S. Banerjee, "On the design of experiments for weighing and making other types of experiments" *Science and Culture*, 1948, **13**, 314. 8. — 'Weighing designs and balanced incomplete blocks," *Annals of Math. Stat.*, 1948, **19**, 394-399.

ON A CERTAIN BASIC THEOREM IN GEOMETRY

IN a recent issue of this journal,[1] I proposed a basic theorem in Geometry as a sort of mock challenge to see if any rigorous geometrical thinking comes out of it. The result was a heap of unnecessary discussions which clearly showed failure to perceive the point in my theorem. If one had recollected David Hilbert's Foundations of Geometry or recognised the need for a definition characterising the interior of an angle formed by two half-rays, the theorem would have been easily disposed of. David Hilbert does not give a definite construction for an interior point of an angle, though he mentions[2] that the half-rays together with their intersection divide the remaining points of the plane containing them into two regions, one of which may be characterised as the interior of the angle from the property that the segment joining any two points of it lies entirely within the region. I give below a construction for determining an interior point.

Let P be any point on a half-ray h and Q any point on a half-ray k, proceeding from a point O. Draw parallels PR, QR to OQ, OP respectively. Then R is an interior point of the angle (h, k).

More briefly, every vertex of a parallelogram is an interior point of the opposite angle.

The above statement may be treated as an axiom for the purpose of my theorem. It is, however, capable of rigorous proof with the help of Hilbert's axioms of order, congruences and parallels.

To proceed to my plane problem, let the given straight lines intersect at O, and R be any point not incident on either of these lines. If S, T be the feet of the perpendiculars on the given straight lines, it is required to show that the angle SRT is the supplement of the angle in which R lies, *viz.*, that which is given by my axiom as the angle POQ where P lies on O S and Q on O T and R P O Q is a parallelogram. Here O S, O T do not signify half-rays but whole straight lines.

The theorem immediately follows if the given straight lines cut at right angles. When the given lines do not cut at right angles, let α be the measure of the acute angle between them.

The triangles RPS, RQT are similar right-angled triangles, since the acute angles RPS, RQT, by the property of parallels, must each be equal to α;

$$\text{further} \quad \sin \alpha = \frac{RS}{RP} = \frac{RT}{RQ} = \frac{RT}{OP},$$

as OP=RQ from the parallelogram RPOQ. (1)

Again, since S, T lie on the circle on OR as diameter, $ST = OR \sin S\hat{O}T = OR \sin \alpha$ (whether $S\hat{O}T = \alpha$ or $\pi - \alpha$) (2)

From (1) and (2), $\frac{RS}{RP} = \frac{RT}{OP} = \frac{ST}{OR}$ which proves that the angle SRT is equal to the angle RPO which is the supplement of the angle POQ.

The theorem is therefore proved.

Cor. R lies in the acute angle or the obtuse angle between the given lines according as

$$ST^2 \lessgtr SR^2 + RT^2.$$

In particular, if the given lines be $x \cos \alpha_i + y \sin \alpha_i = p_i \ (i = 1, 2)$ and R (0, 0), then S is $(p_1 \cos \alpha_1, p_1 \sin \alpha_1)$ and T is $(p_2 \cos \alpha_2, p_2 \sin \alpha_2)$ and the origin will be in the acute or obtuse angle between the lines according as $(p_1 \cos \alpha_1 - p_2 \cos \alpha_2)^2 + (p_1 \sin \alpha_1 - p_2 \sin \alpha_2)^2 \gtrless p_1^2 + p_2^2$

$$i.e., \ p_1, p_2 \cos (\alpha_1, -\alpha_2) \lessgtr 0.$$

The corresponding three-dimensional result is an immediate deduction from the above if one considers the traces of the given planes on the plane through the given point perpendicular to them.

N.B.—No figure is necessary to follow the above proof. Everything follows logically from the axioms and known theorems. My axiom clarifies the location of an interior point of an angle and states in an idealised form a perceptual fact.

A. A. KRISHNASWAMI AYYANGAR.
Andhra University,
Waltair,
December 7, 1948.

1. *Curr. Sci.*, 1948, **17**, 233. 2. David Hilbert. (Translated by Townsend), *Foundations of Geometry*, 2nd Edition, 1910, 14.

NUCLEIC ACID ANTAGONISM OF PENICILLIN BACTERIOSTASIS

IN a 'careful re-investigation' of our experiments[1] on the antagonistic effect of nucleic acid on the bacteriostatic action of penicillin, Ganapathi *et al.*,[2] report that they have failed to repeat our results and offer an explanation for the observations we have made. If the contention is that nucleic acid had no effect on the bactericidal action of penicillin as the title of their report would imply and as repeated in the text (para 4) we have nothing to state except to add that it is well known that organisms which have undergone bactericide by penicillin or any other drug cannot be revived and restored to life by any known process, whatsoever, since the organisms would have lost their lives under the influence of the drug. However, in bacterial cultures kept in the presence of bacteriostatic concentrations of penicillin whose action is reputed to be essentially bacteriostatic, under the conditions, instantaneous death of the organisms never result but only a temporary cessation of respiratory and reproductive processes resulting in inhibition of growth and multiplication. An agent which can sidetract, annihilate or by some means bring down the effective concentration of penicillin when added to the culture, will stimulate almost a sudden restoration of viability and consequently growth is imperative. Our experiments relate only to this latter condition and we repeat our claim. Under the influence of rather high concentrations of penicillin as for example two to sixteen units per ml. not only the organisms suffer death but also undergo lysis. A lack of appreciation of the bacteriostatic and bactericidal effects in relation to drug concentration has brought all the difference and in mode of action studies of drugs, one has very little use of dead organisms. There is no mention in the note about the actual value of the minimum bacteriostatic concentration of penicillin and remembering that the inoculum is comparatively small one can see that even 0·05 unit per ml. is above the bacteriostatic range, as otherwise, there is absolutely no reason why growth did not occur in Ganapathi, *et al.*'s cultures.

In Benedict, *et al.*'s paper[3] quoted by Ganapathi, *et al.* on page 94 is stated 'these data show the rapid increase in stability of penicillin as the hydrogen- ion concentration decreases (pH 2·0 to 6·0) with subsequent increase in decomposition as the concentration of the hydroxyl ions becomes greater, and this fact is illustrated on page 93 of the same paper. This means that penicillin ought to be stable in cultures which reach pH 5·2 to 5·5 at 37° C. But Ganapathi, *et al.* state that at this pH and when incubated at 37° C. small concentrations of penicillin are fairly easily destroyed. We would like to ask (*i*) if penicillin is not destroyed at pH 5·2 to 5·5 as the observations of Benedict, *et al.* claim is it not antagonism of penicillin action which added nucleic acid exerts with subsequent growth in our cultures and (*ii*) if penicillin is destroyed as Ganapathi *et al.*, claim, why they failed to see growth in the presence of nucleic acid, at least visual